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Time delay measurement method using conditional averaging of the delayed signal module

***Abstract**. The paper presents a method of determining time delays of random signals using conditional averaging of the delayed signal module. An evaluation of selected metrological properties was prepared for the proposed statistical models of extreme characteristics. The theoretical relations were compared with the results of modelling and measurements.*

Streszczenie.** W pracy przedstawiono metodę wyznaczania opóźnień czasowych sygnałów losowych wykorzystującą warunkowe uśrednianie modułu sygnału opóźnionego. Dla zaproponowanych modeli statystycznych charakterystyk ekstremalnych dokonano oceny wybranych właściwości metrologicznych. Zależności teoretyczne porównano z wynikami modelowania i pomiarów. **(Metoda pomiaru opóźnienia wykorzystująca warunkowe uśrednianie modułu sygnału opóźnionego).

Keywords: random signals, conditional averaging, transportation time delay estimation.
Słowa kluczowe: sygnały losowe, warunkowe uśrednianie, estymacja czasu opóźnienia transportowego.

Introduction

In the measurement of delay time of random signals one employs statistical methods for the analysis of signals, most often the cross-correlation function [1]. Other less common methods used in the field of time include difference functions [4, 5], the cross-correlation method with the use of the Hilbert transform [2], and methods based on conditional averaging of signals [6–9]. This paper presents the results of selected research into the metrological properties of the method using the function of the conditional expected value of the delayed signal module ($CAEV$). The principle of generating random signals with transport delay is shown in Figure 1.

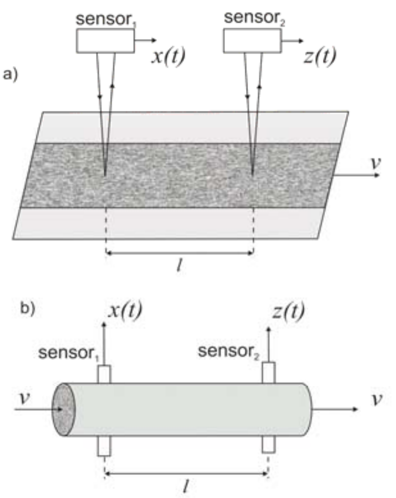


Fig.1. The principle of the generation of signals in the measurement of transport delay: l – the distance between the sensors; v – velocity of the object [8]

In matters concerning time delay estimation in measurement technique (such as detection of interference sources, measurements of transport parameters) the relation between signals $x(t)$ and $z(t)$ is often expressed by the following formula:

$$z(t) = y(t) + n(t) = c \cdot x(t - \tau_0) + n(t), \tag{1}$$

where: $x(t)$ – stationary low-pass random signal with the normal probability distribution $N(0, \sigma_x)$ and autocorrelation function $R_x(\tau)$; y(t) – stationary low-pass random signal with distribution $N(0, \sigma_y)$; c – damping factor; τ_0 = l/v – transportation time delay, $n(t)$ – stationary white noise with distribution $N(0, \sigma_n)$ not correlated with signal $x(t)$; $z(t)$ – delayed and disturbed signal. The cross-correlation functions (CCF) describing the investigated model of signal processing are as follows:

$$R_{xz}(\tau) = cR_x(\tau - \tau_0)\ , \tag{2}$$

$$\rho_{xz}(\tau_0) = c\frac{\sigma_x}{\sigma_z}\ . \tag{3}$$

The conditional expected value of the delayed signal module $|z(t)|$ for the condition $x(t) = 0$ is defined by the relation [6]:

$$A_{|z|}\big|_{x=0} = \int_0^{\infty} |z|\, p\left(|z|\big|_{x=0}\right) \mathrm{d}z \tag{4}$$

where: $p\left(|z|\big|_{x=0}\right)$ denotes conditional probability density for signal $|z|$ for the condition $x = 0$. In order to simplify the visual presentation in relation (4) and following: $A_{|z||x=0}(\tau) = A_{|z|}$, $|z(t)| = |z|$, $x(t) = x$, $c = 1$ were assumed.

