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Numerical modelling of dielecrophoresis (DEP) effect in microchannel flow controller – comparison of calculation methods

Abstract. *Results of calculations using Maxwell stress tensor method for a new DEP based flow control device with the distance between the electrodes equal to 400μm and the relative permittivity of the liquid (2-propanol) equal to 19.9 are presented. The analysis of the influence of the electric field and the shape of the meniscus on the forces acting on the liquid surface and the liquid height-of-rise for several voltage values is presented. The results show good agreement with these obtained using simplified method.*

Streszczenie. *Przedstawiono wyniki obliczeń wykorzystujących metodę tensora napięć Maxwella do analizy nowego typu regulatora przepływu wykorzystującego zjawisko dielektroforezy. Obliczenia wykonano dla odległości elektrod 400μm i względnej przenikalności elektrycznej płynu 19,9. Przedstawiono analizę wpływu pola elektrycznego i kształtu menisku na siły działające na powierzchni płynu i wysokość płynu w mikrokanale dla kilku wartości napięcia. Wyniki analizy pokazują dobrą zgodność z uzyskanymi metodą uproszczoną. (**Modelowanie numeryczne zjawiska dielektroforetycznego w mikrokanałowym kontrolerze przepływu – porównanie metod obliczeń**).*

Keywords: dielectrophoresis, microchannel flow control, numerical modelling.
Słowa kluczowe: dielektroforeza, mikrokanałowy kontroler przepływu, modelowanie numeryczne.

Introduction

Among the most promising passive methods of heat transfer enhancement is the application of microchannel heat exchangers. They can be used especially in small power plants (e.g. ORC systems), heat pumps or cooling and refrigeration systems. This type of heat exchangers allows for reduction of the overall dimensions and gives rise to increasing the efficiency of thermal systems, which results from the possibility of operating at smaller temperature differences.

One of the groups of the microchannel heat exchangers are those utilizing two-phase flows with phase change, and especially boiling. The challenge connected with process of flow in boiling medium is flow instability. Possible effect of this instability is the reverse flow, which can strongly deteriorate heat transfer and such phenomena should be avoided. The possibility of efficient control of liquid flow in mini and micro channels is one of the most important issues to solve for heat transfer equipment and the widespread application of micro heat exchangers in modern thermal systems depends on it.

The authors of the paper proposed a novel flow control device based on the dielectrophoresis (DEP) phenomena [1]. In this flow controller, the dielectric liquid between vertical parallel electrodes rises against gravity in response to the DEP and capillarity forces, which is utilized to force the flow.

The first experiment that showed the influence of the electric field on the dielectric liquid behaviour was presented by Pellat [2]. In this experiment, it was observed that applying the voltage between two parallel electrodes resulted in raising the level of a dielectric liquid between the electrodes (Figure 1). The decrease of the distance between the electrodes caused increasing electric field intensity and consequently the increase of DEP force.

The DEP-based flow controller proposed by Lackowski et al. [1] is presented in Figure 2. The device consists of a microchannel of a rectangular cross-section, two copper electrodes mounted on the opposite sides of the channel, the flow restrictor and HV source. The width and the length of the microchannel are 400 μm and 15 mm, respectively.

A dielectric liquid with high electric permittivity is utilized in the controller. When the high voltage is not applied to the electrodes, the flow of the liquid is restricted by the restrictor. The restrictor defines the basic level (i.e. position) of the liquid surface in the microchannel. After applying the high voltage, the level of the liquid surface rises due to the electric force exerted by the electric field between the electrodes on the liquid surface. This rise of liquid surface causes indirectly an increase of the liquid flow rate. An important part of designing and selection of operating parameters of this originally proposed device is theoretical analysis of the influence of the electric field on the liquid level.

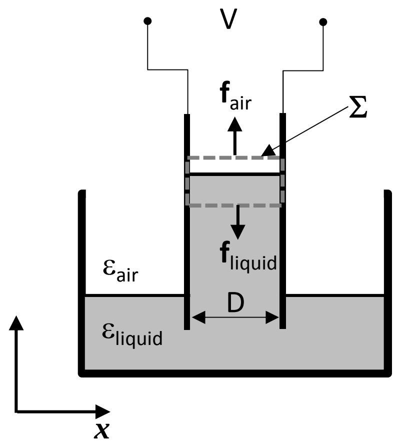
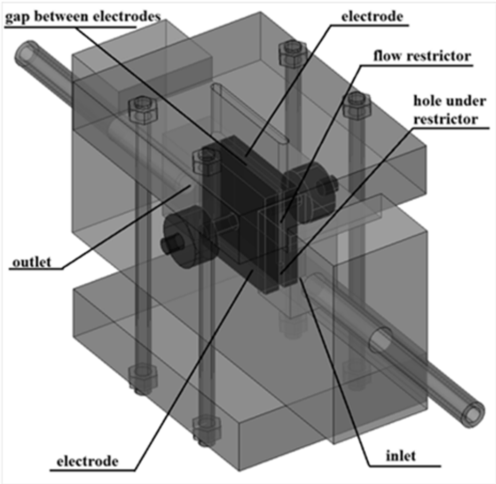


Fig. 1. Classical Pellat's experiment scheme and forces **f** acting on the liquid surface.

