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A new approach to analysing non-linear electrical systems

Abstract. *The Pareto principle has been applied in economic sciences with particular emphasis on the problems associated with optimal decision making. In this paper the Pareto principle is modified in order to study the dynamics of electrical systems described by a nonlinear equation of state. A multi-parameter system with the parameters constituting the elements of the analyzed system has been defined.*

Streszczenie. *Zasada Pareto znalazła zastosowanie głównie w dziedzinie nauk ekonomicznych, ze szczególnym uwzględnieniem problemów związanych z optymalizacją podejmowania decyzji. W tym artykule zasada Pareto została zmodyfikowana w celu zbadania dynamiki układów elektrycznych opisanych przez nieliniowe równania stanu. Zdefiniowano wielo-parametryczny układ elementów analizowanego układu elektrycznego.(**Nowe podejście do analizy nieliniowych układów elektrycznych**)*

Keywords: Pareto principle, non-linear state equation, multi-parameters system, ABC charts, system dynamics.
Słowa kluczowe: Zasada PARETO, nieliniowe równanie stanu, system multi-parametryczny, wykresy ABC, dynamika systemu.

Introduction

The "Pareto principle" (1887) has been used in economics, product quality management as well as decision problems solving [1,2,3,10,16]. The creator of the "Pareto principle", Vilfredo Pareto was a professor at the University of Lausanne who, while studying the distribution of incomes in Italy, found out that 80% of the assets were in the hands of 20% of the entire population. This finding, also called the 80-20 rule, has shown validity, for instance, in the following statements: 80% of a company's sales come from 20%
of its products, 80% of all decisions is determined by 20% of available information, or 80% of shortcomings result from 20% of causes. Further research confirmed the existence of the 80/20 rule in many areas of life and was transferred to a number of technical applications [11-15]. Making use of the "Pareto Principle", it is possible to create ABC charts in order to define parameter classes that exert decisive, medium, and marginal impact on the objective function to describe specific research results, for example to determine the main causes of car accidents. Moreover, the principle was used for the optimization of a permanent-magnet alternator in terms of iron and copper losses, material costs, rated voltage and air-gap induction [11]. In paper [12] an evolutionary-based approach employing the Pareto front was presented in order to minimize the fuel cost and to improve voltage profile as well as enhance voltage stability.

In their papers [13,14] the authors used a mathematical model in the form of a heat conduction equation in solids solved by finite element method to determine the parameters that exert decisive influence on the temperature of high-voltage cable cores located at different depths in the ground. Using both the "Pareto principle" and ABC-analysis, the authors showed how cable core temperature is affected by such parameters as air temperature above the ground T_a, thermal conductivity of the ground λ and current load of the system I. The authors also determined the elements of class *A* having a decisive influence on the temperature field in the cable core and analyzed changes of the elements with respect to the cable cores arranged at different depths in the ground.

One of the primary uses of the principle based on the research of ABC-analysis is the study of inventory items that play a central role in the production or sale of a specific product. The principle is also used in inventory classification of warehouse management systems. In the latter case complex mathematical models have been developed to facilitate decisions concerning stockpiling, monitoring and managing material resources of great industrial corporations. The models are constructed on the principles of linear and nonlinear programming or using optimization methods basing on the theory of genetic algorithms [2,4-9].

The purpose of this paper is not to optimize the systems described here by a linear or nonlinear state equation [18,19,22]. That issue will be dealt with in the future as a continuation of the present research. Specifically, this paper focuses on the results concerning the determination of the impact of individual parameters of the analyzed system on its dynamics defined by means of the objective functions with vector norms used as their constructing elements. The analyzed systems are defined as multi-parameter systems describing the obtained results by elements of linear algebra instead of the classical method of A,B and C classes and the Lorenz curve.

Initial work describing a successful application of this principle to models in the form of partial differential equations was presented in papers [13,14]. The results demonstrate great potential of the method use for the analysis of electrical circuits, which is the reason for undertaking the following study on the nonlinear state equation.

The modification of the Pareto principle for analysis of the nonlinear state equation involves the following aspects:
- application of the method to analyze multi-parameter electrical systems,
- generalized way of investigating the dynamics of electrical systems,
- introduction (and development) of elements of linear algebra to describe the method.

Basic definitions and the algorithm used in the method

The dynamics of nonlinear electrical systems is described by the state equation:

$$\dot{x} = f(x,u,t) \qquad x(0) = x_o \tag{1}$$

where $f(x,u,t)$ is a set of non-linear vector functions, $x(t) \in R^n$ and $u(t) \in R^m$ are state variables and input vectors respectively, x_0 represents the vector of initial conditions. The basic techniques of analyzing equation (1) are numerical methods [19], owing to which it is possible to accomplish the quantitative analysis of the model. They require, however, a lot of experience from the one performing the calculation.

