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Invariant embedding method for rotor parameters identification of induction motors

***Abstract**. The use of theory of invariant embedding for identification of internal parameters of three-phase squirrel-cage induction motor which cannot be directly measured is considered in article. A system of nonlinear differential equations N. Distefano is analytically presented, the numerical solution of which allows to obtain the values of such parameters as resistance and rotor windings inductance.*

***Streszczenie.** W artykule podjęta próba wykorzystania inwariantnej teorii zanurzenia dla identyfikacji wewnętrznych parametrów klatkowego trójfazowego asynchronicznego silnika, których nie da się zmierzyć bezpośrednio. Został przedstawiony system nieliniowych równań różniczkowych N. Distefano, rozwiązanie których pozwala uzyskać wartości takich parametrów jak aktywna rezystancja i indukcyjność rotora. **Wykorzystanie inwariantnej teorii zanurzenia dla identyfikacji wewnętrznych parametrów klatkowego trójfazowego silnika asynchronicznego**.*

Keywords: *induction motors, internal parameters, identification, mathematical model, differential equations.*
Słowa kluczowe: silnik indukcyjny, równania różniczkowe Distefano, identyfikacja parametrów.

Introduction

Three-phase squirrel-cage induction motors (IM) of common use are the most mass production of electric engineering industry. Induction electric drives are nearly 95% of total amount of electric drives, and IM consume more than half of electric power which is produced. That's why efficient estimate of quality performance of these motors during production process and after their production (receiving-delivery trials), timely diagnostics of disorder cause of processing is actual task.

IM's tests performed in modes: idling experiment; short-circuit experiment; dynamic mode; evaluation of the isolation state. determining the parameters of imbalance. Tests performed in induction motors modes: experiment idling; experiment short circuit; dynamic mode; evaluation of the state of isolation; determining the parameters of imbalance. But in tests of induction motors impossible to directly measure the parameters of the rotary circle (rotor winding resistance Rr, inductance winding rotor Lr, the mutual inductance between the stator and rotor windings Lm). Therefore, to determine these parameters using the methods of identification.

Last researches and publications analysis

Identification (in the general) is to determine system parameters based on experimental observations. The vast majority of existing methods for IM's parameters identification and their general characteristics reviewed in the book [1]. These include methods presented below

Methods of identification based on catalog data and circuits. From the equivalent circuit mathematical model of IM are determined parameters to be measured and held converting the original mathematical model of IM of linear equations system for the unknown parameters.

These methods use full of a priori information about the technical condition of IM. The disadvantages of these methods include the use of a simplified circuits, complicated evaluation of the final error parameter identification.

Heuristic methods of identification. The simplest method parameter identification of IM is heuristic (search) methods of identification. They no strictly mathematical formulation, and they should be used only when other methods are ineffective and identify existing essentially nonlinear mathematical models.

Heuristic methods based on finding the minimum of a functional quality. Minimizing may be carried out by different methods (direct search, gradient search).

Because of its simplified scheme, heuristic procedures can long converge to the true values.

When identification of nonlinear systems (multimodality functional quality) heuristic search procedures should include global minimum, and therefore have a very low convergence and significant volume computation.

Identification using Laplace transform. In this method the mathematical apparatus Laplace operator used to the stiff nonlinear differential equations of IM only if $\omega_r = const$, that is in the steady state of the IM, which is achieved by linearization of differential equations. Only under this condition can find an analytical solution of differential equations of IM.

Identification using regression methods. These methods are based on regression procedures using the method of least squares. Nonlinear systems linearized and identified by linear regression provided slow variable output value. Possible use to describe nonlinear system approximation by polynomials and Chebyshev orthogonal polynomials, but this approach has several disadvantages associated with the error of approximation and implicit physical meaning of the coefficients of polynomials.

Identification using sensitivity functions. The general scheme of parametric identification of differential equations of induction motor using sensitivity function is first approximation approach

(1) $$\boldsymbol{U}(\boldsymbol{a}+\Delta\boldsymbol{a},t)=\boldsymbol{U}(\boldsymbol{a},t)+\boldsymbol{S}(t)\cdot\Delta\boldsymbol{a},$$

where $\boldsymbol{S}(t)=\left.\dfrac{\partial \boldsymbol{f}(\boldsymbol{U},\boldsymbol{a})}{\partial \boldsymbol{a}}\right|_{\Delta a=0}$ - sensitivity matrix, which is the solution of the equation sensitivity $\dot{\boldsymbol{S}}=\dfrac{\partial \boldsymbol{f}(\boldsymbol{U},\boldsymbol{a})}{\partial \boldsymbol{U}}\cdot\boldsymbol{S}+\dfrac{\partial \boldsymbol{f}(\boldsymbol{U},\boldsymbol{a})}{\partial \boldsymbol{a}}$ with subsequent determination of (1) additional movement $\Delta\boldsymbol{a}$. For evaluation $\Delta\boldsymbol{a}$ each iteration step used least squares procedure.