Time delay measurement principle

For normal probability distributions, the conditional density $p(y|x)$ is expressed with the formula:

$$p(y|x) = \frac{1}{\sqrt{2\pi}\sigma_y\sqrt{1-\rho_{xy}^2}}\, e^{-\frac{1}{2\left(1-\rho_{xy}^2\right)}\left(\frac{x}{\sigma_x}\rho_{xy} - \frac{y}{\sigma_y}\right)^2}\ . \tag{5}$$

The conditional probability density $p(z|x)$ can be obtained by assigning a convolution of density $p(y|x)$ and $p(n)$:

$$p(z|x) = \frac{1}{2\pi\sigma_y\sigma_n\sqrt{1-\rho_{xy}^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\left(1-\rho_{xy}^2\right)}\left(\frac{x}{\sigma_x}\rho_{xy} - \frac{y}{\sigma_y}\right)^2} \times e^{-\frac{(z-y)^2}{2\sigma_n^2}}\, dy\ . \tag{6}$$

Solving the integral and assuming the condition x=0 gives the result:

$$p(z|x=0) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_n^2 + (1-\rho_{xy}^2)\sigma_y^2}}\, e^{-\frac{z^2}{2[\sigma_n^2 + (1-\rho_{xy}^2)\sigma_y^2]}} = \frac{1}{\sqrt{2\pi}\sigma_z}\, e^{-\frac{z^2}{2\sigma_z^2}} \tag{7}$$

For a normal function of probability density $p\left(z\big|_{x=0}\right)$ based on (4) one obtains $A_{|z|}\big|_{x=0}$ in the form of [6]:

(8) $$A_{|z|}\big|_{x=0} = \sqrt{\frac{2}{\pi}}\sigma_z = \sqrt{\frac{2}{\pi}}\sigma_y\sqrt{(1-\rho_{xy}^2)+\left(\frac{\sigma_n}{\sigma_y}\right)^2}$$

and A_{rel} with $n(\tau)=0$

(9) $$A_{rel} = \frac{A_{|z|}}{\left(A_{|z|}\right)_{\max}} = \sqrt{1-\rho^2 xy}\ .$$

A good estimator of the conditional expected value (4) is the arithmetic conditional average value of the delayed signal module (*CAAV*) [3, 6, 7]. Appointing a *CAAV* estimator consists in determining mutually uncorrelated at the time of passing through zero: signal *x,* registering fragments of the behaviour of signal $|z|$ of appropriate length and their averaging in a set of implementations.

The variance of the *CAAV* estimator for M averages is determined by the relation:

(10) $$Var\left[\hat{A}_{|z|}\right] = \frac{Var\left[|z|\right]}{M} = \frac{\sigma_y^2\left[\left(1-\frac{2}{\pi}\right)\left(1-\rho_{xy}^2\right)\right]+\sigma_n^2}{M}\ .$$

Due to the fact that for $n(t)$ = 0 the value of the function $A_{|z|}(\tau_0)=0$, the relative variance (11) can be represented with a formula using square normalisation of the maximum value of characteristic $(A^2|z|)_{max}$:

(11) $$\varepsilon^2\left[\hat{A}_{|z|}\right] = \frac{Var\left[A_{|z|}\right]}{\left(A_{|z|}^2\right)_{\max}} = \frac{\left(\frac{\pi}{2}-1\right)\left(1-\rho_{xy}^2\right)+\frac{\pi}{2}\frac{\sigma_n^2}{\sigma_y^2}}{M\left(1+\frac{\sigma_n^2}{\sigma_y^2}\right)}\ .$$

An example of the behaviour of function $A_{rel}(\tau)$ for mutually delayed stochastic signals $x(t)$ and $z(t)$ with $n(t)$ = 0 was presented in Figure 2. Transport delay $\tau=\tau_0$ determines the argument of the main minimum of the *CAAV* function and the main maximum of the correlation function $R_{xz}(\tau)$.

