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Determination of similarity criteria in optimization tasks by means of neuro-fuzzy modelling

***Abstract**. A method is proposed for solving optimization problems with high complexity when searching for the function minimum by using methods and means of similarity theory and neuro-fuzzy modelling. The problem with nonlinear objective function and constraints is transformed into a task with a nonlinear objective function and linear constraints. In this task, the basic similarity criteria are presented in the form of membership functions. Dependent similarity criteria are defined through the base with the use of standard computational procedures.*

***Streszczenie.** Zaproponowano metodę do rozwiązywania problemów optymalizacyjnych o wysokiej złożoności przy poszukiwaniu minimum funkcji za pomocą metod i środków teorii podobieństwa i modelowania -rozmytego. zadanie z nieliniowymi funkcją celu i ograniczeniami jest przekształcana na zadanie z nieliniową funkcją celu i liniowymi ograniczeniami. W zadaniu tym podstawowe kryteria podobieństwa są przedstawiane w postaci funkcji przynależności. Zależne kryteria podobieństwa zdefiniowane są przez podstawę, przy użyciu standardowych procedur obliczeniowych.. (**Określenie kryteriów podobieństwa w zadaniach optymalizacyjnych za pomocą modelowania rozmytego**).*

Keywords: similarity theory, neuro-fuzzy modelling, basic similarity criteria, membership functions, uncertainty.
Słowa kluczowe: teoria podobieństwa, modelowanie rozmyte, podstawowe kryteria podobieństwa, funkcje przynależności, niepewność.

Introduction

Means of similarity theory, in particular the criterion method (CM), can effectively solve and analyze optimization problems [1, 2]. Criteria-based method can be defined as a set of techniques and principles, according to which the analysis, comparison and interpretation of baseline data to provide scientific and practical conclusions. The ultimate goal of studies using the criterial method is to reveal regularities, which under certain conditions can be represented as the law of optimal control [2]. The main purpose of KM is to find the optimal variant of the process or the object. Most often, in the further analysis, the optimal parameters are used as reference. According to his idea, KM is close to the geometrical programming [3, 4]. This use of duality of optimization problems is the replacement of the direct problem for the corresponding dual. The main difference is that the basis for geometric programming is inequality between geometric and arithmetic averages, and the background of the CM – matrix properties of dimensions or indicators [1]. This is the meaning of dual variables. In geometric programming are weighting factors, in KM – similarity criteria. That is, the result of solving the tasks of the KM are optimal values of criteria of similarity or, in other words, the optimum ratio of the individual parameters, and not they themselves. This specific feature characteristic only of KM, determines its scope.

Problem statement

The direct problem is formulated like this: To minimise:

$$(1)\quad y(x)=\sum_{i=1}^{m_1} a_i \cdot \prod_{j=1}^{n} x_j^{\alpha_{ji}}$$

given

$$(2)\quad q_k(x)=\sum_{i=m_k+1}^{m_{k+1}} a_i \prod_{j=1}^{n} x_j^{\alpha_{ji}} \le G_k ,$$

$$k=\overline{1,p} \quad x_j \rangle 0,\ j=\overline{1,n}$$

where $y(x)$ is some generalised optimum criterion; and a_i, α_{ji} are constant factors, x_j denote the variable parameters, m_1 - amount of criterion function components (1) and m_{k+1} - amount of compound restrictions for direct problem function of CP (1) - (2);

The corresponding dual tasks can be formulated like this [4, 5]: To maximise (3)

$$(3)\quad d(\pi_o)=\prod_{i=1}^{m}\left(\frac{a_i}{\pi_{io}}\right)^{\pi_{io}} \prod_{k=1}^{p}\left(\frac{\lambda_k}{G_k}\right)^{\lambda_k} ,$$

given orthogonally (4)

$$(4)\quad \sum_{i=1}^{m} \alpha_{ti}\pi_i = 0 ,\ s=\overline{1,n}$$

and normalization (5)

$$(5)\quad \sum_{i=1}^{m_1} \pi_i = 1$$

where π_i is the similarity criterion and

$$\lambda_k = \sum_{i=m_k+1}^{m_{k+1}} \pi_{io}$$

denote LaGrange (correlation) coefficients.