On the other hand, the qualitative analysis of equation (1) is no less interesting as it defines the influence of individual factors from equation (1) on the dynamics of the modeled system. To this end, we analyze a nonlinear equation with three state variables $x_1(t)$, $x_2(t)$, $x_3(t)$ in order to study the impact of factors b_k (k = 1,…,9) on the dynamics of the system.

$$\begin{aligned} \dot{x}_1 &= b_1x_1 + b_2x_2 + b_3x_3x_2^2 \\ \dot{x}_2 &= b_4x_1 + b_5x_2 + b_6x_3 \\ \dot{x}_3 &= -b_7x_1 - b_8x_2 - b_9x_3 + E, \quad x(0) = 0 \end{aligned} \tag{2}$$

where:

$$\begin{aligned} &b_1 = 1.2, \qquad b_2 = 1.5, \qquad b_3 = 1.4, \\ &b_4 = 2.0, \qquad b_5 = 1.0, \qquad b_6 = 1.3, \\ &b_7 = 2.0, \qquad b_8 = 40.0, \qquad b_9 = 16.0. \end{aligned} \tag{3}$$

In this case E is an input function and its value is changed in order to perform a global analysis of the system's dynamics. We assume that in this case parameters b_k (k = 1,…,9) are also the rated coefficients. The system of equations (2) forms a multi-parameter system with parameters b_k (in *Def. 1* p_k). The following definition of the multi-parameter system was developed to carry out an original analysis of electrical systems described by a nonlinear state equation. The analysis employs the Pareto principle, which, as mentioned before, is used primarily in economic sciences. Introducing Pareto's 80/20 law in technical sciences with emphasis on electrical systems is an original development. It makes use of the following definitions and algorithm for the analysis of multi-parameter systems [15].

Def. 1. A multi-parameter system is defined as a physical system that can be described by a function dependent on a number of parameters

$$F_p = f(p_1,..,p_n) \tag{4a}$$

Function Fp depends on vector $\boldsymbol{P}$,

$$\boldsymbol{P} = [p_1,...,p_n] \tag{4b}$$

whose elements are parameters $p_1,...,p_n$,
where in the changes in the value of $p_1,...,p_n$, are limited,

$$m_k \le p_k \le n_k, \quad k = 1,2,\ldots,n \tag{4c}$$

The changes of parameter values result from the previously made assumptions, the technological process employed in the production of the system elements or changes in the physical environment in which the system operates, while m_k, n_k constitute the lower and upper limits of these changes.

The parameters $p_1,...,p_n$, on whose values the objective function remains dependent, are called *base parameters.*

Definition 1 applies to electrical systems described by partial differential equations, for example, cable systems laid at different depths in the ground [13,14] as well as electrical systems described by a nonlinear and linear state equation [18,19,22]. In the latter case, the parameters are formed by elements a_{ij} that belong to matrix $\boldsymbol{A}$ found in the linear state equation or they are the coefficients in the nonlinear equation.

Function F_p dependent on the value of parameters $p_1,...,p_n$ ($m_k \le p_k \le n_k$, k = 1,2,…,n) is defined as an objective function with its primary aim of analysis being the definition of A,B and C classes as having a decisive, medium and marginal impact on the function's value.

Def. 2. The range of base parameter changes affecting the value of the objective function resulting from the changing conditions of the physical environment in which the analyzed system operates, production technology system components, or the assumed changes in the values of the system are called *base change ranges*.

Def. 3. Parameter p_b determining further analysis of the impact of individual parameters $p_1,...,p_n$ on the function's value is called *basic parameter.*

Using the methodology for determining the ABC charts [13,14], the following are the basic formulas developed to analyze the impact of various system parameters on the value of the objective function:

$$F_{k,pw} = \frac{F_{k,p\,max} - F_{k,p\,min}}{F_{k,p\,max}} \tag{5}$$

where $F_{k,pw}$ is a relative objective function depending on parameter k (k = 1,2,…,n), $F_{k,pmax}$ - $F_{k,pmin}$ is the difference between the maximum and minimum value of the objective function F_p determined for parameters m_k and n_k (i.e. for the assumed range of base changes of parameter k).

$$F_s = \sum_k F_{k,pw} \quad (k = 1,\ldots,n) \tag{6}$$

wherein F_s is the sum of $F_{k,pw}$, whereas *n* is the number of parameters present in formula (3). Introducing an element of the cumulative value a_k (weight ratio) corresponding to parameter p_k, for each basic parameter p_b (b = 1,…,M)

$$a_k = \frac{F_{k,pw}}{\sum_k F_{k,pw}} \quad (k = 1,\ldots,n) \tag{7}$$

we obtain the relative total value of weighting factors S = 1.

$$S = \sum_k a_k = \sum_l a_l + R \tag{8}$$

The weight coefficients a_k determine the impact level of individual parameters on the relative value of the objective function $F_{k,pw}$. Some part a_k of the weight coefficients represent elements of A and B classes that exert a decisive and a medium impacts on the relative value of objective function $F_{k,pw}$, R denotes the elements of class C that have a marginal impact.

