Edmarcio A. BELATI[1], Lina P. GARCES[2], William M. DA ROSA[1], Igor F. do PRADO[3] and Priscila ROSSONI[1]

UFABC (1), UFG (2), UESC (3)

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Distributed Generation Allocation Using the Genetic Algorithm of Chu-Beasley and Sensitivity

***Abstract**. This paper presents a methodology for the allocation of distributed generation (DG) units to minimize active power losses in distribution networks. This methodology is based on the genetic algorithm of Chu-Beasley (GACB) and first-order sensitivity (FOS). To evaluate the different proposals of solution, instead of using Load Flow (LF), a FOS technique was used in order to directly estimate the solution of the LF. The proposed methodology was applied to three distribution systems, containing 34, 70 and 126 buses, respectively. Results obtained for the 34 bus system using GACB and FOS technique were compared with that obtained using GACB but solving the LF via Newton-Raphson (NR) method, showing the computational time gain when the FOS technique is used. For the three systems, the best locations for allocation two DG units are shown and the technical impacts in the network, i.e., active power losses and voltage profiles, are verified.*

Streszczenie.** W artykule opisano metodologię lokalizacji rozproszonych źródeł energii przy kryterium minimalizacji strat mocy czynnej. Metoda bazuje na algorytmie genetycznym Chu-Beasley i czułości pierwszego rzędu. Metodę sprawdzono na przykładzie trzech różnych sieci dystrybucyjnych.* ***Analiza lokalizacji rozproszonych generatorów przy wykorzystaniu algorytmu generycznego Chu-Beasley

Keywords: Distributed generation; Sensitivity analysis; Genetic algorithm of Chu-Beasley.
Słowa kluczowe: generacja rozproszona, algorytm genmetyczny Chu-Beasley.

1. Introduction

The efforts towards the expansion of the participation of alternative energy in the electricity generation are growing in the world. Diversifying the energy matrix by implementing new sources of electric generation is the object and desire of many countries. These new forms of power generation may be connected to distribution and transmission networks. The renewable resources [1], e.g., wind, solar, biomass, among others, are classified as environmentally friendly. These resources can be identified as distributed generation (DG) [2].

The insertion of renewable resources in a power system is increasing, especially in power distribution networks. In general, connecting the power generator near the load brings benefits to the distribution networks. Thus, it is possible to reduce losses and improve voltage profiles in the feeders, allowing postponement of investments in an infrastructure.

In specialized literature, there are several papers that discuss this problem from different perspectives. In [3] an algorithm for allocation of DG units is presented. The goal was to reduce power losses in the system and ensure that the voltage profile remains within acceptable levels. This work presents an algorithm based on an Optimal Power Flow (OPF), divided into two phases. In the first phase, the classification of the buses is performed according to loss reduction criterion. The second phase is responsible for allocating and calculating new voltage levels after allocation of the DG units. On the other hand, [4] presents a methodology that uses exhaustive enumeration to calculate the optimal size of the DG units minimizing losses in the distribution system. This proposition is attractive when evaluating systems of small size, but applied to larger systems may lead to greater processing time. In [5], an approach to determine the best location and size of a DG unit using Genetic Algorithm (GA) is presented. In this work, Linear Programming (LP) is used to confirm the optimization results obtained by GA and to investigate the influence in the objective function of the allocation of DG units in different places. This approach was tested in a distribution network in Egypt. In the work proposed in [6], a procedure based on GA and Decision Theory is used. The objective is to establish the best location and the capacity power of a DG unit in systems of medium voltage. This study includes the technical limitations of the system, such as transmission capacity of the feeder, voltage profile and short-circuit currents at various points of the feeder. The objective function seeks to evaluate the best way to the network planning considering wind turbines to meet the load with minimal cost.

For allocating DG units in electrical distribution systems, and in order to evaluate the network performance in relation to the active power losses and voltage profile it is necessary to solve the power balance equations with a Load Flow (LF). This evaluation consumes a good portion of the processing time of the algorithms based on metaheuristics to the allocation of DG units.