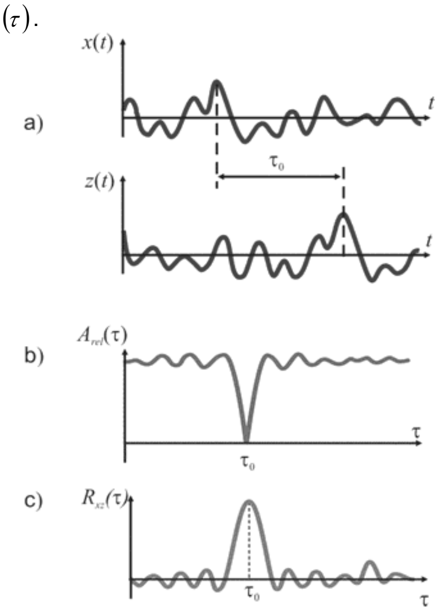


Fig.2. Examples of behaviours of:
a) stochastic signals $x(t)$ and z(t); b) function $A_{rel}(\tau)$ for $n(t)$ = 0;
c) cross-correlation function $R_{xz}(\tau)$

Simplified theoretic and experimental models of a *CAAV* characteristic

By modelling the input signal $x(t)$ with low-pass white noise in band B with a normalised autocorrelation function:

(12) $$\rho_{x(\tau)} = \frac{\sin 2\pi \mathrm{B}\,\tau}{2\pi \mathrm{B}\,\tau}$$

and by expressing the cross-correlation function with the relation:

(13) $$\rho_{xy}(\tau) = \rho_{x(\tau-\tau_0)}$$

followed by expanding it by means of Taylor's series approximation:

(14) $$\rho_{xy(\tau)} \approx \left[1-\frac{2\pi^2B^2(\tau-\tau_0)^2}{3}\right]$$

one obtains a simplified model of the *CAAV* relation (6):

(15) $$A_{|z|}\,|_{x=0} \approx \sqrt{\frac{2}{\pi}}\sigma_y\sqrt{13{,}15B^2(\tau-\tau_0)^2+\left(\frac{\sigma_n}{\sigma_y}\right)^2}\ .$$

With $\sigma_n=0$ and for $\tau=\tau_0$ $A_{|z|}|_{x=0}$=0, and the characteristic in neighbourhood τ_0 changes in a linear way. By applying similar simplifications for expression (10) one can represent the variance of the *CAAV* estimator with the following relation:

(16) $$Var\left[\hat{A}_{|z|}\right] \approx \frac{\sigma_y^2}{M}\left[4{,}73B^2(\tau-\tau_0)+\left(\frac{\sigma_n}{\sigma_y}\right)^2\right].$$

With $\sigma_n=0$ and for $\tau=\tau_0$ the variance equals zero.

During the *CCF* estimation with pairs of non-correlated samples divided into N cycles, a relation for relative deviation [3] was obtained for the model of signals (1):

(17) $$\sigma_{rel}\left[\hat{R}_{xz}(\tau_0)\right] = \frac{\sigma\left[\hat{R}_{xz}(\tau_0)\right]}{R_{xz}(\tau_0)} \approx \left[\frac{1}{N}\left(2+\frac{1}{\left(\frac{S}{N}\right)}\right)\right]^{1/2}.$$

where $\left(\frac{S}{N}\right)=\left(\frac{\sigma_x}{\sigma_z}\right)^2$ – signal-to-noise relation.

The relative standard deviation of the *CAAV* estimator for $\tau=\tau_0$ and M averages can be represented with the formula:

(18) $$\sigma_{rel}\left[\hat{A}_{|z|}(\tau_0)\right] = \frac{\sigma\left[\hat{A}_{|z|}(\tau_0)\right]}{A_{|z|}(\tau_0)} \approx \left[\frac{1}{M}\left(\frac{\pi}{2}-1\right)\frac{1}{1+\left(\frac{S}{N}\right)}\right]^{1/2}.$$

The increase of the disturbance value causes the main maximum of *CCF* to decrease and the main minimum of *CAAV* to increase respectively.