Elements of the cumulative value a_k form matrix $\boldsymbol{A} = [a_{ij}]$, (i =1,…,M; j = 1,…,n) whose number of rows depends on particular values of basic parameters p_b (b = 1,…,M) and the number of columns depends on n number of parameters $p_n = b_n$.

$$\begin{array}{cccccc} & b_1 & b_2 & \cdots & b_n & \\ A = & \left[\begin{matrix} a_{11} \\ a_{21} \\ \cdots \\ a_{M1} \end{matrix}\right. & \begin{matrix} a_{12} \\ a_{22} \\ \cdots \\ a_{M2} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} & \left.\begin{matrix} a_{1n} \\ a_{2n} \\ \cdots \\ a_{Mn} \end{matrix}\right] & \begin{matrix} p_{b1} = E_1 \\ p_{b2} = E_2 \\ \cdots \\ p_{bM} = E_M \end{matrix} \end{array} \tag{9}$$

wherein, for example element a_{11} of matrix $\boldsymbol{A}$ corresponds to the weight coefficient a_1 for input function E_1 ($E_1 = p_{b1}$).

Distinction must be made here between relative objective function F_{pw} and objective function F_p. The former represented by formula (5) is used to determine the weight coefficients found in (9) whereas the latter is used to calculate F_{pw} and may be represented, for example, by one of the vector norms. The procedure for the method is as follows [15]:

Step 1. Given equation (1), we can determine the following: vector $\boldsymbol{P}$ depending on parameters p_k (k = 1,…,n), objective function F_p with the constraints for parameters p_k, relative objective function F_{pw}, and basic parameter p_b.

Step 2. For maximum and minimum values p_k (k = 1,2,…,n), we resolve 2n times equation (1) describing the system [15]. We determine weight coefficients a_k and cumulative value S.

Step 3. We set matrix $\boldsymbol{A}$ in order to determine the content of $a_k = a_{ij}$ in cumulative value S for each individual value of basic parameter p_b.

Step 4. On the basis of the elements of matrix $\boldsymbol{A}$, we determine parameters p_k which define elements of A,B and C sets.

Step 5. We create charts to illustrate the arrangement of elements in sets A,B and C depending on the value of basic parameter p_b.

In order to illustrate the developed algorithm we present two examples of nonlinear systems described by equation (1). The examples illustrating the developed procedure show the normalized system described by the equation with dimensionless variables and dimensionless input function, and the electrical circuit which is characterized by a normalized objective function by reference of the integral of the absolute value x_k to k^{th} established value of the state variable.

Dimensionless nonlinear system

The first example illustrating the above algorithm involves analysis of equation (1) with parameters (2). The base changes of parameters $b_1,\ldots,b_9$ in relation to rated values equal ±10%. The basic parameter $p_b = E_b$ (b = 1,…,8) assumes the following values: 5, 10, 15, 20, 25, 30, 35, 40.

Relative objective function F_{pw} is defined as follows:

(10) $$F_{pw} = \frac{||x||_{1,max} - ||x||_{1,min}}{||x||_{1,max}}$$

(11) $$F_p = ||x||_1 = \int_0^{t_1} |x_1(t)|\,dt + \int_0^{t_2} |x_2(t)|\,dt + \int_0^{t_3} |x_3(t)|\,dt$$

where t_1, t_2, t_3 are the setting times of individual state variables, which are defined for t_l (l = 1,2,3) satisfying the condition of one percentage deviation from steady state. State variables x_l are dimensionless. In relation to function F_p, it is assumed that it is a monotonic function and there are no local extrema in the $[m_k, n_k]$ interval.