In [7], one of the most efficient methods used for solving the problem of LF for transmission systems, the Newton-Raphson (NR) method, is proposed. Although the NR method is used mostly in transmission systems, it can be applied to distribution systems when sparsity techniques are used, [8].

Finally, [9] presents a first-order sensitivity (FOS) technique applied to distribution networks. This technique has demonstrated good performance in estimating active power losses and will be utilized in this work.

Thus, in this work we present a methodology for allocation of DG units based on the genetic algorithm of Chu-Beasley (GACB). The GACB is an improvement in the conventional technique of GA, [10]. The active power loss reduction is used for allocating two DG units in the distribution systems, i.e., the objective function is to minimize the active power losses. To improve the performance of GACB, the FOS technique presented in [9] was incorporated in the simulation process. This technique is used to evaluate the candidate solutions, in a direct way, eliminating the utilization of the LF, making the algorithm faster. Thus, the contribution of this paper is to apply the GACB with the FOS technique in allocation of DG units, providing a quick and efficient method.

This paper has the following organization: Section 2 describes the genetic algorithm of Chu-Beasley. Section 3 shows the first-order sensitivity technique. Section 4 presents the results obtained from tests on three systems (34, 70, 126-bus). Finally, some concluding remarks are made in Section 5.

2. Genetic Algorithm of Chu-Beasley

In this work, a specialized algorithm based on Genetic Algorithm of Chu-Beasley is proposed to solve the distributed generation allocation problem. This specialized algorithm was initially applied to the generalized assignment problem [11], and has been modified and successfully applied in the solution of different problems in engineering, mainly, in solving the transmission network planning problems [12-14].

The GACB has been shown to be effective in various applications due to its particular characteristics:

a) it uses a fitness function that identifies the individual's quality through the objective function values;
b) an unfitness function quantifying the unfeasibility of the individual is also used;
c) it substitutes only one individual in the population at each iteration depending on the substitution criteria;
d) it runs an efficient strategy of local improvement for each tested individual.

Each step of the CBA is detailed below.

2.1 Codification

Codification is one aspect that is most important in the structure of the Genetic Algorithm because it is the step in which a possible solution is represented by the algorithm.

In this work, binary codification is used. As the focus of the problem is the allocation of the DG units in the system, an individual is represented by the numbers of buses that correspond to the best locations to install the DG units. The proposed codification, when considering the allocation of two DG units, is shown in Figure 1. As shown, it can be observed that each position bit in the individual represents the number of the bus where a DG unit must be allocated, for example, in Figure 1 the solution shown proposes the installation of one DG unit in bus 2 and the other in bus 4 of the system considered. As the number of the bus is an integer number, these values are converted into binary numbers. The vector representing the individual should contain a number of elements as many bits are required for expressing, in binary, the total number of buses. For the allocation of two DG units, it is necessary to generate a single vector formed by the composition of the two integers, corresponding to the number of the buses, converted to binary, as shown in Figure 1.

2.2 Objective Function

For each individual, the objective function is estimated via the FOS technique. In this case, the objective function corresponds to the total active power losses considering the addition of the DG units in different buses of the system.

2.3 Initial Population

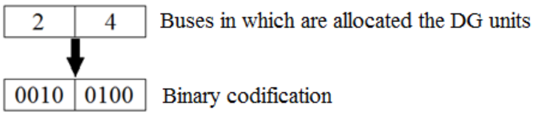

Fig. 1. Binary codification used in the algorithm

The initial population was randomly generated. The size of the population is equal to 50% of the total number of buses of the system, for example, the size of the initial population for the 24-bus systems is 12. Thus, initially, it is generating a matrix *N*x*M* where *N* is the size of the population and *M* corresponds to the number of bits that represents the integer number corresponding to the number of buses where DG units will be allocated.

2.4 Selection

The method used for selection is a roulette-wheel. In this method, a fictitious roulette-wheel is created such that each individual is represented by a portion of the roulette-wheel. The size of the portion is proportional to the contribution of the individual losses in the total value of active power losses. After, the roulette-wheel is rotated twice to select the two parents that will be used in the recombination mechanism.