The disadvantages of this method of identification include insufficient justification for of acceptability of the first approximation $\Delta\boldsymbol{U}(t,\boldsymbol{a})\approx\Delta^{(1)}\boldsymbol{U}(t,\boldsymbol{a})$ in case of hard nonlinear differential equations of IM and the need for joint solution of a mathematical model of the IM and system sensitivity functions.

Identification using linear filtering techniques based on the use of filters Wiener, Kalman to determine the parameters of the linearized mathematical model of IM. The advantages of these methods include consideration of measurement uncertainties of input parameters. Disadvantages - additional uncertainty in the determination of IM's parameters due to linearization mathematical model.

Identification by methods invariant deepening and nonlinear filtering. These methods used to identify the parameters and simultaneously consistent evaluation of linear or nonlinear observable systems.

Convergence identification according to the methods provided in a wide range of initial estimates, but required a priori information about the range of values.

The identification based on the integration time system of nonlinear differential equations, the solution of which should converge to the parameter estimates and variable time. Since measurements are in the right part of the mathematical model of IM, then the longer the measurement process, the better the solution converges to the true values.

Since these methods can provide optimal parameter estimation and all state variables, it is one of the most powerful mathematical methods of identification.

Methods of identification based on fuzzy-algorithms using fuzzyfication and defuzzyfication procedures, fuzzy-logic rules to determine the IM's parameters.

Fuzzy rules are very difficult to determine for stiff systems to identify and apply more than 2 parameters, so this algorithm is used only to identify a small number of IM's parameters.

Methods of identification based on neural networks using artificial neural networks, which consist of individual neurons working in parallel. Neurons carefully summarize and self-stimulating activity of other neurons in excess of the threshold value known input signal. Thus, the knowledge that is in neural networks consist of weight distribution of signals between neurons. These scales are set in advance in the phase of learning simple, independent of the particular task rules.

Neural networks used primarily to identify sampled systems. In the case of nonlinear rigid system of IM appears more error associated with inadequate discrete mathematical model of the real IM. In addition, due to the large amount of computation and the need for prior training of the neural network used to identify a small number of parameters.

The task

Main results of use invariant embedding method during parameters identification of rotor circuit squires-cage IM considered in this paper.

The main material

Let's suppose that conducted observations one or more component state vector I during time T contain errors. For these observations and dynamical equations of the process

$$\frac{d\boldsymbol{I}}{dT} = g(\boldsymbol{I}). \tag{2}$$

According to N. Distefano [2], we define the optimal estimation at time t, improving the estimation by increasing the number of observations.

When $t < T,$ then the problem called interpolation or smoothing. When $t = T$ it is called filtering problem, and when $t > T$ – prediction task.

In identification problems, where the main objective is to define a set of constants a_i , it is convenient to operate with these constants as with additional state coordinates which satisfy the obvious differential equation

$$\frac{da_i}{dt} = 0, \quad i = 1, 2, \ldots, k \,. \tag{3}$$

Then constants a_i can be included into extended state vector. Clearly, extended state vector filter gives not only the optimal estimation, but also optimal estimate of vector $\boldsymbol{a}$ – the main purpose of identification. According to Bellman [3], the filtering problem solved by using invariant embedding ideas. Define the vector of observation $\boldsymbol{\Gamma w}$

$$\boldsymbol{w} = [w_1; w_2; w_3; w_4]^T == [i_A; i_B; i_C; \omega_r]^T = \boldsymbol{\Gamma I} + \boldsymbol{\eta}, \tag{4}$$

where $\boldsymbol{\Gamma}$ - rectangular matrix of full rank; $\boldsymbol{\eta}$ - observation errors vector.