Theoretical model of the DEP in microchannel

DEP can be analyzed using different approaches, such as classical thermodynamics, minimal energy approach or electromechanical approach [3]. All they yield virtually the same results, however the electromechanical approach gives the best insight into physical phenomena existing on the liquid-air interface.

The basic quantity used in electromechanical analysis is the Maxwell stress tensor [4], **T**. In general, the tensor **T** represents forces acting on a unit surface: its diagonal elements represent pressure, whereas off-diagonal ones represent shear strain.



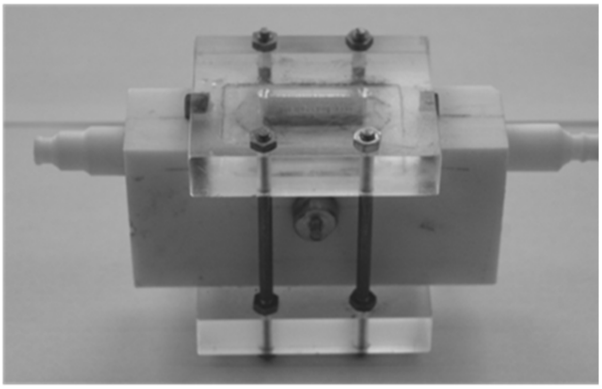


Fig.2. Proposed flow controller; a) schematic diagram, b) photograph.

To introduce the Maxwell tensor, one can analyze the motion of a material point. It is governed by the equation

(1) $$\rho \frac{d^2\mathbf{r}}{dt^2} = \nabla \cdot \mathbf{T} + \mathbf{f}_{ext}$$

where: ρ – the mass density, r – the position vector of the material point, $\mathbf{T}$ – the Maxwell stress tensor, $\mathbf{f}_{ext}$ – force volume density (e.g. for gravitation force $\mathbf{f}_{ext} = \rho\mathbf{g}$). For a stationary case, equation (1) takes the form:

(2) $$\nabla \cdot \mathbf{T} + \mathbf{f}_{ext} = 0 .$$

In terms of the components of the electric field, the components of the Maxwell stress tensor can be expressed as:

(3) $$T_{ij} \equiv \varepsilon_0 \varepsilon_r (E_i\, E_j - \frac{1}{2}\delta_{ij} E^2) ,$$

where: δ_{ij} – the Kronecker delta, ε_0 and ε_r – the electric permittivity of vacuum and relative permittivity of a medium, respectively, the indices i and j denote coordinates of a coordinate system.

In Cartesian coordinate system (x, y, z), when there is no electric field component along z-axis (E_z = 0), the nine components of the Maxwell stress tensor can be expressed as:

(4a) $$T_{xx} = \frac{1}{2}\varepsilon_0\varepsilon_r (E_x^2 - E_y^2)$$

(4b) $$T_{xy} = \varepsilon_0\varepsilon_r (E_x E_y)$$

(4c) $$T_{xz} = 0$$

(4d) $$T_{yx} = \varepsilon_0\varepsilon_r (E_x E_y)$$

(4e) $$T_{yy} = \frac{1}{2}\varepsilon_0\varepsilon_r (E_y^2 - E_x^2)$$

(4f) $$T_{yz} = 0$$

(4g) $$T_{zx} = 0$$

(4h) $$T_{zy} = 0$$

(4i) $$T_{zz} = \frac{1}{2}\varepsilon_0\varepsilon_r (-E_x^2 - E_y^2) .$$

The value of the force $\mathbf{F}$ caused by the electric fields, which acts on the surface of a body contained in a volume V, can be determined as a volume integral:

(5) $$\mathbf{F} = \int_V \nabla \cdot \mathbf{T}\, dV .$$

For the case shown in Fig. 1, after applying the Gauss theorem, the force acting on the liquid surface can be found by integrating $\mathbf{T}$ over a closed surface Σ:

(6) $$\mathbf{F} = \oint_\Sigma (\mathbf{T} \cdot \mathbf{n})\, d\Sigma ,$$

where $\mathbf{n}$ is the normal vector directed outside the surface. It is seen that the product $\mathbf{T}\cdot\mathbf{n}$ represents a force surface density $\mathbf{f}_s$. Formula (6) can be used to find the forces in a general case.