2.5 Recombination

The two chosen configurations in the selection mechanism are considered in the recombination step. In (GA) theory, the combination consists of interchanging information between two vectors to form two new vectors, known as *offspring*. Thus, the new vectors will have elements of the previous ones. So it can be said that the two vectors are crossed or recombined to form two new vectors. There are several ways that recombination can be accomplished, and the simplest one is the recombination of one point, that consists of selecting only one point to perform the recombination.

Thus, once the individuals to be recombined are selected in the previous phase, and given a configuration of *M* elements for each individual, a random number between 1 and (*M*-1) is generated. This number will indicate the point of recombination. The right block of both configurations is interchanged to generate the two new configurations (known as *offsprings*), as shown in Figure 2.

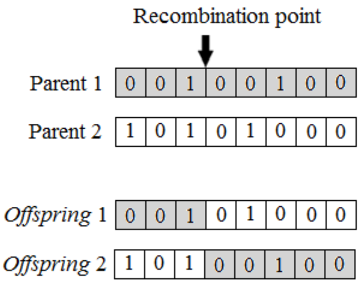

Fig. 2. Recombination mechanism

The offspring that has the better objective function is taken to the mechanism of mutation; the other is discarded.

2.6 Mutation

In this work, the mutation consists in the random choice of an element of the *offspring* selected in the previous step, and then, its current value is changed from 0 to 1, or from 1 to 0.

2.7 Local Improvement Phase

Once the selection, recombination and mutation operators are completed, there is a new individual. This individual is always feasible. Therefore, an improvement can only be achieved in the optimality of the current solution.

To improve the quality of the current solution, a search of the neighboring buses is done and the objective functions are analyzed, such that the configuration that presents better function objective is considered to be introduced in the new population.

This local improvement phase is one of the main contributions of the genetic algorithm of Chu-Beasley.

2.8 Population Substitution

The objective function is considered to substitute the new individual into a population. Those individuals that have the worst value in the objective function are removed yielding place to the new individual who will have a better value in the objective function. If the new individual is of poorer quality than any individuals in the population, or is already present in the current population, it is discarded.

2.9 Stopping Criteria

If the better solution found remains unchanged for 20 iterations, the convergence of the algorithm is achieved and, therefore this solution corresponds to the best locations to install the DG units.

3. First-Order Sensitivity

To evaluate each individual in the GACB, it is necessary to calculate the active power losses. In this work, the FOS technique was used instead of the LF to evaluate the fitness function of the individuals. The FOS technique is based on the LF equations and uses the information obtained in the base case solution. The FOS technique is detailed below.

3.1 Load Flow Equations

Injections of active and reactive power are obtained by imposing the Kirchhoff's Currents Law at each bus and can be calculated in polar form by Eq. (1) and Eq. (2), respectively.

$$P_k(V,\theta) = V_k \sum_{m\in\kappa} V_m (G_{km} cos\theta_{km} + B_{km} sen\theta_{km}) \tag{1}$$

$$Q_k(V,\theta) = V_k \sum_{m\in\kappa} V_m (G_{km} sen\theta_{km} - B_{km} cos\theta_{km}) \tag{2}$$

where: G_{km} – real element of the matrix Y_{BUS} associated with the bus *k* and *m*; B_{km} – imaginary element of the matrix Y_{BUS} associated with the bus *k* and *m*; κ – set formed by the bus *k* and the bus *m* connected to it.

The solution of the LF problem is obtained using the equations of balance of active and reactive power given, respectively, by Eq. (3) and Eq. (4).

$$\Delta P_k = P_k^{esp} - P_k^{calc}(V,\theta) = 0 \tag{3}$$

$$\Delta Q_k = Q_k^{esp} - Q_k^{calc}(V,\theta) = 0 \tag{4}$$

Where ***esp*** superscript represents the values specified of the power injections at the load buses that are considered constant and *calc* superscript is the calculated values of power injections obtained from the state variables vector (V,θ) and system parameters.