Based on these observations in the interval $[0..T]$ optimal estimation state vector I is determined at $t = T$ in a such way to minimize squared error function $f(\boldsymbol{I}(T), T)$, given as

$$f(\boldsymbol{I}(T), T) = \int_0^T (w - \boldsymbol{\Gamma I}, w - \boldsymbol{\Gamma I})dt + \\ +(\boldsymbol{I}(0) - \boldsymbol{b}, -\boldsymbol{\Lambda}(\boldsymbol{I}(0) - \boldsymbol{b})), \tag{5}$$

where $\boldsymbol{b}$ - the best a priori estimation $\boldsymbol{I}(0)$; $\boldsymbol{\Lambda}$ - nondegenerate matrix, which sets as confidence in the given estimation.

According to [2], the minimization of (5) achieved in solving differential equations of the optimal nonlinear filter

$$\frac{d\boldsymbol{e}}{dT} = \boldsymbol{g}(\boldsymbol{e}) + \boldsymbol{Q}(T)\boldsymbol{\Gamma}^T(\boldsymbol{w} - \boldsymbol{\Gamma e}); \quad \boldsymbol{e}(0) = \boldsymbol{b}, \tag{5}$$

and correction coefficients matrix $\boldsymbol{Q}(T)$ satisfies the equation

$$\frac{d\boldsymbol{Q}}{dT} = \boldsymbol{g}_c(\boldsymbol{e})\boldsymbol{Q} + \boldsymbol{Q}\boldsymbol{g}_c^T(\boldsymbol{e}) - \boldsymbol{Q}\boldsymbol{\Gamma}^T\boldsymbol{\Gamma Q}; \boldsymbol{Q}(0) = \boldsymbol{\Lambda}^{-1}. \tag{7}$$

Here, to simplify writing shows $\boldsymbol{c} = \boldsymbol{I}(T);$ $\boldsymbol{e} = argmin\ f(\boldsymbol{c}, T);$ $\boldsymbol{g}_c(\boldsymbol{e})$ - Jacobian $\boldsymbol{g}(\boldsymbol{e})$ on **c**; "upper T" – transpose sign.

Identification problem by N. Distefano reduced to solving a system of two differential equations (6) and (7).

Let $\tilde{\boldsymbol{I}} = \boldsymbol{I} - \boldsymbol{e}$ is estimation filter error. Then the error of filter estimation $\tilde{\boldsymbol{I}}$ has this kind of

$$\tilde{\boldsymbol{I}} = \int_0^T \boldsymbol{X}(T)\boldsymbol{X}^{-1}(\boldsymbol{S})\boldsymbol{Q}(\boldsymbol{S})\boldsymbol{\Gamma}^T\boldsymbol{\eta}(\boldsymbol{S})d\boldsymbol{S}, \tag{8}$$

where $\boldsymbol{X}(T)$ - solution of equation.

$$\frac{d\boldsymbol{X}(T)}{dT} = -[\boldsymbol{Q}(T)\boldsymbol{\Gamma}^T\boldsymbol{\Gamma} - \boldsymbol{g}_c(\boldsymbol{e})]\boldsymbol{X}(T). \tag{9}$$

Thus, the identification problem with simultaneous finding of identification error by N. Distefano is to solve equations system (6) - (8).

We will carry out identification by using the method of invariant embedding for active resistance R_r and inductance L_r of rotor.

Let's present first four equations of mathematical models of IM [4] in the Cauchy form.

(10) $$A\frac{di}{dt} = Bi + u_m ,$$

where $$A = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix};$$

$$B = \begin{bmatrix} -R_s & 0 & 0 & 0 \\ 0 & -R_s & 0 & 0 \\ 0 & -\omega_r L_m & -R_r & -\omega_r L_r \\ \omega_r L_m & 0 & \omega_r L_r & -R_r \end{bmatrix};$$

$$i = [i_{s\alpha}; i_{s\beta}; i_{r\alpha}; i_{r\beta}]^T; \quad u_m = [u_{s\alpha}; u_{s\beta}; u_{r\alpha}; u_{r\beta}]^T.$$

Inverse matrix A^{-1} will have the form

(11)
$$A^{-1} = \begin{bmatrix} \frac{L_r}{L_sL_r - L_m^2} & 0 & \frac{-L_m}{L_sL_r - L_m^2} & 0 \\ 0 & \frac{L_r}{L_sL_r - L_m^2} & 0 & \frac{-L_m}{L_sL_r - L_m^2} \\ \frac{-L_m}{L_sL_r - L_m^2} & 0 & \frac{L_s}{L_sL_r - L_m^2} & 0 \\ 0 & \frac{-L_m}{L_sL_r - L_m^2} & 0 & \frac{L_s}{L_sL_r - L_m^2} \end{bmatrix}$$