By comparing the relations (17) and (18), one obtains:

(19) $$\frac{\sigma_{rel}\left[\hat{A}_{|z|}(\tau_0)\right]}{\sigma_{rel}\left[\hat{R}_{xz}(\tau_0)\right]} \approx \left[\frac{N}{M}\frac{\left(\frac{\pi}{2}-1\right)\left(\frac{1}{1+c^2\left(\frac{S}{N}\right)}\right)}{\left(2+\frac{1}{c^2\left(\frac{S}{N}\right)}\right)}\right]^{1/2}.$$

The diagram of relation (19) for c = 1 and several values of quotient N/M is represented in Figure 3.

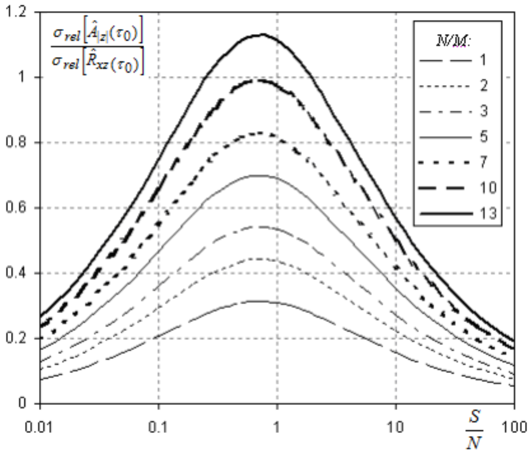


Fig.3. Diagrams of relation (19) for given values of N/M

As can be noticed, in the considered scope of S/N the relative standard deviation of $CAAV$ is always smaller than the respective deviation of CCF for the value of N/M smaller or equal to 10. In practice, the value of the quotient N/M depends on the measurement signals correlation interval, which is determined by the selection of non correlated samples.

Simulation research

Simulation research was conducted with the use of an application developed in the LabVIEW environment. Mutually delayed stochastic signals reflecting model (1) were generated. Next, using non correlated pairs of samples, discreet CCF and $CAAV$ estimators were determined for given values of S/N. The number of samples was assumed as 200 000, standard deviation of the signal $\sigma_x = 1$ and discrete transportation time delay l_0 equal to 100 samples.

Figure 4 represents the diagram of the following relation:

(20) $$\hat{\sigma}_{rel}\left[\hat{A}_{|z|}(l_0)\right] / \hat{\sigma}_{rel}\left[\hat{R}_{xz}(l_0)\right] = f\left(\frac{S}{N}\right),$$

in comparison to the theoretical behaviour of (19) for N/M = 1 and N/M = 2. The results of simulations in the whole investigated scope of S/N deviate to a small degree from the calculations (in favour of the $CAAV$ characteristic), which becomes evident in particular in the scope of S/N values close to one.