To solve the LF problem, the NR method was developed in [7]. Although the NR method is not the most utilized in distribution systems, it has good performance when sparsity techniques are utilized in the solution process [8]. Another justification for using the NR method is due to the use of data from the last iteration of the Jacobian matrix in the sensitivity analysis technique.

3.2 Sensitivity Analysis

The sensitivity analysis is of great importance in studies of the power system operation. It helps in understanding the relationship existing between cause and effect of system parameters and can be used in applications in real time to estimate new solutions after the occurrence of perturbations in the system.

Considering two types of variables: operational variables indicated by the vector u; and controlled variables indicated by the vector x.

x – state vector of the problem (V,θ);
u – operational vector (P_k^{esp}, Q_k^{esp}).

Then, the active and reactive LF equations, Eq. (3) and Eq. (4) can be rewritten compactly as:

$$g(x,u) = 0 \tag{5}$$

Suppose this $x = x^*$ is the solution to the operational vector specified $u = u^*$ that satisfies Eq. (5), then:

$$g(x^*,u^*) = 0 \tag{6}$$

Knowing that a change Δu in u^*, causes a change Δx in x^*, a Taylor series expansion in equation (6) is applied, obtaining:

$$g(x^*+\Delta x, u^*+\Delta u) = g(x^*,u^*) + S_x\Delta x + S_u\Delta u \tag{7}$$

where:

$$S_x = J = \begin{bmatrix} \frac{\partial\Delta P}{\partial\theta} & \frac{\partial\Delta P}{\partial V} \\ \frac{\partial\Delta Q}{\partial\theta} & \frac{\partial\Delta Q}{\partial V} \end{bmatrix} \text{ and } S_u = \begin{bmatrix} \frac{\partial\Delta P}{\partial P} & \frac{\partial\Delta P}{\partial Q} \\ \frac{\partial\Delta Q}{\partial P} & \frac{\partial\Delta Q}{\partial Q} \end{bmatrix}.$$

Matrix S_u results in an identity matrix, when considering the constant power load model, which is the case adopted in this study.

Combining equations (6), and (7) gives:

$$S_x\Delta x + S_u\Delta u = 0 \tag{8}$$

And rearranging for Δx, gives:

$$\Delta x = -S_x^{-1} S_u \Delta u = 0 \tag{9}$$

S_u is an identity matrix and ${S_x}^{-1}$ is equal to J^{-1}, used in the last iteration of the NR method. The expression for the correction vector Δx is given by:

$$\Delta x = J^{-1}\Delta u \tag{10}$$

Eq. (10) can be rewritten in matrix form as (11), where NPQ is the total number of load buses of the power distribution system.

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_{(2NPQ)} \end{bmatrix} = \begin{bmatrix} \frac{\partial\Delta P}{\partial\theta} & \frac{\partial\Delta P}{\partial V} \\ \frac{\partial\Delta Q}{\partial\theta} & \frac{\partial\Delta Q}{\partial V} \end{bmatrix}^{-1} \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{(2NPQ)} \end{bmatrix} \tag{11}$$

Eq. (11) considers that the distribution networks are composed of a substation and load buses. As the vector u consists of the independent variables that are active and reactive injections at the load buses, and the vector x considers the controlled variables that are voltage magnitudes and phase angles at the load buses, the matrix system (11) can be rewritten as (12).

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \vdots \\ \Delta\theta_{(NPQ)} \\ \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_{(NPQ)} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P_1^{esp} \\ \Delta P_2^{esp} \\ \vdots \\ \Delta P_{NPQ}^{esp} \\ \Delta Q_1^{esp} \\ \Delta Q_2^{esp} \\ \vdots \\ \Delta Q_{NPQ}^{esp} \end{bmatrix} \tag{12}$$

The matrix system, shown in (12), has on the left side of the equality the vector correction of state variables and, on the right side the inverse of the matrix *J* multiplied by the perturbation vector. In this application, the perturbations are new power injections at load buses, represented by DG units of each individual in the GACB.