In a particular case, when the liquid surface is flat, there is no meniscus and the unit vector $\mathbf{n}$ has only the y-component, the product can be expressed as:

(7) $$\mathbf{T} \cdot \mathbf{n} = \left[\varepsilon_0\varepsilon_r (E_x E_y), \frac{1}{2}\varepsilon_0\varepsilon_r (E_y^2 - E_x^2), \varepsilon_0\varepsilon_r (E_z E_y) \right].$$

Additionally, when E_y = 0, equation (6) can be reduced to the simple form known from literature

(8) $$\mathbf{F} = \frac{1}{2}\varepsilon_0 \left(\varepsilon_{liquid} - \varepsilon_{air}\right) E_x^2\, Dw\, \hat{\mathbf{y}} ,$$

where: $\hat{\mathbf{y}}$ – the unit vector along the y-axis, D – the distance between the electrodes, and w — the microchannel length.

Results

The analysis presented above was used in calculations employing COMSOL Multiphysics software, by means of Electrostatic interface of AC/DC branch of the main module [5]. The calculations were performed for the distance between the electrodes D = 400 μm, the relative permittivity of the liquid (2-propanol) ε_r = 19.9, four values of applied voltage, and two cases: with and without the meniscus. Uniformity along the microchannel length was assumed, so the calculations were performed in 2D geometry. For the analyzed range of parameters, the results do not depend on the voltage frequency [6], so it was set to zero in our calculations. Extremely fine, physics-controlled mesh was used.

Figure 3 shows the distribution of the electric potential U between the electrodes (the grayscale is seen on the right) and the vectors of force surface density $\mathbf{f}_s$ acting on the liquid, for the case with the meniscus, for two values of V_0 (500 V and 800 V). It is seen that the forces increase with the applied potential.

Figure 4 presents how the y-components of forces $\mathbf{f}_{s,air}$, and $\mathbf{f}_{s,liquid}$ (defined in Fig. 1) change along the meniscus line for the same two values of V_0 (500 V and 800 V). Only these components have influence on the fluid level position, because the x-components are balanced. It appears that the force acting on the liquid is maximal in the centre of the meniscus, and increases with the applied voltage.

The y-component of the total force acting on the liquid surface across the x-axis unit length can be found by integrating the force density over the whole interface length [Eq.(6)]. The values obtained by the integration are: 0.0523 N/m, 0.753 N/m, 0.1026 N/m, 0.1340 N/m for V_0 = 500 V, 600 V, 700 V and 800 V, respectively. They almost do not differ from values obtained from simplified formula (8), which are 0.0523 N/m, 0.753 N/m, 0.1025 N/m and 0.1339 N/m, respectively. The obtained results showed that the influence of the meniscus can be neglected and the simplified formulas can be used.

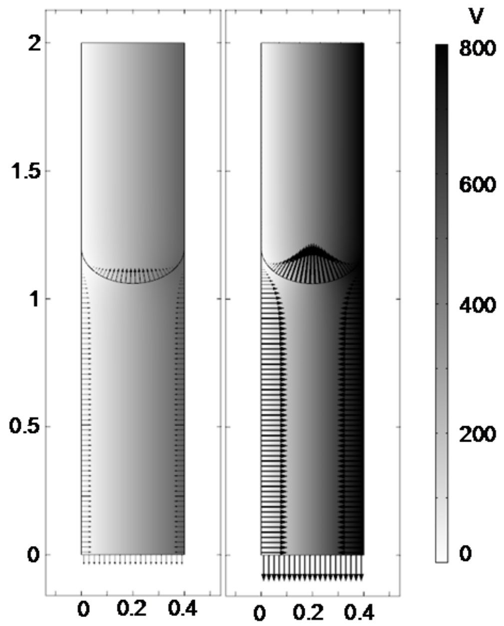


Fig. 3. Distribution of the electric potential between the electrodes (colour scale on the right) and forces $\mathbf{f}_s$ acting on the liquid surface (arrows), left V_0=500V, right V_0=800V; axis scale in mm.

The liquid level, also called the dielectrophoretic height-of-rise h_{DEP} can be found taking into account that the electrical force should balance the liquid weight W, which is

(9) $$W = \rho g h D w ,$$

where g is the local gravitational acceleration.

Because the dielectrophoretic force does not depend on the meniscus shape, we can use Eq.(8) instead of Eq.(6) and compare it with the force (9). Taking into account that the electric field between the electrodes is related to the potential difference U by relationship $E_x = U / D$, one can obtain:

(10) $$h_{DEP} = \frac{\varepsilon_0 \left(\varepsilon_{liquid} - \varepsilon_{air}\right) U^2}{2 \rho g D^2} .$$

Values of h_{DEP} are 1.7 cm and 4.3 cm for the potential difference 500 V and 800 V, respectively. They are greater than the capillary height-of-rise h_{cap}, which can be found from

(11) $$h_{cap} = \frac{2 \sigma \cos \theta}{\rho g D} .$$

where σ is the surface tension, and θ is the contact angle, which depends on electrodes material and surface quality. For fully wetted condition ($\cos\theta$ = 1), when the h_{cap} is maximal, its value is 1.4 cm, whereas for θ = 45° it is 1 cm.