In criterion programming the orthogonality and normalization system of equations can be written as:

$$(6)\quad \alpha \cdot \pi = b ,$$

where α is the matrix of indicators

$$\alpha = \begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \dots & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \dots & \alpha_{2m} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{n1} & \alpha_{n2} & \alpha_{n3} & \dots & \alpha_{nm} \\ 1 & 1 & 1 & \dots & 1 \end{vmatrix}$$

and π is the vector of similarity criteria

$$\pi = \begin{vmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ ... \\ \pi_m \end{vmatrix}; \; b = \begin{vmatrix} 0 \\ 0 \\ 0 \\ ... \\ 1 \end{vmatrix}$$

When α is a square matrix, and it can be only when the total amount of criterion function members and restrictions per unit is more than the amount of variables the system of equations (6) easily solved by any known method. In any other cases the system of equations is not defined or has a lot of solutions.

In all other cases, for example when α – rectangular matrix, the system of equations not defined or has lot solutions. In CM, the value s=m–n–1 is called the degree of complexity of the task. For such problems, orthonormal systems of equations (6) is written in the form:

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} & ... & \alpha_{1m_1} & \alpha_{1m_1+1} & \alpha_{1m_1+2} & ... & \alpha_{1m} \\ \alpha_{21} & \alpha_{22} & ... & \alpha_{2m_1} & \alpha_{2m_1+1} & \alpha_{2m_1+2} & ... & \alpha_{2m} \\ ... & ... & ... & ... & \alpha_{3m_1+1} & \alpha_{3m_1+2} & ... & \alpha_{3m} \\ \alpha_{n1} & \alpha_{n2} & ... & \alpha_{nm_1} & \alpha_{nm_1+1} & \alpha_{nm_1+2} & ... & \alpha_{nm} \\ 1 & 1 & ... & 1 & 0 & 0 & ... & 0 \end{vmatrix} \times \cdot$$

$$\times \begin{vmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ ... \\ \pi_m \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ ... \\ 1 \end{vmatrix}$$

where $m_1+1;\; m_1+2$ etc. – the indices of the members of the system of equations, which correspond to the members of the constraint (2).

Analysis of existing methods for challenging problems solution

In [1] it is shown that similarity criteria in the system of normalized orthogonally equations (4) - (5), are defined as:

$$\pi_i = \beta_{i0} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \tag{7}$$

where β_{i0} is the normalisation vector, β_{ij} the nullity vector, π_j is the basic criteria of similarity and $s=m\text{-}n\text{-}1$ the degree of complexity of a criteria programming problem.

If we express similarity criteria through basic criteria and normalisation and nullity vectors dual function will be written in such a way (8). Here T_k - set which consists of indexes of members of k- restriction.

As we consider problems where function $d(\pi)$ is convex then it is always possible to replace definition $d(\pi)_{\max}$ by definition of a stationary function point because sets of maximizing points of these functions coincide [1]. Let's find algorithm of expression (8) in (9):

Take derivative of function (9) by basic criteria of similarity π_j (10). Having set it equal to zero and having found antilogarithm of it, we will receive a system of equations from which it is possible to receive maximum function conditions $d(\pi_j)$. They will be the following (11) to (12).