Using the matrix system in (12), new solutions for the state variables of the problem, x_{new}, can be obtained when perturbations are performed in the load buses as follows:

$$x_{new} = x^* + \Delta x \tag{13}$$

3.3 First-Order Sensitivity Algorithm

The algorithm proposed to solve the problem consists of the following steps:

(i) Enter system data;
(ii) Obtain operational point by LF through NR method;
(iii) Obtains J^{-1} of the last iteration of the LF;
(iv) Enter with perturbation of the power injections with Δu;
(v) Use (10) to compute Δx;
(vi) Use equation (13) to update x_{new};
(vii) If a new perturbation is desired go to item (iv);
(viii) Otherwise - End.

4. Case Studies

The studies were carried out on the 34, 70, and 126 bus distribution systems. The data systems may be found in [15], [16] and [17], respectively. The data of the 126 bus system shown in [17] is the same presented in [8], only with a change in the number of the buses used. The simulations were performed on a platform consisting of a MatLab® with Intel® Core ™ i5-2410M CPU@2.30 GHz, 4GB RAM, Windows 7 Home Premium Operating System Processor - 64 Bit.

4.1 Validation of the First-Order Sensitivity Technique

A comparative test was carried out with LF via NR method to validate the FOS technique. The active and reactive power in the load buses were increased simultaneously in increments of 2%, 4%, 6%, up to a maximum of 100%, maintaining a constant power factor. For each perturbation of 2%, the FOS technique was applied, and the active power losses were estimated. Figure 3 shows the results obtained using the FOS technique and the LF via NR method. The curves demonstrate that the FOS technique can be applied to calculate active power losses. The gain in processing time by using FOS compared to the NR method is approximately 99% [8].

4.2 Allocation of DG units

In this work, the DG units were dispatched in fixed mode with active and reactive power of 1MW and 1MVar respectively. It should be noted that others values of power injections coming from of the DG in the bus of the system can be assumed and easily incorporated in the proposed methodology. Initially, the status of the system was obtained, called the base case solution, with the LF via NR method, adopting an accuracy of 10^{-5} p.u. for power balance equations. With the base case solution, new solutions were estimated directly using the FOS technique to evaluate the fitness function of each individual in the GACB. For each test system, ten simulations of the algorithm were performed. Different solutions were obtained due to the probabilistic characteristics involving in the generation of the initial population and in the evolutionary process of the GA.

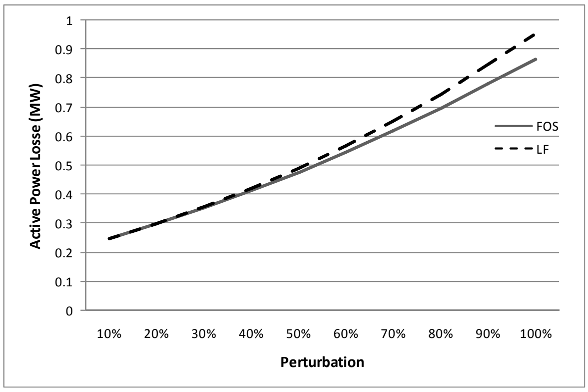

Fig. 3. Comparison of the Active Power Losses in the 126 bus system - LF and FOS

4.2.1 34 bus System

In Table 1, one comparative test of the GACB using the NR method and the FOS technique to evaluate active power losses, for 10 simulations (Sim.), is shown. In this Table, the number of the simulation, the selected buses for DG installation, the number of iterations (It.) and the time spent in each simulation can be seen.

