Mykhaylo ZAGIRNYAK, Mariia MALIAKOVA, Andrii KALINOV
Kremenchuk Mykhailo Ostrohradskyi National University

doi:10.15199/48.2017.12.49

An analytical method for calculation of passive filter parameters with the assuring of the set factor of the voltage supply total harmonic distortion

Abstract. *An analytical method for calculation and choice of passive filter parameters by the analytical expressions of the nonlinear load current harmonic components with the assuring of the set value of the factor of the supply network voltage total harmonic distortion is offered. It is proposed to obtain the current expressions with the use of the small parameter method realized in the frequency domain. The analytical calculation is performed with the use of the automated method for generation of the electrical values components on the basis of the discrete convolution method. The operability of the proposed method is demonstrated by the example of an electricity supply network section with a nonlinear load and a linear reactor.*

Streszczenie. *W artykule opisano analityczną metodę obliczania i doboru parametrów filtra pasywnego z wykorzystaniem wyrażeń analitycznych opisujących nieliniowe składowe harmoniczne przy zapewnieniu zadanej wartości współczynnika całkowitego zniekształcenia harmonicznych napięcia sieci zasilającej. Proponowane jest uzyskanie wyrażeń za pomocą metody małego parametru zrealizowanej w dziedzinie częstotliwości. Obliczenia analityczne przeprowadza się za pomocą zautomatyzowanej metody otrzymywania wartości wielkości elektrycznych w oparciu o metodę splotu dyskretnego. Możliwości proponowanej metody wykazano na przykładzie części sieci zasilającej z obciążeniem nieliniowym i reaktorem liniowym.* ***(Analityczna metoda obliczania parametrów filtra pasywnego przy zadanym współczynniku całkowitego zniekształcenia harmonicznych napięcia zasilania)***

Key words: *passive filter, algorithm, analytical method, frequency-domain*
Słowa kluczowe: pasywny filtr, algorytm, metoda analityczna, domena częstotliwości

Introduction

Decrease of the higher harmonics (HH) level in electrical networks is a part of the general problem of reduction of nonlinear load influence on the supply network and improvement of the quality of electrical energy in electricity consumption systems [1]. Nowadays the solution to this problem is based on the use of passive filter-compensating devices (FCD). Unlike power active filters [2-3], FCDs are the simplest and most economical filters, which caused their widespread use. FCDs are designed individually for every particular case of their application [4-7]. It guarantees the possibility for attaining the highest parameters of HH filtration.

The data necessary for FCD design: 1) rated voltage; 2) the required compensation for the reactive power at the reference frequency; 3) the values of the currents of nonlinear load harmonic components; 4) the network short circuit power; 5) the required parameters of the electrical energy quality at the nonlinear load power lines (or at another point of contiguity) [4-7].

It should be noted that the correct calculation of the passive filter parameters is impossible without a qualitative analysis of the currents harmonic composition which is generated by the nonlinear load [8-9]. Also, it should be mentioned that most methods for calculation of the passive filter parameters are numerical. An essential disadvantage of such calculation of the filter optimal parameters consists in the fact that it is not always possible to unambiguously determine the degree of reduction of voltage supply total harmonic distortion (THD_U).

Problem statement

Development of the method for calculation and choice of passive filtration devices that will enable determination of the passive filter parameters depending on the required factor of distortion of the THD_U.

Material and results of the research

As the problem of the analysis of electricity supply systems can be reduced to the calculation of electric circuits with lumped parameters, it is necessary to work out or develop the methods for the analysis and parameters of nonlinear electric circuits (NEC).

Paper [7] contains examples of obtaining the analytical expressions for the load currents harmonic components in the function of nonlinear load parameters and the parameters of the circuit linear elements with the use of the small parameter method (SPM). It allows creating the background for the calculation and choice of the devices for passive filtration of the supply network voltage and current higher harmonic components in such a way that the required value of THD_U be provided.