The given approach offered in [1], can be applied only when t is small. When t is about 10 its use is in doubt as it is necessary to solve systems of the nonlinear equations of a high order.

$$d(\pi_j) = \prod_{i=1}^{m} \left(\frac{a_i}{\beta_{i0} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j} \right)^{\beta_{i0} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j} \times \tag{8}$$
$$\times \prod_{k=1}^{p} \left(\sum_{i \in T_k} \left(\beta_{i0} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) \right)^{\sum\limits_{i \in T_k} \left(\beta_{i0} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j \right)}$$

$$\ln d(\pi_j) = \ln \prod_{i=1}^{m} a_i^{\beta_{io}} + \sum_{i=1}^{s} \pi_j \ln \prod_{i=1}^{m} a_i^{\beta_{ij}} - \tag{9}$$
$$- \sum_{i=1}^{m} \left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) \cdot \ln \left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) +$$
$$+ \sum_{k=1}^{p} \sum_{i \in T_k} \left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) \cdot \ln \sum_{i \in T_k} \left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right)$$

$$\left(\ln d(\pi_j)\right)' = \ln \prod_{i=1}^{m} a_i^{\beta_{ij}} - \sum_{i=1}^{m} \beta_{ij} \cdot \ln\left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) -$$
$$- \sum_{i=1}^{m} \left(\frac{\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j}{\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j} \right) \cdot \beta_{ij} + \tag{10}$$
$$+ \sum_{k=1}^{P} \sum_{i \in T_k} \beta_{ik} \cdot \ln \sum_{i \in T_k} \left(\beta_{io} + \sum_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) +$$
$$+ \sum_{k=1}^{P} \left(\frac{\sum\limits_{i \in T_k} \beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j}{\sum\limits_{i \in T_k} \beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j} \right) \cdot \beta_{ik}.$$

$$d(\pi_j) = \frac{\prod\limits_{i=1}^{m} \left(\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j \right)^{\beta_{ij}}}{\prod\limits_{i=1}^{m} a_i^{\beta_{ij}}} \times \qquad j = \overline{1,s}, \tag{11}$$
$$\times \frac{1}{\prod\limits_{k=1}^{P} \left(\sum\limits_{i \in T_k} \left(\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) \right)^{\sum\limits_{i \in T_k} \beta_{ik}}} = 1$$

or

$$\frac{\prod\limits_{i=1}^{m} \left(\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j \right)^{\beta_{ij}}}{\prod\limits_{k=1}^{P} \left(\sum\limits_{i \in T_k} \left(\beta_{io} + \sum\limits_{j=1}^{s} \beta_{ij} \cdot \pi_j \right) \right)^{\sum\limits_{i \in T_k} \beta_{ik}}} = \prod_{i=1}^{m} a_i^{\beta_{ij}}, \quad j = \overline{1,s}, \tag{12}$$

Determination of optimal criteria for similarity method of neuro-fuzzy modelling

Membership function, are similar to similarity criteria π which are a dimensionless system parameters correspondence. And in that case when they are defined by a method of integrated analogues, they also are weight coefficients of criterion function components (rated to unit) [5, 6].

Membership function and similarity criterion change from 0 to 1. Hence it is possible to draw analogy between membership function and similarity criterion [5, 7].

Similarity of membership function and similarity criterion allows to use membership function instead of similarity criterion defining similarity criterion optimising vector in challenging problems.

We offer to replace basic similarity criteria by membership functions

$$\pi_i = \beta_{i0} + \sum_{j=1}^{s} \beta_{ij} \cdot \mu_j \tag{13}$$

where β_{i0} is the normalisation vector, β_{ij} the nullity vector, μ_j are the membership functions for basic similarity criteria and $s=m\text{-}n\text{-}1$ the degree of complexity of a criteria programming problem.