Table 1. Comparative test between GACB-NR and GACB-FOS

Sim.	GACB-NR			GACB-FOS		
	Buses	Time (s)	It.	Buses	Time (s)	It.
1	9 and 24	6.5796	33	9 and 24	2.2247	24
2	9 and 25	8.5377	44	10 and 24	4.5438	49
3	9 and 26	4.5507	23	9 and 24	2.6744	27
4	9 and 24	6.7935	37	9 and 24	4.2893	46
5	10 and 24	11.5073	61	9 and 24	2.5234	25
6	9 and 24	5.4119	26	10 and 24	4.1053	41
7	10 and 24	5.8972	32	9 and 25	4.5458	46
8	9 and 24	6.0859	33	9 and 26	3.9697	39
9	10 and 24	7.8855	41	9 and 25	2.7050	31
10	9 and 24	8.1258	44	9 and 24	3.0433	33

The results show that the best points to allocate the DG units are at the buses 9 and 24, as illustrated in Figure 4. In this configuration, the system presents 0.04115 MW of active power losses. Without the DG units, the system presents 0.22229 MW of active power losses. In relation to the computational time, on average the GACB-FOS was 50% faster than the GACB-NR. To obtain the solution, the GACB-FOS spent on average 3.5 s. The Figure 5 shows the voltage profile before and after in DG units allocation. In the system without DG units, the lowest voltage level was, approximately, 0.94 p.u. However, with DG units connected at the buses 9 and 24, it is 0.98 p.u.

Table 2 - Simulations of the 70 bus and 126 bus systems

Simulations	70 bus system				126 bus system			
	Buses	Time (s)	Losses (MW)	Iterations	Buses	Time (s)	Losses (KW)	Iterations
1	64 and 11	28.5235	0.02574	33	18 and 12	138.9696	0.27188	67
2	64 and 11	38.9177	0.02457	44	11 and 16	132.5528	0.27340	67
3	64 and 10	52.8158	0.02888	23	10 and 18	255.5294	0.27432	60
4	67 and 62	20.5267	0.02585	28	12 and 19	150.5844	0.27500	35
5	63 and 9	24.7952	0.02849	33	12 and 19	246.1911	0.27500	58
6	63 and 11	27.5393	0.02457	37	11 and 17	126.7365	0.26894	31
7	64 and 12	28.1997	0.02518	38	49 and 17	108.7882	0.28325	25
8	63 and 11	32.7642	0.02457	44	15 and 10	190.0090	0.28347	44
9	63 and 11	36.4176	0.02457	49	57 and 11	337.7807	0.28555	79
10	63 and 14	17.8408	0.03095	24	11 and 17	243.8951	0.26894	57