By way of demonstration of the proposed method for determination of the passive filter parameters let us consider the operation of a nonlinear electric circuit with an active-inductive load in fig. 1. In reality this type of nonlinearity may be caused by welding outfit or induction furnaces.

A Load currents calculation in analytical form

To make it possible to calculate the filter parameters in such a way that the required value of THD_U is provided, at first it is necessary to determine the currents flowing in the analyzed system using SPM in the frequency domain and representing the nonlinear dependence in the form of instantaneous conductivity.

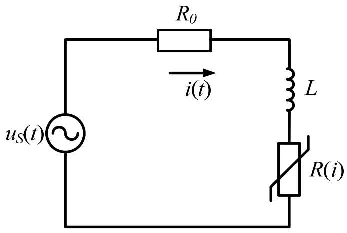

Fig. 1. Electric circuit with an active-inductive load

It was assumed for the analysis that supply voltage $u_s(t)$ of the researched circuit is set by the first harmonic of the cosine component: $u_s(t) = U_{a1} cos(\omega t)$. To simplify the mathematical expressions and to reduce the awkwardness of the calculation dependences we will take into consideration for the analysis only ponderable first and third harmonic components of the current. It should be mentioned that analytical generation of the electric values, required for the calculation, in the frequency domain was realized with the use of the automated method based on the discrete convolution algorithm [9].

The differential equation describing the operation of the analyzed circuit is of the form:

(1) $$u_s(t) = iR + L\frac{di(t)}{dt} + R(i)i(t).$$

Nonlinear resistance is described by dependence: $R(i) = R_0 + R_2 i^2$. The expression for the nonlinear conductivity of the analyzed circuit is written down:

(2) $$Y(I) = \frac{1}{R(I)} = \frac{1}{R_0 + R_2 I^2}.$$

According to the above mentioned method, nonlinear conductivity can be presented in the form of Tailor series: $Y(i) = \frac{1}{R_0} - \frac{R_2}{R_0^2}i^2$ or with the use of numerical approximation: $Y(i) = Y_0 + Y_2 i^2$. The voltage at the nonlinear element of the analyzed electric circuit is equal to the difference of the supply voltage and the voltage at the inductive element:

(3) $$u(t) = u_s(t) - u_L(t),$$

where $u_L(t) = L\frac{di(t)}{dt}$, L – inductance.

Let us represent the analytical expressions of the arrays of the cosine Y_{ak}, U_{an} and sine Y_{bk}, U_{bn} components of instantaneous conductivity and voltage in the frequency domain, respectively:

$$Y_{ak} = \begin{pmatrix} Y_0 + Y_2\left(2I_{b3}^2 + 2I_{a1}^2 + 2I_{b1}^2 + 2I_{a3}^2\right) \\ 0 \\ Y_2\left(I_{a1}^2 - I_{b1}^2 + 2I_{a1}I_{a3} + 2I_{b1}I_{b3}\right) \\ 0 \\ Y_2\left(2I_{a1}I_{a3} - 2I_{b1}I_{b3}\right) \\ 0 \\ Y_2\left(I_{a3}^2 - I_{b3}^2\right) \end{pmatrix};$$

$$Y_{bk} = \begin{pmatrix} 0 \\ 0 \\ Y_2\left(2I_{a1}I_{b1} + 2I_{a1}I_{b3} - 2I_{a3}I_{b1}\right) \\ 0 \\ Y_2\left(2I_{a3}I_{b1} + 2I_{a1}I_{b3}\right) \\ 0 \\ Y_2\left(2I_{a3}I_{b3}\right) \end{pmatrix},$$

$$U_{an} = \begin{pmatrix} 0 \\ U_{a1} - I_{b1}L\omega \\ 0 \\ -I_{b3}L\omega \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad U_{bn} = \begin{pmatrix} 0 \\ I_{a1}L\omega \\ 0 \\ I_{a3}L\omega \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where k, n – number of the harmonic of conductivity and voltage, respectively.