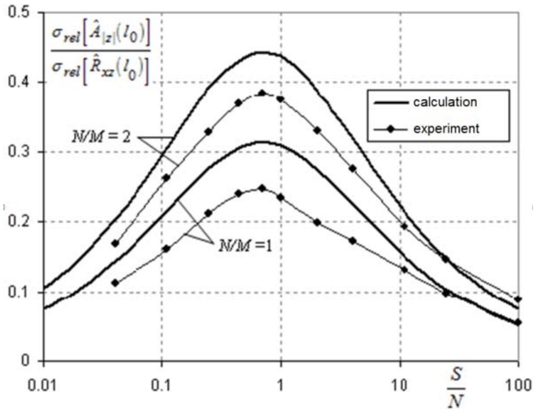


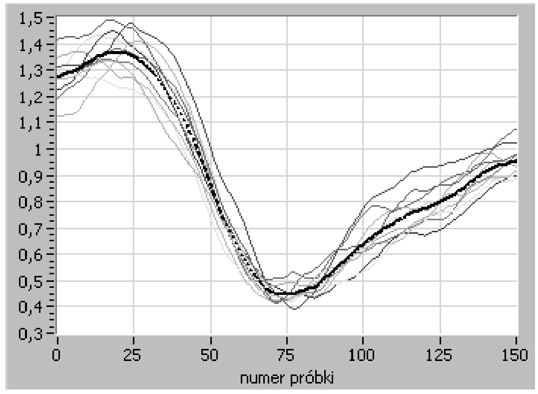
Fig.4. Diagrams of relations (19) and (20) for N/M = 1 and N/M =2

Experimental research

The variance of extreme characteristics was studied practically. The measurement was carried out at an experimental site with random voltage signal generators, signal delay systems, data acquisition system, function generator and a digital oscilloscope. Signals were sampled with a fixed interval $T_p = 0{,}2\ \text{ms}$.

In experiments carried out for the parameters: $M = 30$; $\sigma_{x_i} \approx \sigma_{ni} \approx 0{,}3\ \text{V}$; $\tau_0 = 75 \cdot 0{,}2 \cdot 10^{-3} = 15\ \text{ms}$ ten estimates of CCF and $CAAV$ characteristics were determined along with average behaviours of estimates (lines marked in bold in Fig. 5). The variance of the $CAAV$ characteristic is visible as clearly smaller than the correlative one at the point and in the neighbourhood of the transport delay [6].

a)



b)

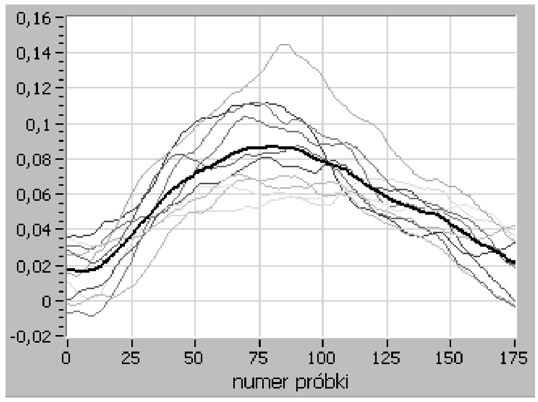


Fig.5. Experimental characteristics of: a) $CAAV$; b) CCF [8]

Figure 6 presents the random signals $x(t)$ and $n(t)$ used in the experiments. The random signal $x(t)$ is a low-pass white noise in band B=500Hz, signal $n(t)$ is a white noise in band B=20kHz. Figure 7 shows the $CAAV$ characteristic in the absence of the disturbing signal n(t). Figure 8 illustrates the $CAAV$ characteristic at given levels of disturbing signal $n(t)$: a) $\sigma_n = 0{,}7V$; b) $\sigma_n = 0{,}22V$; c) $\sigma_n = 0{,}07V$. Experiments were carried out with the number of averages M=256.

For the experiments, processing systems and signals were used which were not completely deprived of constant components, and the digital oscilloscope with a real quantisation error, which is visible in the autocorrelation and $CAAV$ characteristics. The vertical displacement of the $CAAV$ characteristic caused by the constant component in the input and output signals and disturbances acting at the output do not change the position of the main minimum of the $CAAV$ function in time.