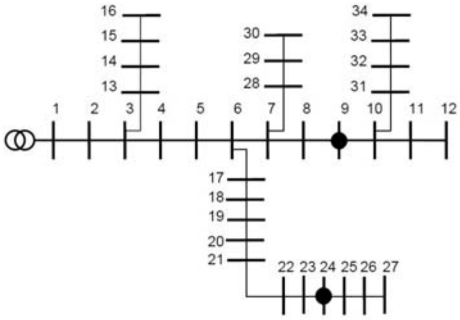

Fig. 4. 34 bus system with DG's units in the buses 9 and 24

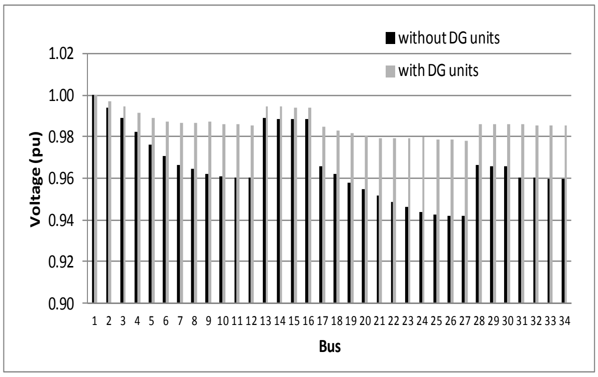

Fig. 5. Voltage profile of the 34 bus system without and with the DG´s units

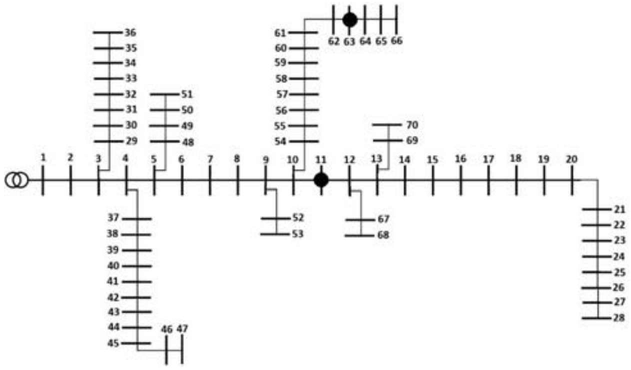

Fig. 6. 70 bus system with DG units in the buses 11 and 63

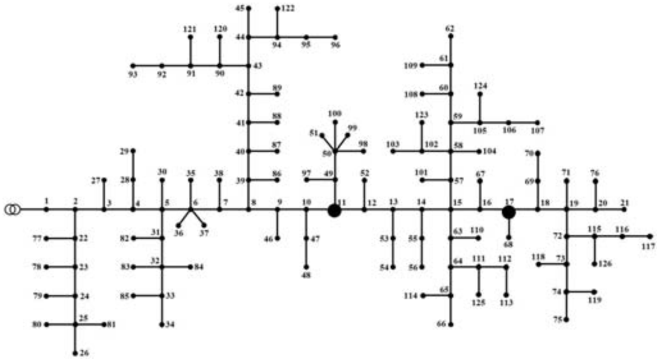

Fig. 7. 126 bus system with DG units in the buses 11 and 17

4.2.2 70 and 126 bus Systems

Using the GACB-FOS algorithm, 10 simulations were performed to allocate two DG units in the 70 bus and 126 bus systems. In Table 2, the number of the simulations, the selected buses to allocate two DG units, the number of iterations in the GACB, and the time spend in each simulation are shown.

The best solutions obtained propose to installing one unit at bus 11 and the other at bus 63 for the 70 bus system; and to install DG units at buses 11 and 17 for the 126 bus system, as illustrated in Figures 6 and 7 respectively. These simulations were performed 4 times and the time spent in each was, approximately, 30.8 s and 40 s, respectively. The 70 bus system presents 0.024570 MW and 0.22502 MW of active power losses with and without DG units, respectively. Now, the 126 bus system gives 0.268944 kW and 1.709832 kW of active power losses with and without DG units, respectively. The Figures 8 and 9 show, respectively, the voltage profile before and after of the allocation of DG units in the systems of 70 and 126 buses. In the 70 bus system without DG units, lowest voltage level is, approximately, 0.91 p.u. and with DG units connected at the buses 11 and 63, is 0.98 p.u. In the 126 bus system, an improvement in the voltage profile can also be observed.