As a result of the operation of discrete convolution in the frequency domain [9], which is equivalent to the operation of multiplication of trigonometric series in the time domain of the above stated arrays, analytical expressions of the electric circuit current are obtained. According to SPM, current is represented in the form of a power series of the first power as to conductivity Y_2:

(4) $$I = a_0 + a_1 Y_2,$$

and corresponding coefficients a_0 and a_1 of the power series are determined. Thus, a_0 equals:

(5) $$a_{a0} = A_0;\ a_{b0} = B_0.$$

To determine the harmonic composition of coefficient a_1, a_0^3 is analyzed in the frequency domain. The performed analysis revealed that a_1 contains the third harmonic apart from the first one. So, the cosine and sine component:

(6) $$a_{a1} = \begin{pmatrix} 0 \\ A_{11}Y_2 \\ 0 \\ A_{13}Y_2 \end{pmatrix}; \quad a_{b1} = \begin{pmatrix} 0 \\ B_{11}Y_2 \\ 0 \\ B_{13}Y_2 \end{pmatrix}.$$

The arrays of the cosine and sine harmonic components of current are written down:

(7) $$I_{am} = \begin{pmatrix} 0 \\ A_0 + A_{11}Y_2 \\ 0 \\ A_{13}Y_2 \end{pmatrix}; \quad I_{bm} = \begin{pmatrix} 0 \\ -\left(B_0 + B_{11}Y_2\right) \\ 0 \\ -B_{13}Y_2 \end{pmatrix}.$$

where m – number of the load current harmonic.

To determine the coefficients the arrays of the cosine and sine orthogonal components of current, obtained by the convolution of the arrays of instantaneous conductivity and voltage, are compared with the arrays obtained as a result of representation of the desired current in the form of an instantaneous conductivity power series. Then coefficients in the right and left parts of the obtained equations are equated at equal powers of Y_2 and an equation system for determination of the sought coefficients is created.

(8) $$\begin{cases} A_0 = \dfrac{U_{a1}Y_0}{\left(1 + Y_0^2 L^2\omega^2\right)}; \\ B_0 = \dfrac{U_{a1}L\omega Y_0^2}{\left(1 + L^2\omega^2 Y_0^2\right)}; \\ A_{11} = -3U_{a1}^3 Y_0^2 \dfrac{L^2\omega^2 Y_0^2 - 1}{\left(1 + L^2\omega^2 Y_0^2\right)^3}; \\ B_{11} = 6Y_0^3 L\omega \dfrac{U_{a1}^3}{\left(1 + L^2\omega^2 Y_0^2\right)^3}; \\ A_{13} = U_{a1}^3 Y_0^2 \dfrac{-6L^2\omega^2 Y_0^2 + L^4\omega^4 Y_0^4 + 1}{\left(1 + L^2\omega^2 Y_0^2\right)^4}; \\ B_{13} = -4U_{a1}^3 Y_0^3 L\omega \dfrac{L^2\omega^2 Y_0^2 - 1}{\left(1 + L^2\omega^2 Y_0^2\right)^4}. \end{cases}$$

As the current is represented in the form of a power series of the first power (4), only coefficients at Y_2 zero and first powers are taken into account. The obtained system of algebraic equations is used to determine coefficients depending on the circuit parameters, A_0, B_0, A_{11}, B_{11}, A_{13}, B_{13} and analytical expressions for the orthogonal components of the current of the researched nonlinear circuit are generated:

(9) $$I_{am} = \begin{pmatrix} 0 \\ \dfrac{U_{a1}Y_0}{\left(1 + Y_0^2 L^2\omega^2\right)} - 3U_{a1}^3 Y_0^2 \dfrac{L^2\omega^2 Y_0^2 - 1}{\left(1 + L^2\omega^2 Y_0^2\right)^3} Y_2 \\ 0 \\ U_{a1}^3 Y_0^2 \dfrac{-6L^2\omega^2 Y_0^2 + L^4\omega^4 Y_0^4 + 1}{\left(1 + L^2\omega^2 Y_0^2\right)^4} Y_2 \end{pmatrix};$$

$$I_{bm}=\begin{pmatrix} 0 \\ \dfrac{U_{a1}L\omega Y_0^2}{\left(1+L^2\omega^2Y_0^2\right)}+6Y_0^3L\omega\dfrac{U_{a1}^3}{\left(1+L^2\omega^2Y_0^2\right)^3}Y_2 \\ 0 \\ -4U_{a1}^3Y_0^3L\omega\dfrac{L^2\omega^2Y_0^2-1}{\left(1+L^2\omega^2Y_0^2\right)^4}Y_2 \end{pmatrix}.$$

