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A Scalable Soft Richardson Method for Detection in a Massive MIMO System

Abstract. *The exponential growth in traffic data transmission rates is outpacing current technologies. To overcome these barriers, we propose to use the modified Richardson method, which offers a lower number of iterations and an optimal scalability condition for parallel architecture. Research indicates that the convergence provided by the channel hardening effect offers a good performance in Richardson detection. We then show a simulation of the proposed detector that allows the iteration methods to be set in systems with a large number of antennas.*

Streszczenie. *Metoda Richardsona został użyta do rozwiązania problemu wzrostu wielkości przesyłu danych. Metoda ta umożliwia użycie mniejszej liczby iteracji i optymalizację skalowalności w architekturze równoległej. Przedstawiono symulacje wykazującą że zaproponowana detekcja umożliwia iterację z duzą liczbą anten.* ***(Skalowalna metoda Richardsona do detekcji w dużych systemach MIMO)***

Keywords: soft Richardson method, massive MIMO systems, parallel signal detection, channel hardening
Słowa kluczowe: metoda Richardsona, systemy MIMO, równoległa detekcja sygnałów

Introduction

The exponential growth of mobile services such as cloud computing, software defined by networks (SDN), the internet of things, and video-on-demand has increased the demand for high-speed data transmission in the next generation of communication standards. In this evolution, the infrastructure for the next standards of communication must be able to support a large number of antennas (more than 128) at the base station (BS) [1]. With so many antennas, massive multiple-input and multiple-output (MIMO) systems are recognized as a promising tool for the fifth generation (5G) of wireless data networks [2, 3].

The data transfer rates in such systems is getting near the theoretical limit. It made the researchers focus on new architectures and methods to handle this rapid data traffic growth.

The multiplicity of mobile devices and the size of wireless data transfers have become the most important basis for the evolution of modern communication systems. In general terms, the capacity of a massive MIMO system is defined by the minimum ratio of antennas involved in the communication transceiver [4]. The performance of a MIMO system is intrinsically related to the number of antennas involved, having a proportional relationship with the data transfer performance [5]. The prospect of increasing communication capacity has prompted numerous research endeavors focusing on the feasibility of implementing such arrays [6].

New architectures, proposed in the literature, employing massive MIMO systems combined with spatial multiplexing, present a significant increase in data transmission rates by sending concurrent data streams. Despite the benefits offered in terms of spectral and energy efficiency, massive MIMO detection is considered a critical task in terms of the complexity of computational execution [7, 8]. However, several low-complexity algorithms have shown promising results as an alternative to matrix inversion.

Tree-search algorithms that use the maximum likelihood, such as sphere decoding, have demonstrated progress with fixed complexity by using a termination criterion to avoid exponential growth [9, 10, 11]. Algorithms based on the trellis reduction methods have an average polynomial complexity and a robustness to anomalies that require numerous iteration cycles. However, their performances tend to exceed detection methods based on local searches [12, 13]. Other types of detections methods that have been less exploited in the literature including approaches based on heuristic optimizations, such as the reactive Tabu [14], simulated annealing [15], and particle swarm optimization [16]; theses methods are used to obtain a transmission rate that is independent of the number of antennas. Moreover, stochastic methods, such as Monte Carlo Markov chains (MCMCs) [17], use probabilistic strategies or random sampling to reduce the number of iterations.

Thus far, the algorithms based on the Gauss-Seidel method, successive over-relaxation (SOR) and Richardson methods can be exploited when considering the effect of channel hardening in massive MIMO systems [18, 19, 20]. However, these classical methods offer a less effective convergence compared with nonstationary iterative methods, which are based on iterative Krylov methods [21, 22, 23, 24]. Despite the results based on superficial analysis reported in the literature, these methods present a low-complexity execution with a large parallelizable part, which enables the acceleration of detection algorithms for GPGPU or FPGA implementations.

In this paper, we propose an approach that focuses on approximate detection by using the soft Richardson method. In this context, the channel hardening that occurs in massive MIMO systems highlights the main terms of the diagonal matrix. This particularity will precondition the channel matrix to allow for the exploited Richardson method, which has the potential to parallelize the code. Within this proposal, methods for selecting how to execute a massive MIMO system can be suitable for parallel computing. Simulation results demonstrate that the method is well scalable for large dimensions and provides an optimum performance when subjected to a concurrent transition with a large number of antennas.

Large-Scale MIMO System Model

The wireless communication system considered in this work comprises a MIMO array with input and output dimensions represented by N_r and N_t, respectively. Furthermore, assume that $N_r \geq N_t$ is the minimum requirement for a parallel balanced transmission. The *k* stream sequences of **b** bits transmitted concurrently are encoded into a sequence of *n* bits of information at a rate $R = k/n$.