If we express similarity criteria through membership functions for basic similarity criteria and normalisation and nullity vectors dual function will be written in the following way (15).

$$d(\mu_j) = \prod_{i=1}^{m} \left(\frac{a_i}{\beta_{i0} + \sum_{j=1}^{t} \beta_{ij} \cdot \mu_j} \right)^{\beta_{i0} + \sum_{j=1}^{t} \beta_{ij} \cdot \mu_j} \times \tag{14}$$

$$\times \prod_{k=1}^{p} \left(\sum_{i \in T_k} \left(\beta_{i0} + \sum_{j=1}^{t} \beta_{ij} \cdot \mu_j \right) \right)^{\sum_{i \in T_k} \left(\beta_{i0} + \sum_{j=1}^{t} \beta_{ij} \cdot \mu_j \right)}$$

Defining the membership function form the alternative approach was used. The best results were received using π function: m (x) = мS (x) мZ (x), where мS (x) - S-function, мZ (x) - Z-function. This function in Matlab has a name pimf, order of its parameters: [and b c d], where [a d] is the carrier of fuzzy set; [b c] - core of fuzzy set; [and b] - smf-function parameters smf, [c d] - zmf-function parameters zmf.

Let membership function maximum for similarity criterion optimum meaning is defined as:

$$\mu^{\pi_j}(\pi_j) = \frac{a_j}{\sum_{i=1}^{m1} a_i} \tag{15}$$

Using formula (14) it is possible to calculate approximate optimum meaning of optimum criterion.

For improved similarity criterion calculation:

1. Let's construct membership function with a core calculated using (14).
2. Then normalise fuzzy set, dividing all membership functions into maximum value of membership function.
3. Fuzzy set $\tilde{\pi}_j$ we transform into a set of α-level to reduce area of possible values.
4. Specified value μ^{π_j} is found using methods of linear programming in an interval between transition points.

The offered approach has the following advantages:

1. It does not depend on the degree of complexity of a problem.
2. It is not necessary to solve equations of (10)–(12).
3. Considering specific character of μ^{π_j} it is possible to represent $\tilde{\pi}_j$ with the help of linguistic variables, that is to involve experts.

If the algorithm is used to solve a specific technical problem with the presence of the sample of retrospective data, to find the basic criteria you can use a simplified approach, which is shown in Fig. 1.

The first algorithm is used when there is a fuzzy set of values of the basic criteria of similarity.

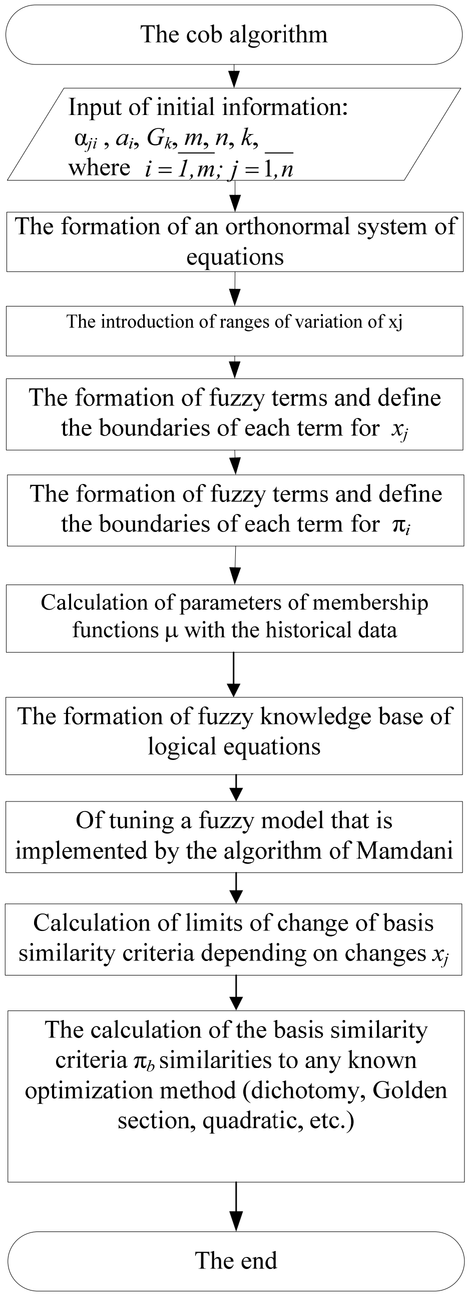

Fig.1. Block diagram of algorithm for determining the basic similarity criteria

The advantages of this algorithm include the possibility of incorporating the experience of experts and retrospective data.