B Calculation and choice of the linear reactor parameters

Using the obtained analytical expressions of the load current components (9) of the researched circuit (fig. 1), let us analyze the realization of the algorithm of the calculation of the passive filter parameters by the example of an electricity supply network section with a nonlinear load and a linear reactor. The analyzed system is represented as a single-phase electric circuit with lumped parameters, undistorted voltage supply $e(t)$ and a reactor connected in series (fig. 2).

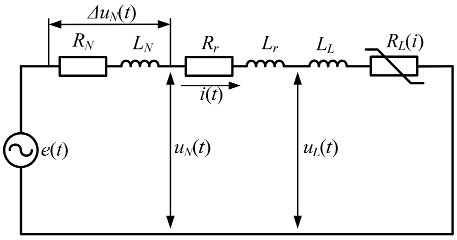

Fig. 2. Equivalent circuit of the electricity supply network section with a nonlinear load

It consists of the network resistive impedance R_N and inductance L_N the reactor resistive impedance and inductance L_r active nonlinear $R_L(i)$ and inductive linear L load. In most cases the value of the reactor resistive impedance is very small in comparison with the inductance. Thus, resistive impedance R_r can be neglected during the calculation. The following parameters were taken for the calculation: supply network resistive impedance $R_N=2$ Ohm, network inductance – $L_N=0{,}007$ Hn, nonlinear load resistance $R_L=3$ Ohm, the sinusoidal supply voltage was represented by a cosine component $U_{a1}=220\sqrt{2}$ V.

The algorithm of the calculation and choice of the linear reactor parameters is given in fig. 3. In accordance with the algorithm and using SPM, the analytical expressions for the weighty cosine and sine components of the first and third current harmonics of the analyzed circuit in the frequency domain in the function of reactor inductance L_r were determined (10).

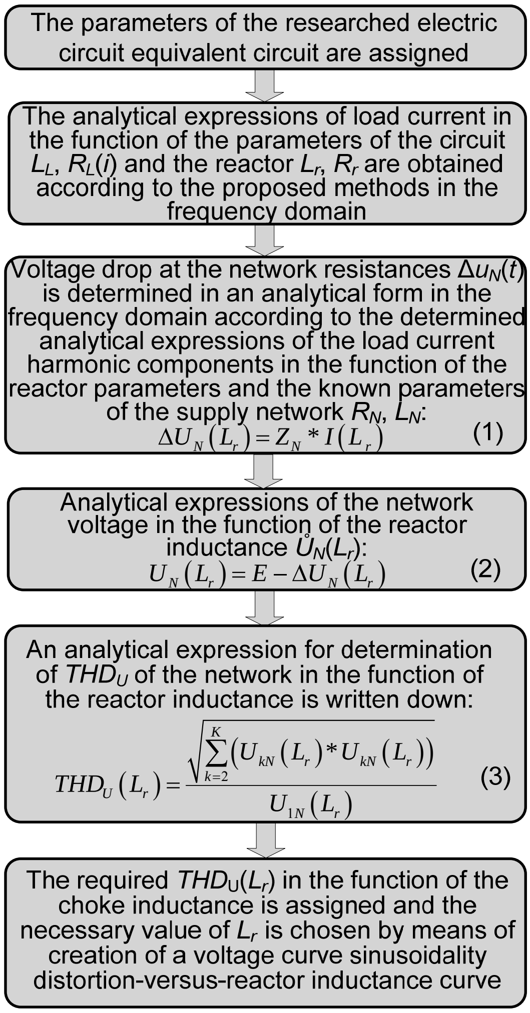

Fig. 3. Algorithm of the calculation and choice of the linear reactor parameters by the set factor of voltage supply total harmonic distortion