Therefore, in accordance with the algorithm described in subsection 2 above, the results of individual stages of the procedure are as follows:

Step 1. Vector $\boldsymbol{P}$ has the following components:

$$\boldsymbol{P} = [b_1,\ldots,b_9]$$

(12) $$F_{k,pw} = \frac{||x||_{1k,max} - ||x||_{1k,min}}{||x||_{1k,max}}$$

$$F_{k,p} = ||x||_1 = \int_0^{t_1} |x_1(t)|\,dt + \int_0^{t_2} |x_2(t)|\,dt + \int_0^{t_3} |x_3(t)|\,dt$$

$$0.9 \cdot b_k \le b_k \le 1.1 \cdot b_k, \quad k = 1,\ldots,9$$

Step 2. It is assumed that parameters b_k are subjected to change by ±10%, the cumulative value $S = \sum_k a_k$, and individual weighting coefficients are determined by solving $2n$ times equation (2). Exemplary weight coefficients a_k and their contribution to the cumulative value S for $E_7 = 35$ is shown below. The remaining R contains $a_k \le 0.05$. The number of the parameter is given in parentheses:

$E_7 = 35$

(13a) $$S = \sum_k a_k = 0.08(b_1) + 0.05(b_2) + 0.01(b_3) + 0.04(b_4) + 0.09(b_5) + 0.28(b_6) + 0.02(b_7) + 0.28(b_8) + 0.15(b_9) = 1.0$$

(13b) $$S = \sum_l a_l + R = 0.08(b_1) + 0.09(b_5) + 0.28(b_6) + 0.28(b_8) + 0.15(b_9) + R = 0.88 + R$$

Step 3. Matrix $\boldsymbol{A} = [a_{ij}]$ is as follows:

(14)
$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccccccccc} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 & b_8 & b_9 \end{array}$$
$$A = \begin{bmatrix} 0.14 & 0.09 & 0.07 & 0.01 & 0.05 & 0.22 & 0.02 & 0.20 & 0.20 \\ 0.13 & 0.08 & 0.09 & 0.02 & 0.02 & 0.20 & 0.07 & 0.17 & 0.22 \\ 0.11 & 0.08 & 0.06 & 0.01 & 0.02 & 0.25 & 0.03 & 0.22 & 0.22 \\ 0.22 & 0.21 & 0.19 & 0.09 & 0.07 & 0.07 & 0.06 & 0.05 & 0.04 \\ 0.10 & 0.10 & 0.08 & 0.06 & 0.03 & 0.22 & 0.01 & 0.20 & 0.21 \\ 0.09 & 0.09 & 0.04 & 0.03 & 0.07 & 0.24 & 0.02 & 0.24 & 0.18 \\ 0.08 & 0.05 & 0.01 & 0.04 & 0.09 & 0.28 & 0.02 & 0.28 & 0.15 \\ 0.17 & 0.03 & 0.005 & 0.06 & 0.17 & 0.19 & 0.005 & 0.19 & 0.18 \end{bmatrix} \begin{array}{l} E_1 = 5.0 \\ E_2 = 10 \\ E_3 = 15 \\ E_4 = 20 \\ E_5 = 25 \\ E_6 = 30 \\ E_7 = 35 \\ E_8 = 40 \end{array}$$

Step 4. For basic parameter $E_7 = 35$ in A,B and C we have the following items:

E_7 = 35

(15) $$A = \{b_6, b_8, b_9, b_5\},\ B = \{b_1, b_2\},\ C = \{b_3, b_4, b_7\}$$

Step 5. The curves defining the element arrangement in A,B and C classes depending on the value of the basic parameter are shown in Figure 2.

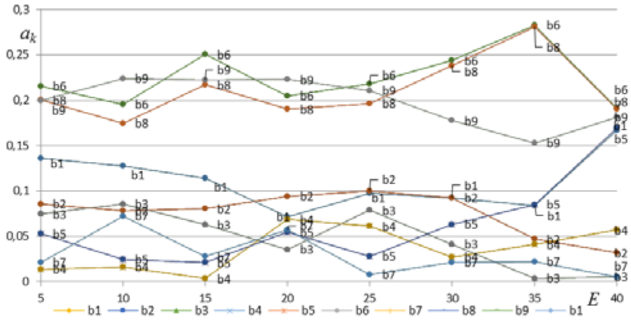


Fig. 2. Impact of the basic parameter E_b on a_k value for different b_k

By analyzing the curves shown in Figure 2, it must be concluded that the greatest impact on the system dynamics is exerted by parameters b_6, b_8, b_9 throughout the whole range of the basic parameter changes ($A = \{b_6, b_8, b_9\}$ for E from 5 to 40). The curves of parameters b_6, b_8, b_9 intersect at several points causing changes in the arrangement of class A elements that have the decisive influence on the dynamics of the system.