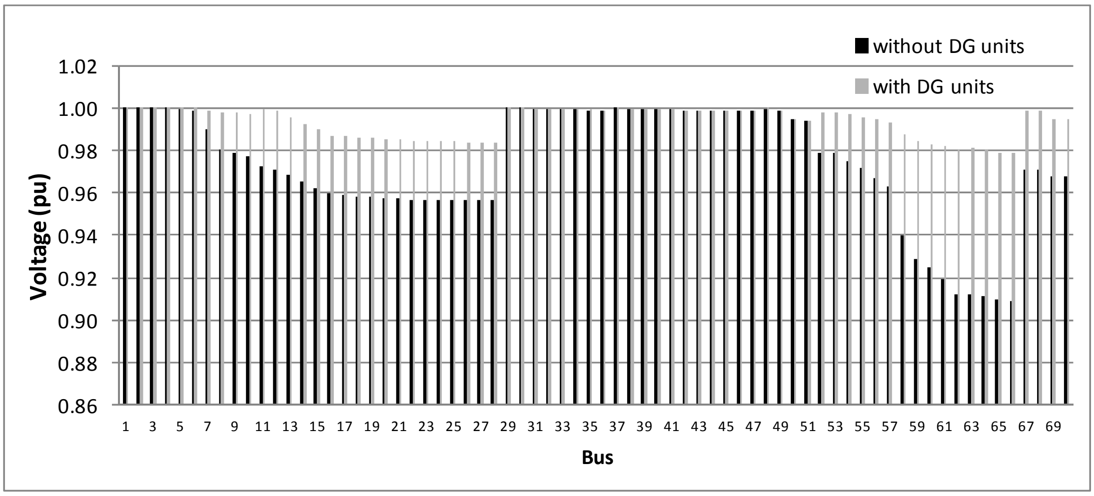

Fig. 8. Voltage profile of the 70 bus system without and with the DG units

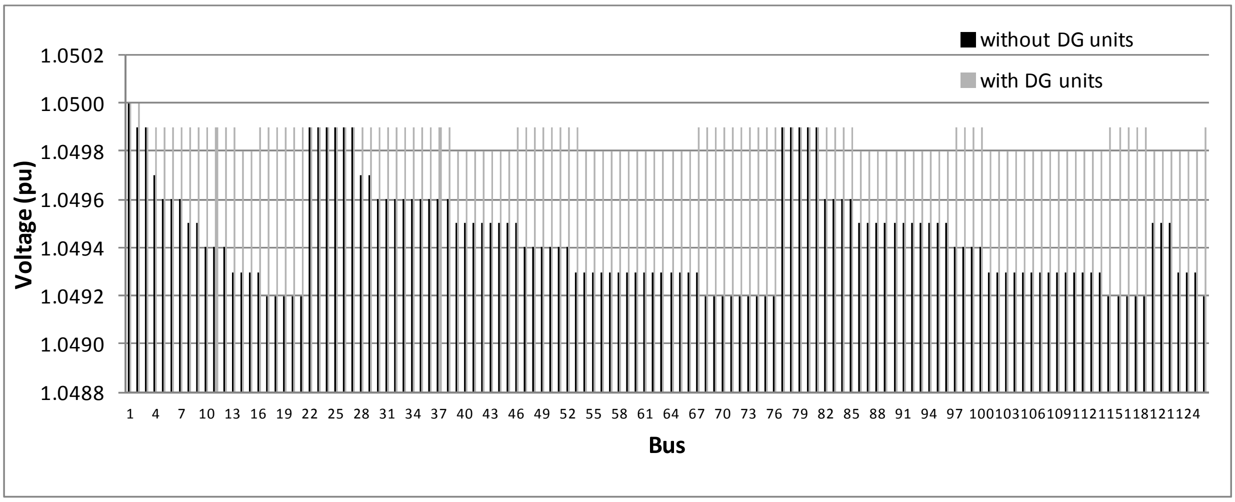

Fig. 9. Voltage profile of the 126 bus system without and with the DG units

5. **Conclusions**

In this paper, a proposal for allocation of DG units based on GACB using FOS technique is presented. The GACB has characteristics that improves the search process through local search, and combined with the utilization of the FOS technique bring computational gain and improvement in the simulation process. The FOS technique was validated in a distribution system showing the viability to obtain active power losses. The performance of the GACB using FOS was compared with the utilization of the NR method. The GACB with FOS was faster than GACB via NR-LF, leading to a gain in the order of 50% in processing time in the 34 bus system. The integration of DG units is highly effective in reducing active power losses and improving the voltage profile in a distribution network. The tests performed with the 34, 70 and 126 bus systems confirm these statements.

***Authors**:*

Prof. Dr. Edmarcio Antonio Belati, doctoral student William Moreti da Rosa and masters student Priscila Rossoni - CECS, Federal University of ABC, Rua Santa Adélia, 166, CEP 09,210-170, Santo Andre - SP, Brazil. E-mail: edmarcio.belati@ufabc.edu.br, william.moreti@ufabc.edu.br, priscila.rossoni@ufabc.edu.br.

Profa. Dra. Lina Paola Garces - EMC, Federal University of Goias (UFG), Av. Universitária, 1488, 74605-010, Setor Leste Universitário, Goiânia, GO, Brazil. E-mail: linitagarces@gmail.com.

Prof. Msc. Igor Ferreira do Prado - DCET, State University of Santa Cruz (UESC), Rod. Jorge Amado, km 16, 45662-900, Bairro: Salobrinho, Ilhéus, BA, Brazil. E-mail: igorfprado@yahoo.com.br.

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