$$I(L_r)_{am}=\begin{pmatrix} 0 \\ \dfrac{U_{a1}Y_0}{\left(1+Y_0^2\left(L_R+L_N\right)^2\omega^2\right)}-3U_{a1}^3Y_0^2\dfrac{\left(L_R+L_N\right)^2\omega^2Y_0^2-1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^3}Y_2 \\ 0 \\ U_{a1}^3Y_0^2\dfrac{-6\left(L_R+L_N\right)^2\omega^2Y_0^2+\left(L_R+L_N\right)^4\omega^4Y_0^4+1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^4}Y_2 \end{pmatrix};$$

(10)

$$I(L_r)_{bm}=\begin{pmatrix} 0 \\ \dfrac{U_{a1}\left(L_R+L_N\right)\omega Y_0^2}{\left(1+L^2\omega^2Y_0^2\right)}+6Y_0^3\left(L_R+L_N\right)\omega\dfrac{U_{a1}^3}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^3}Y_2 \\ 0 \\ -4U_{a1}^3Y_0^3\left(L_R+L_N\right)\omega\dfrac{\left(L_R+L_N\right)^2\omega^2Y_0^2-1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^4}Y_2 \end{pmatrix}.$$

Using expression (1) in algorithm (fig. 3) voltage drop $\Delta U_N\left(L_r\right)$ in the frequency domain was determined:

$$\Delta U\left(L_r\right)_{an}=\begin{pmatrix} 0 \\ \dfrac{U_{a1}Y_0}{\left(1+Y_0^2\left(L_R+L_N\right)^2\omega^2\right)}-3U_{a1}^3Y_0^2\dfrac{\left(L_R+L_N\right)^2\omega^2Y_0^2-1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^3}Y_2 \\ 0 \\ U_{a1}^3Y_0^2\dfrac{-6\left(L_R+L_N\right)^2\omega^2Y_0^2+\left(L_R+L_N\right)^4\omega^4Y_0^4+1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^4}Y_2 \end{pmatrix}\begin{pmatrix} 0 \\ \sqrt{R_N^2+\left(\omega L_N\right)^2} \\ 0 \\ \sqrt{R_N^2+\left(3\omega L_N\right)^2} \end{pmatrix};$$

(11)

$$\Delta U\left(L_r\right)_{bn}=\begin{pmatrix} 0 \\ \dfrac{U_{a1}\left(L_R+L_N\right)\omega Y_0^2}{\left(1+L^2\omega^2Y_0^2\right)}+6Y_0^3\left(L_R+L_N\right)\omega\dfrac{U_{a1}^3}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^3}Y_2 \\ 0 \\ -4U_{a1}^3Y_0^3\left(L_R+L_N\right)\omega\dfrac{\left(L_R+L_N\right)^2\omega^2Y_0^2-1}{\left(1+\left(L_R+L_N\right)^2\omega^2Y_0^2\right)^4}Y_2 \end{pmatrix}\begin{pmatrix} 0 \\ \sqrt{R_N^2+\left(\omega L_N\right)^2} \\ 0 \\ \sqrt{R_N^2+\left(3\omega L_N\right)^2} \end{pmatrix}.$$

According to expression (2) in fig.3 and (11) the supply network voltage $U_N\left(L_r\right)$ in the frequency domain was determined:

(12)
$$U_N\left(L_r\right)_{an}=U_{a1}-\Delta U_N\left(L_r\right)_{an};$$
$$U_N\left(L_r\right)_{bn}=-\Delta U_N\left(L_r\right)_{bn}.$$

Factor THD_U of the voltage harmonic distortion curve in the function of the reactor inductance was determined by the obtained analytical expressions for the supply network voltage harmonic composition (3) fig. 3.