We multiply equation (10) on A^{-1}

(12) $$\frac{di}{dt} = A^{-1}Bi + A^{-1}u_m,$$

where

$$A^{-1}B = \frac{1}{L_sL_r - L_m^2} \times$$
$$\times \begin{bmatrix} -L_rR_s & L_m^2\omega_r & L_mR_r & L_mL_r\omega_r \\ -L_m^2\omega_r & -L_rR_s & -L_mL_r\omega_r & L_mR_r \\ L_mR_s & -L_sL_m\omega_r & -L_sR_r & -L_sL_r\omega_r \\ L_sL_m\omega_r & L_mR_s & L_sL_r\omega_r & -L_sR_r \end{bmatrix}.$$

For squirrel-cage IM $u_{r\alpha} = u_{r\beta} = 0$. Then $A^{-1}u_m$ will be written as

(13)
$$\mathbf{A}^{-1}\mathbf{u}_m = \left[\frac{L_r u_{s\alpha}}{L_sL_r - L_m^2}; \frac{L_r u_{s\beta}}{L_sL_r - L_m^2}; \frac{-L_m u_{s\alpha}}{L_sL_r - L_m^2}; \frac{-L_m u_{s\beta}}{L_sL_r - L_m^2}\right]^T$$

Let's extend state vector u with fifth equation of IM mathematical model and with parameters R_r and L_r

$$u = [u_1; u_2; u_3; u_4; u_5; u_6; u_7]^T =$$
$$= [i_{s\alpha}; i_{s\beta}; i_{r\alpha}; i_{r\beta}; \omega_r; R_r; L_r]^T,$$

$$\frac{du}{dt} = g(u, u_{s\alpha}, u_{s\beta}), \quad u \in \Re^7, \quad g : \Re^7 \times \Re^7 \to \Re^7,$$

where g - next vector-function:

$$g_1 = \frac{-L_sR_su_1 + L_m^2u_5u_2 + L_mu_6u_3 + L_mu_7u_5u_4 + u_7u_{s\alpha}}{L_su_7 - L_m^2};$$

$$g_2 = \frac{-L_m^2u_5u_1 - u_7R_su_2 - L_mu_7u_5u_3 + L_mu_6u_4 + u_7u_{s\beta}}{L_su_7 - L_m^2};$$

$$g_3 = \frac{L_mR_su_1 - L_sL_mu_5u_2 - L_su_6u_3 - L_su_7u_5u_4 - L_mu_{s\alpha}}{L_su_7 - L_m^2};$$

$$g_4 = \frac{L_sL_mu_5u_1 + L_mR_su_2 + L_su_7u_5u_3 - L_su_6u_4 - L_mu_{s\beta}}{L_su_7 - L_m^2};$$

$$g_5 = \frac{p}{J}\left(\frac{mp}{2}L_m(u_2u_3 - u_1u_4) - M_0\right); \quad g_6 = g_7 = 0,$$

where p - number of pole pairs; m - number of phases; J - moment of inertia of the rotor; M_0 - rotor resistance moment; L_m - mutual inductance between stator and rotor.

Let's define observations vector w

(14) $$w = [w_1; w_2; w_3; w_4]^T = [i_A; i_B; i_C; \omega_r]^T = \Gamma u + \eta,$$

where i_A, i_B, i_C - currents in stator winding of IM.

We will write matrix of full rank Γ for the equation (14). To do this, we will use transfer equation from the system of coordinates $\alpha, \beta, 0$ in the real coordinate system [4]

(15)
$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Thus, the identification problem by using the invariant embedding method of N. Distefano reduced to solving system of three differential equations (6) - (8). But these equations have complex matrix form, which is making it almost impossible to simplify them.

The solution of system (6) - (7) can be carried out by the Runge-Kutta method of fourth order with constant step h.

For numerical research of identification algorithm IM 4A71A4 was selected with parameters $p = 2$, $m = 3$, $R_S = 16.39\ \Omega$, $R_r = 11.08\ \Omega$, $L_S = 0.663\ H$, $L_r = 0.7015\ H$, $L_m = 0.624\ H$, $J = 0.011\ kg \cdot m^2$. The noise η modeled by a random variable with normal distribution. Fig. 1-2 presented a dynamic process of convergence of IM`s internal parameters R_r and L_r to its nominal value. Fig. 3-4 presented the dependence of the relative error identification R_r and L_r on time, and fig. 5 - total relative error identification.