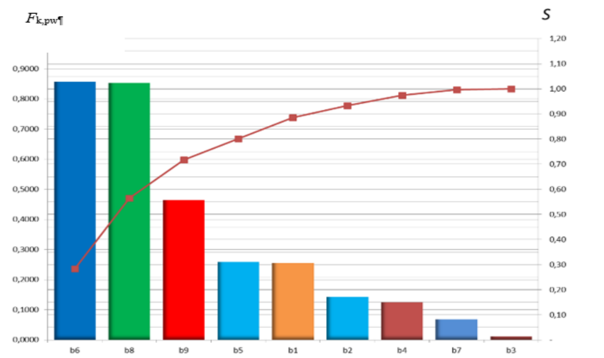


Fig. 3. ABC chart of the system described by a nonlinear equation of state (2) for E_7 = 35 [15]

For comparison's sake, a classical interpretation the Pareto principle based on the *ABC* charts and Lorenz curve is presented in Figure 3, (E_7 = 35). They can also be used to specify the weight coefficients of a_k, which are the percentage rates of cumulative value *S* listed on the right axis of the Pareto chart.

Figure 4 shows the norm curves $||x||_1$ for $E_7 = 35$ depending on the assumed changes of parameter b_6, b_8 as well as the changes of parameter b_3. The curves illustrate, respectively, decisive and marginal impacts of values b_6, b_8 and b_3 on norm $||x||_1$ which is in good agreement with the bar charts shown in Figure 4.

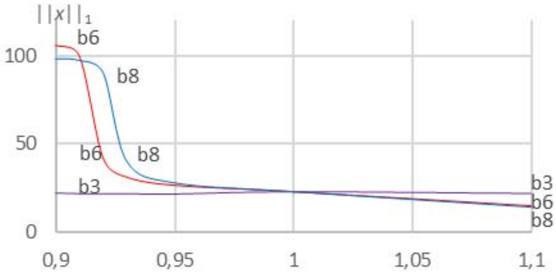
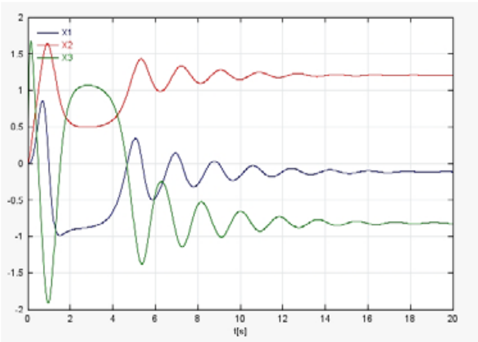


Fig. 4. Norm charts $||x||_1$ depending on parameters b_8,b_6 and b_3, $E_7 = 35$

a)



b)

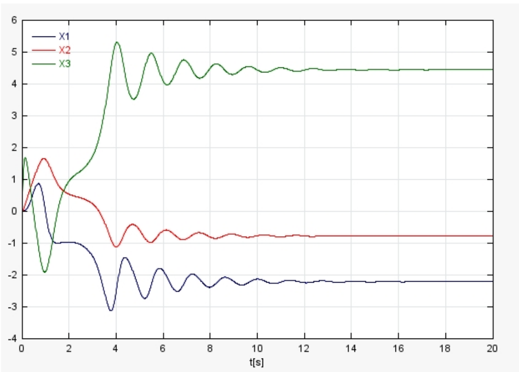


Fig. 5. Curves of state variables $x_l = x_l(t)$ for change of parameter value from b_6 1.19 into 1.18; a) $x_3 < 0$ and b) $x_3 > 0$ ($E_7 = 35$) [15]

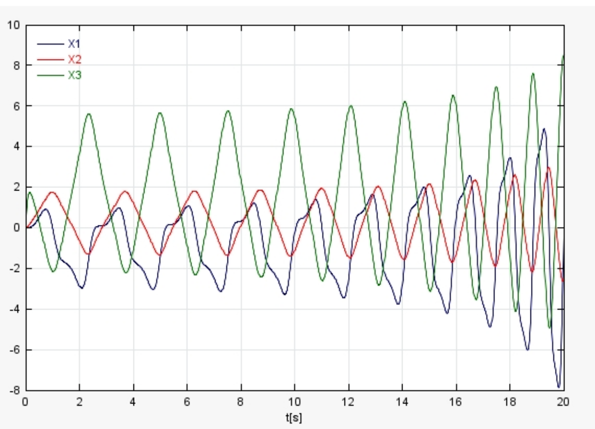


Fig. 6. Curves for state variables $x_l = x_l(t)$ for b_6 = 0.90 (E_7 = 35) [15]

The value of norm $||x||_1$ depends indirectly on $\sum_l t_l$ of setting times t_l of the state variables. Figure 5 shows the curves of state variables $x_l = x_l(t)$ (l = 1,2,3) for E_7 = 35. With practically a step change of $||x||_1$ in the range of b_6 changes from 1.18 to 1.19, it is possible to observe changes of the steady state of state variable x_3 from $x_3(t) < 0$ into $x_3(t) > 0$ (from negative to positive). It is also of interest to determine the state of the system at higher changes of parameter b_6. For $b_6 < 0.89$ the system becomes unstable in the *Lapunov* sense i.e. vector $\boldsymbol{x}$ does not return to the equilibrium point, as illustrated in Figure 6.