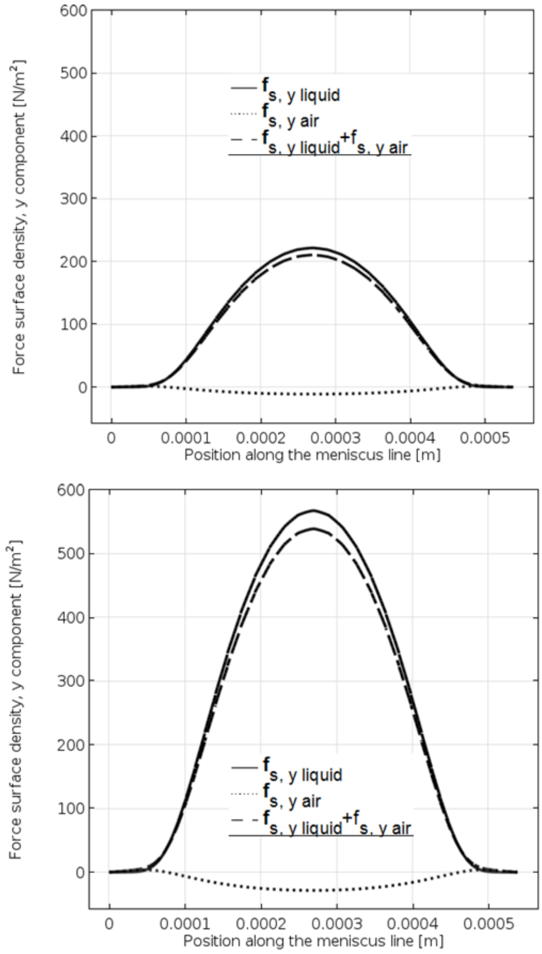


Fig. 4. The y-components of $\mathbf{f}_{s,air}$, $\mathbf{f}_{s,liquid}$ and $\mathbf{f}_{s,air}$+$\mathbf{f}_{s,liquid}$ forces along the liquid-air interface, upper V_0=500V, lower V_0=800V

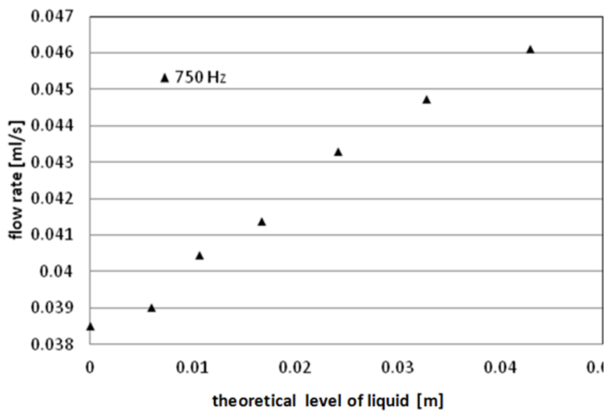


Fig.5. Correlation between the flow rate of the 2-propanol and theoretical level of liquid (h_{DEP}) in the microchannel section, frequency of applied voltage 750 Hz.

The dependence between the theoretical dielectrophoretic height-of-rise and measured values of the flow rate in the flow controller are shown in Figure 5. It is seen that the correlation between the data is almost linear. Adding a constant equal to the capillary height-of-rise to h_{DEP} would not change the type of this dependence. It means that the theoretical liquid level h_{DEP} can be used to predict behaviour of the flow controller, and especially to assess the flow rate, which is an important parameter of the proposed flow controller.

Summary

The model and calculations presented in the paper allow to determine the value of the liquid level in microchannel between two parallel rectangular electrodes where DEP occurs. This microchannel is a part of a new kind of device intended for use in microchannel heat exchange technology to control the liquid flow rate.

The analysis of theoretical based on the electromechanical approach using the Maxwell stress tensor shows that the influence of the shape of meniscus can be neglected. This implies that in order to find dielectrophoretical forces and the dielectrophoretic height-of-rise, the simplified formulas can be used.

The observed linear correlation between the experimentally obtained flow rate and the calculated level of liquid can be used to estimate the flow rate with good accuracy. A complete model from which the flow rate can be obtained directly should be a full 3D one, and take into account not only the electric (Maxwellian) but also the flow (Navier-Stokes) equations.

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