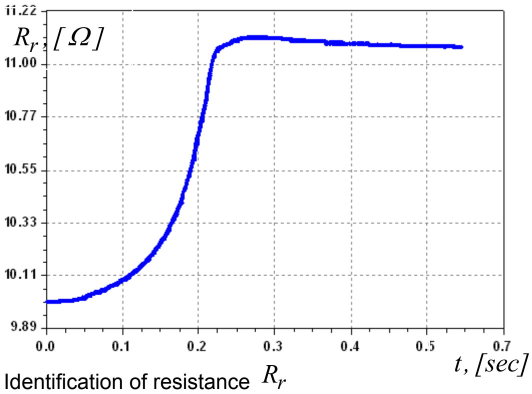

Fig. 1 – Identification of resistance R_r

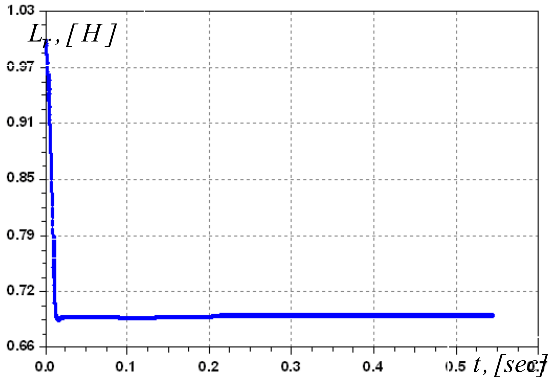

Fig. 2 - Identification of inductance L_r

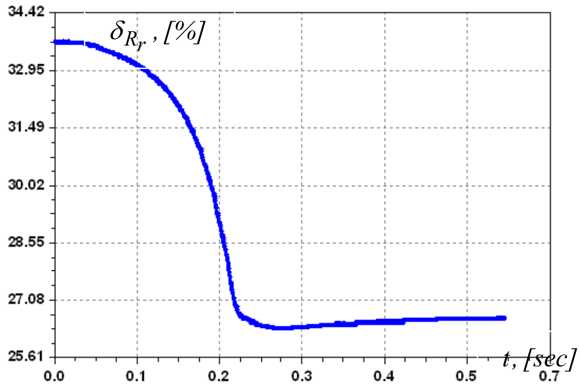

Fig. 3 - R_r identification error

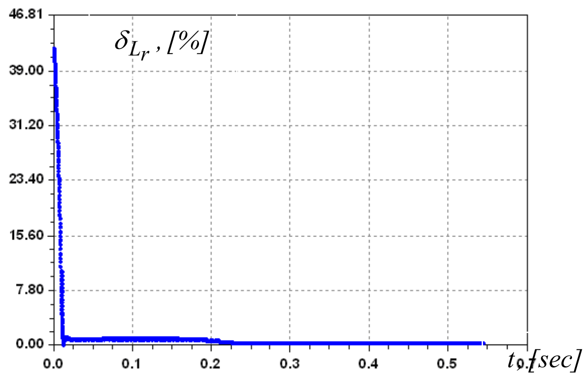

Fig. 4 - L_r identification error

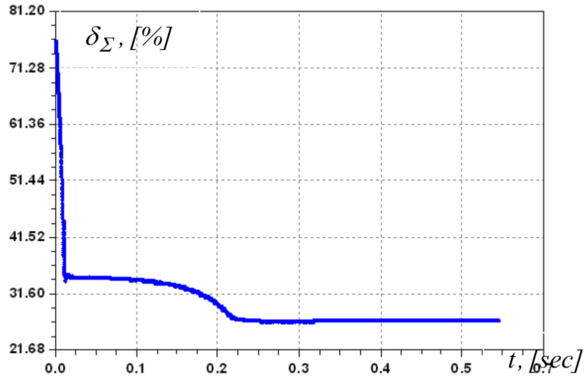

Fig. 5 - Total identification error

Summary

Analysis of the identification process modeling showed that the area of convergence of internal parameters of the induction motor to the nominal values of about 50%. But for its success requires very small discretization step in time. Convergence of the algorithm depends on the choice of initial values. This is caused by the fact that the mathematical model of IM is classified as a rigid system of differential equations.

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