The electrical circuit with a nonlinear coil

Another example illustrating the algorithm is the analysis of an electric circuit with a nonlinear coil in which magnetic flux Φ is approximated by dependence $\Phi = b \cdot arctg(a \cdot i)$, where a and b are coefficients [21]. In this case,

$$u_L = z \cdot \frac{d\phi}{dt} = z \cdot \frac{d\phi}{di}\frac{di}{dt} = zba \cdot \left(\frac{1}{1+(ai)^2} \right) \cdot \frac{di}{dt}$$

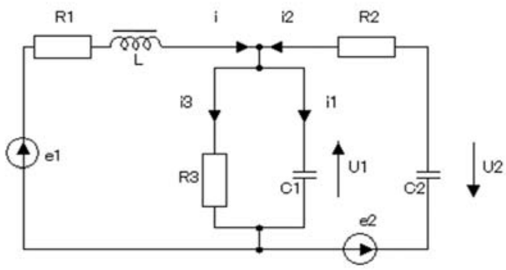


Fig. 7. The electrical circuit with a nonlinear coil [15]

The circuit is shown in Figure 7, which after transformations, can be described by the following set of equations [22]:

$$\begin{aligned} \dot{x}_1 &= -b_1 x_1 - b_2 x_1^3 - b_3 x_2 - b_4 x_1^2 x_2 + b_4 x_1^2 e_1 + b_3 e_1 \\ \dot{x}_2 &= b_5 x_1 - (b_6 + b_7) x_2 - b_6 x_3 + b_6 e_2 \\ \dot{x}_3 &= -b_8 x_2 - b_8 x_3 + b_8 e_2 \\ x_1(0) &= 0, \quad x_2(0) = 0, \quad x_3(0) = 0 \end{aligned} \tag{16}$$

where $x_1 = i\,(t)$, $x_2 = u_1(t)$, $x_3 = u_2(t)$

In this case:

$$b_1 = \frac{R_1}{zba} \quad b_2 = \frac{R_1 a}{zb} \quad b_3 = \frac{1}{zba}$$
$$b_4 = \frac{a}{zb} \quad b_5 = \frac{1}{C_1} \quad b_6 = \frac{1}{R_2 C_1} \tag{17}$$
$$b_7 = \frac{1}{R_3 C_1} \quad b_8 = \frac{1}{R_2 C_2}$$

with the values of the elements in the circuit:

R_1 = 10Ω $C_1 = 5 \cdot 10^{-3}$F e_1 = 200V ÷ 800V

(18) R_2 = 10Ω $C_2 = 5 \cdot 10^{-3}$F E_2 = 200V = *const*

a = 3.0A^{-1} b = 2.0Wb z = 500 (number of turns)

R_3 = 200Ω

The parameters used for analysis are assumed to be the rated parameters of the circuit. Parameter e_1 is the basic parameter set to range from 200V to 800V, E_2 = const.

In the analyzed example, in order to determine the impact of individual circuit elements on the value of the objective function $F_{k,p}$, the assumed p_k parameters are the ones given in (18) above.

The results obtained according to the procedure given in subsection 2 are presented below.

Step 1.
Vector $\boldsymbol{P}$ has the following components:

$$\boldsymbol{P} = [a,b,C_1,C_2,R_1,R_2,R_3,z]$$

(19) $$F_{k,pw} = \frac{||x||_{1k,max} - ||x||_{1k,min}}{||x||_{1k,max}}$$

$$F_{k,p} = ||x||_1 = \left(\int_0^{t_1} |x_1(t)|\,dt\right) / w_{1u} +$$

$$\left(\int_0^{t_2} |x_2(t)|\,dt\right) / w_{2u} + \left(\int_0^{t_3} |x_3(t)|\,dt\right) / w_{3u}$$

$$0.9 \cdot p_k \le p_k \le 1.1 \cdot p_k, \quad k = 1,2,\ldots,8$$

where w_{lu} $(l = 1,2,3)$ is a steady value determined for individual state variables and components $||x||_1$ are time-dependent on their steady state times t_l.