Using the obtained analytical expression (3) a plot of voltage total harmonic distortion value curve against the reactor inductance was built (fig. 4 a).

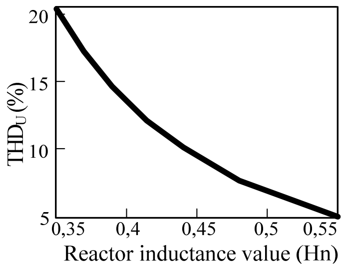

Fig. 4. *THD_U* versus reactor inductance L_r curve

Conclusions

A method for the calculation and choice of the passive filter parameters by the analytical expressions of the nonlinear load current harmonic components with the possibility for the assuring of the required level of the supply load distortion has been proposed. The mentioned components of current are obtained with the use of the proposed small parameter method realized in the frequency domain. As a result of the research, it has been demonstrated that due to the creation of the voltage supply harmonic distortion factor plot against the reactor inductance it is possible to choose the required value of the filter parameter to provide the necessary level of the supply voltage distortion. The proposed method can be applied to more complicated filter structures.

***Authors**: Rector of Kremenchuk Mykhailo Ostrohradskyi National University and the Chairman and the Professor of Electric Machines Department Mykhaylo Zagirnyak, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: mzagirn@gmail.com; Senior Lecture of Electric Machines Department of Kremenchuk Mykhaylo Ostrohradskyi National University Mariia Maliakova, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: mariia.maliakova@gmail.com; Associate Professor of Electric Drive and Control Systems Department of Kremenchuk Mykhaylo Ostrohradskyi National University Andrii Kalinov, Pershotravneva str. 20, Kremenchuk, Ukraine, 39600, e-mail: andrii.kalinov@gmail.com*

REFERENCES

[1] IEEE Std. 1459-2010 Trial-Use Standard, IEEE Standard Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions.

[2] Zagirnyak M., Maliakova M., Kalinov, A. (2015). Analysis of operation of power components compensation systems at harmonic distortions of mains supply voltage, Proceedings 2015 ACEMP–OPTIM–Electromotion 2015 Joint Conference. Side, Turkey, pp. 355–362.

[3] Zagirnyak M. V., Prus V. V., Nikitina A. V. (2006) Grounds for efficiency and prospect of the use of instantaneous power components in electric systems diagnostics Przeglad Elektrotechniczny (Electrical Review), Vol.82, pp. 123-125.

[4] Ginn H. L., Czarnecki L. S. (2006) An optimization based method for selection of resonant harmonic filter branch parameters, IEEE Trans. on Power Delivery, 2006, Vol. 21, no 3, pp.1445-1451.

[5] Salmerón P., Litrán S. P. (2010) Improvement of the Electric Power Quality Using Series Active and Shunt Passive Filters, IEEE Transactions On Power Delivery, Vol. 25, no. 2.

[6] Kawann, C., Emanuel, A. E. (1996) Passive shunt harmonic filters for low and medium voltage: A cost comparison study, IEEE Trans. on Power Systems, 1996, Vol. 11, no 41825-1331.

[7] Zagirnyak, M. V., Mamchur, D. G., Kalinov, A. P. (2010) Elimination of the influence of supply mains low-quality parameters on the results of induction motor diagnostics, in Proc. XIX Int. Conference on Electrical Machines – ICEM 2010, Rome. IEEE Catalog Number: CFP1090B-CDR. ISBN: 978-1-4244-4175- 4. Library of Congress: 2009901651. RF-009474.

[8] Zagirnyak M., Maliakova M., Kalinov A. (2015) Analysis of electric circuits with semiconductor converters with the use of a small parameter method in frequency domain, COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Eng., Vol. 34, no. 3, pp. 808–823.

[9] Zagirnyak, M., Kalinov, A., Maliakova, M. (2011). An algorithm for electric circuits calculation based on instantaneous power component balance, Przegląd Elektrotechniczny (Electrical Review), Vol. 87, no. 12b, pp. 212–215.