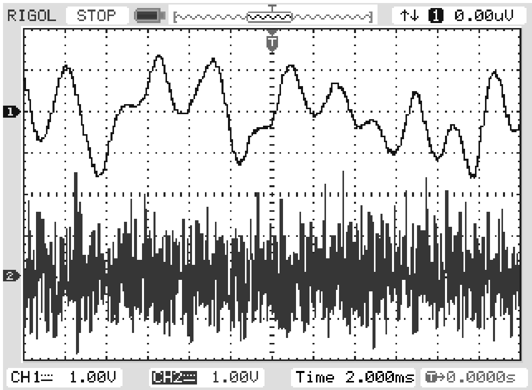


Fig.6. Random signals: signal $x(t)$-1; disturbing signal $n(t)$-2

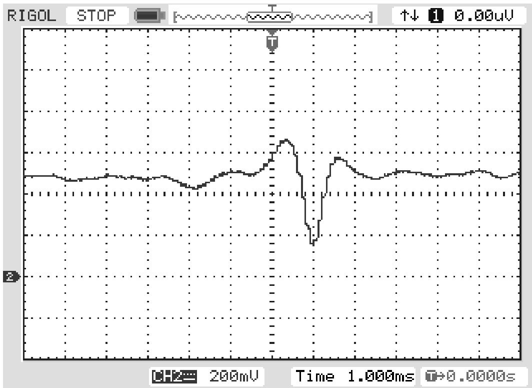


Fig.7. $CAAV$ characteristic: $\sigma_x = 0{,}7V$; $\sigma_n = 0V$; $\tau_0 = 1\text{ms}$; M = 256

Summary

The $CAEV$ method is characterised by a relatively simple probabilistic model and a simple practical implementation. The advantages of the characteristic $A_{|z|}(\tau)$ include a large steepness and small variance in the neighbourhood of the minimum in situations in which the distortion level of the delayed signal is not high.

Simulation tests and measurements confirm the results of the theoretical calculations and the beneficial properties of the $CAAV$ characteristic compared to the correlation characteristic in situations in which the correlation level of the input signal and the delayed signal is large.

Distortions of the delayed signal, both natural and those introduced by processing systems, cause an elevation of the main minimum of the $CAAV$ characteristic, a reduction in steepness of the characteristic in the neighbourhood of the minimum and a deterioration of accuracy in determining the value of the transport delay.

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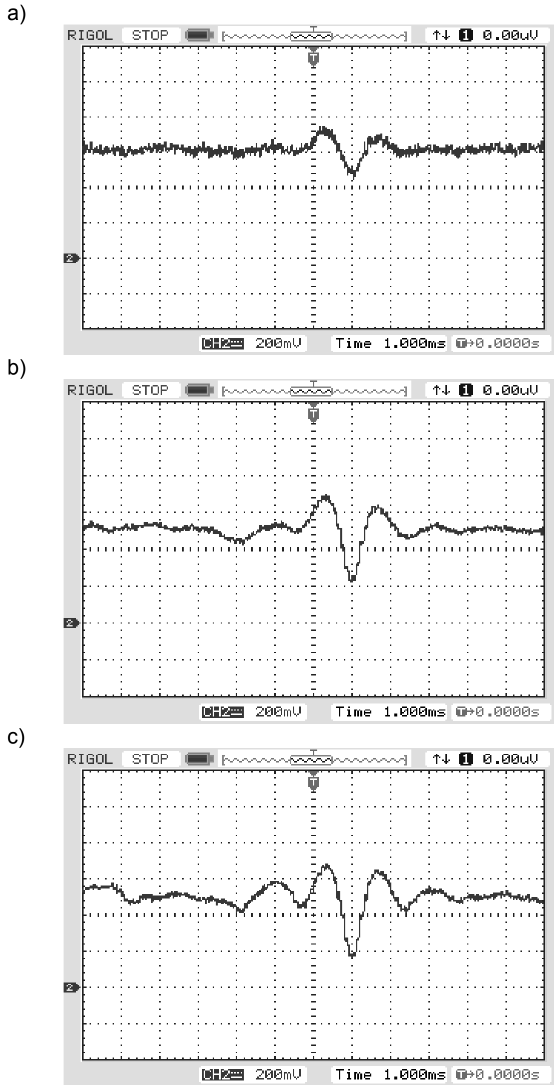

Fig.8. $CAAV$ characteristics: $\sigma_x = 0{,}7V$; $\tau_0 = 1ms$; $M = 256$; a) $\sigma_n = 0{,}7V$; b) $\sigma_n = 0{,}22V$; c) $\sigma_n = 0{,}07V$

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