Step 2.
The circuit parameters are assumed to change ±10%. The cumulative value S is determined by formula (8). Individual weight coefficients a_k are calculated by solving $2n$ time's equation (16). Exemplary weight coefficients a_k and their share in the cumulated value S is shown below. The remaining R contains $a_k \le 0.05$. The parameter number is given in parentheses:

$e_{1.4}$ = 500V

$$S = \sum_k a_k = 0.17(a) + 0.20(b) + 0.07(C_1) + 0.09(C_2) +$$

(20a) $$0.01(R_1) + 0.01(R_2) + 0.25(R_3) + 0.20(z) = 1.0$$

$$S = \sum_l a_l + R = 0.17(a) + 0.20(b) + 0.079(C_1) + 0.09(C_2) +$$

(20b) $$0.25(R_3) + 0.20(z) + R = 0.98 + R$$

Step 3.
Knowing the weight coefficients a_k, it is possible to determine matrix $\boldsymbol{A}$:

(21) $$\begin{array}{cc} & \begin{array}{cccccccc} a & b & C_1 & C_2 & R_1 & R_2 & R_3 & z \end{array} \\ A = & \left[\begin{array}{cccccccc} 0.06 & 0.28 & 0.08 & 0.07 & 0.10 & 0.01 & 0.12 & 0.28 \\ 0.01 & 0.25 & 0.14 & 0.14 & 0.05 & 0.01 & 0.15 & 0.25 \\ 0.14 & 0.11 & 0.05 & 0.10 & 0.21 & 0.01 & 0.28 & 0.11 \\ 0.17 & 0.20 & 0.07 & 0.09 & 0.01 & 0.01 & 0.25 & 0.20 \\ 0.13 & 0.14 & 0.13 & 0.01 & 0.01 & 0.06 & 0.38 & 0.14 \\ 0.13 & 0.14 & 0.07 & 0.08 & 0.07 & 0.01 & 0.36 & 0.14 \\ 0.16 & 0.13 & 0.11 & 0.01 & 0.09 & 0.06 & 0.31 & 0.13 \end{array}\right] \begin{array}{l} e_{1,1} = 200V \\ e_{1,2} = 300V \\ e_{1,3} = 400V \\ e_{1,4} = 500V \\ e_{1,5} = 600V \\ e_{1,6} = 700V \\ e_{1,7} = 800V \end{array} \end{array}$$

Step 4.
For example, for basic parameter $e_{1,4}$ = 500V in A,B and C classes we have the following elements:

$e_{1,4}$ = 500V

(22) $$A = \{a,b,R_3,z\},\ B = \{C_1,C_2\},\ C = \{R_1,R_2\}$$

Step 5.
Using matrix $\boldsymbol{A}$, we produce curves depicting the rearrangement of elements A,B and C sets depending on the basic parameter values (Fig. 8).

Figure 9 shows the results for $e_{1,4}$ = 500V expressed by using the ABC charts and Lorenz curve.

Figure 10 shows exemplary norm charts $||x||_1$ for $e_{1,4}$ = 500V depending on the assumed parameter changes R_3,b,a and R_2. Analyzing the charts we can notice a significant impact of parameter R_3 and a marginal impact of R_2 on the value of objective function $F_p = ||x||_1$.

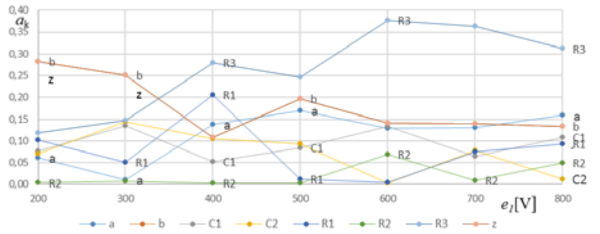


Fig. 8. Impact of the basic parameter e_1 on a_k value for different parameters given by (18)

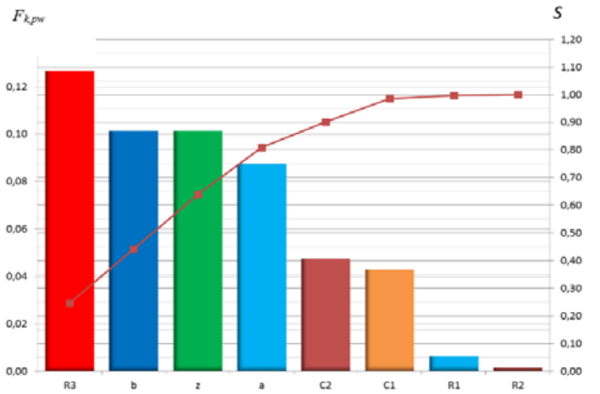


Fig. 9. The *ABC* chart and Lorenz curve of the nonlinear state equation (16) for $e_{1,4}$ = 500V [15]