The drawbacks include the need to clarify the solution methods of the dichotomy of the Golden section, and so on.

In Fig. 2 shows a generalized block diagram of the algorithm for solving optimization problems of large complexity when searching for the minimum of function using the methods and means of similarity theory and neuro-fuzzy modeling.

In Fig. 3 is a block diagram of the algorithm that implements the procedure for determining optimum values of membership functions, which after the substitution in (13) allow to determine the optimal values of criteria of similarity.

The advantages of the third algorithm include the ability to use existing software tools to automate the process of determining the parameters of the membership function]. The disadvantage is the increase of calculation errors while reducing the sample size of the original data.

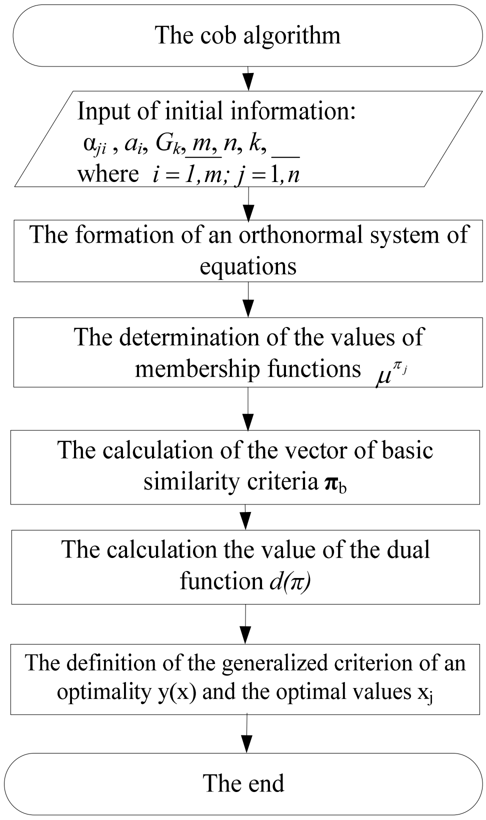

Fig. 2. Generalized block diagram of the algorithm for solving optimization tasks of large complexity

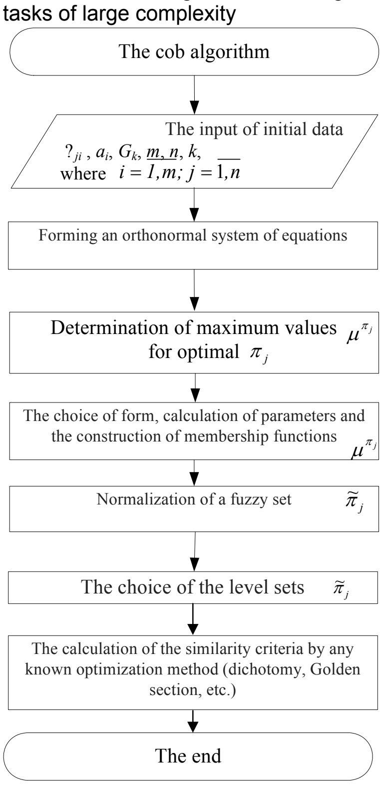

Fig. 3. Block diagram of algorithm for determining optimum values of membership functions

Conclusions

The proposed approach of determining similarity criteria in optimization problems with application of neuro-fuzzy modeling allows to widen the criteria-based method, relatively large-scale problems.

The algorithm of definition of criteria of similarity in this case can be constructed using standard computational procedures. To do this task with a nonlinear objective function and constraints is transformed into a task with a nonlinear objective function and linear constraints. In this task, the basic similarity criteria are presented in the form of membership functions. In turn dependent similarity criteria are defined through the base.

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