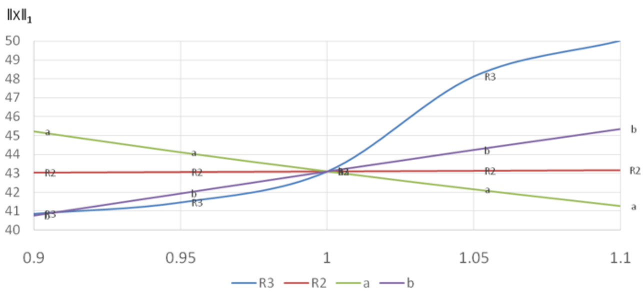


Fig. 10. Charts for norm $||x||_1$ dependent on parameters R_3,b *a* and parameter R_2 ($e_{1,4}$ = 500V)

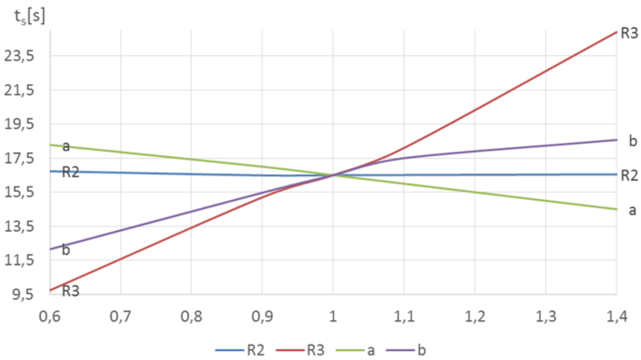


Fig. 11. Impact of parameter R_3 (basic) and R_2 (marginal) on the mean time t_s ($e_{1,4}$ = 500V) [15]

The value of norm $||x||_1$ depends on $\sum_l t_l$ of times t_l.

Figure11 illustrates the changes of mean time $t_s = \sum_l \frac{t_l}{3}$ for state variables x_l (l = 1,2,3) depending on the assumed changes of parameter R_3,b,a and R_2, $e_{1,4}$ = 500V.

Comments and Conclusions

The paper presents a modified Pareto principle for the analysis of nonlinear electrical systems described by a state equation. The result of the modification is a development of a multi-parameter system and original algorithmic procedures to test non - linear systems described by a nonlinear state equation. This leads to a generalized analysis of a nonlinear system's dynamics dependent on basic parameter p_b. The ABC bar charts and Lorenz curve have been replaced by a matrix model providing the same interpretation of the results.

The two systems described by a nonlinear equation of state are analyzed to determine ABC sets of parameters having decisive, medium and marginal impacts on the dynamics of the system as a result of a detailed analysis

of an example described by equation (2) it was found that:
for input function $E_7 = 35$, parameters b_6, b_8, b_9, b_5 have a maximum impact on the dynamics of the system wherein, for $a_6 \leq 0.89$, the system is unstable in the Lapunov sense, for the same input function the impact of parameter a_3 is marginal.

In the example described by equation (16) parameters R_3,a,b *and* z exert a decisive influence on the system. Figure 11 shows the impact of R_3 (basic) and R_2 (minimum) on the system's transient state.

The relative objective function F_{wp} used to determine A,B and C classes can be defined according to various criteria. We can define the function by one of the basic norms of the vector [19,22], whose components comprise the values

of state variables in time t_1, mean setting time, or time t_1 of a chosen state variable. The actual choice of objective function F_p depends on the chosen criterion, or in the further proceeding, the selected procedure of system optimization.

The parameters used in the analyzed examples of multi-parameter systems were rated parameters, and their deviations were ±10%. In the future, another interesting aspect of the study will be concerned with procedures for creating a generalized dependence of the objective function on the parameters of the system using the principles of linear programming [5,8,9].

The paper however, does not give a thorough stability analysis of the nonlinear system (in the sense of *Lapunov*). It is not concerned with optimization procedures of the analyzed systems, in which class A parameters having a decisive influence on the dynamics of the system are used. The issues will be investigated in future research. However, the approach shows its originality and possibility for studies of nonlinear circuits.

A detailed literature review presented in this paper and in [15] has shown the originality of the author's algorithm as well as an innovative application of the modified Pareto principle for the analysis of electrical circuits modeled by the non-linear state equation. Another important aspect of the paper that should be emphasized here is the introduction of linear algebra elements that, in future, will make it possible to develop a digital filter in order to precisely define the elements of ABC sets.

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