Encoded bits are then mapped into a set of finite alphabet points belonging to the set $\mathcal{S}$ (such as 64 quadrature amplitude modulation (QAM)), whose average transmission energy is given by $\mathbb{E}\{|\ \mathbf{s}_c\ |^2\} = E_s$. The $\mathbf{s}_c = [s_1, ..., s_{N_t}]^T$ generated vector terms, where $\mathbf{s}_c \in \mathcal{S}^{N_t}$, are then transmitted by a channel whose relationship can be modeled as

$$\mathbf{y_c} = \mathbf{H_c s_c} + \mathbf{n_c}, \tag{1}$$

where $\mathbf{y_c} \in \mathbb{C}^{N_r}$ is the reception vector and $\mathbf{H_c} \in \mathbb{C}^{N_r \times N_t}$ is the matrix interleaving channel. In the analysis, the $\mathbf{H_c}$ matrix was considered to follow the Rayleigh flat fading model. The noise vector, given by $\mathbf{n_c} \in \mathbb{C}^{N_r}$, follows a complex circularly symmetric Gaussian distribution with zero mean and a variance given by σ^2, i.e., $\mathbf{n_c} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I}_{N_r})$.

Both the channel model and the components of the complex constellation have independent terms. This feature allows for the use of a model that produces actual values of the analysis with a more flexible equation [25]; thus, (1) can be rewritten as a real value model defined by

$$\text{(2a)}\quad \mathbf{y} = \begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} \Re\{\mathbf{s}\} \\ \Im\{\mathbf{s}\} \end{bmatrix}, \quad \mathbf{n} = \begin{bmatrix} \Re\{\mathbf{n}\} \\ \Im\{\mathbf{n}\} \end{bmatrix},$$

and

$$\text{(2b)}\quad \mathbf{H} = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix}.$$

In particular, the $\mathbf{H}$ matrix represents the channel values for all pairs of transmitters and receiver antennas in a spatial multiplexing systems.

Soft Detection by the Richardson Method

The signal received follows a linear definition, and the relationship between the observed signal $\mathbf{y}$ and the channel matrix inverse $\mathbf{H}$ is formulated by $\mathbf{x} = \mathbf{H}^{-1}\mathbf{y}$. However, the resolution, which determines the matrix inversion, is a costly process, particularly when applied to high-order matrices.

The detection process used in MIMO communication systems is based on correctly determining the information received by each antenna involved. Specifically, the detection process seeks to minimize the residual error problem by means of

$$\text{(3)}\quad \tilde{\mathbf{x}} = arg\,min \|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|.$$

The classic iterative methods, such as the Richardson, Jacobi, Gauss Seidel and SOR methods, are low-complexity algorithms and can be used to solve this residual problem. The methods are defined by

$$\text{(4)}\quad \mathbf{x} = \mathbf{H}^{-1}\mathbf{y} \Longleftrightarrow \tilde{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_i + \mathbf{S}^{-1}\mathbf{r},$$

where $\tilde{\mathbf{x}}$ is a computer estimates of the transmission terms for *i* iterations. The residual error is $\mathbf{r} = \mathbf{y} - \mathbf{H}\tilde{\mathbf{x}}_i$ and $\mathbf{S}$ is a split matrix that determines the complexity level of each method, as depicted in Table 1.

Table 1. Splitting matrix involved in each iterative method

Method	Splitting Matrix
Richardson	$\mathbf{I}$
Jacobi	$\mathbf{D}$
Gauss Seidel	$\mathbf{L} + \mathbf{D}$
SOR	$\mathbf{L} + \omega^{-1}\mathbf{D}$

The proposed use of the Richardson method arose by defining the simple splitting matrix that avoids matrix inversion, where the method only performs multiplication by $\mathbf{H}$ and certain vector operations. Despite the low complexity provided by the iterative algorithm, this method only uses symmetric matrices defined as positive at their execution and can be slow as it approaches the exact solution over time [21]. Moreover, the eigenvalues of $\mathbf{H}$ determine the running times of these methods.

Channel hardening effect

Basically, the Richardson method applied in massive MIMO detection requires that the channel matrix used in detection become more influential with an increasing number of antennas. In massive MIMO systems, the channel suffers from the hardening effect during propagation, which is useful for linear preconditioning of the interlacing matrix.

The *Marchenko-Pastur* concept, which defines the channel hardening by the eigenvalues distribution, allows for a more consistent convergence in the solution by choosing the best spectral radius. A viable solution for using this effect is an approach that seeks to minimize the error by forcing it to zero, as shown by

$$\text{(5)}\quad \begin{aligned} \frac{\partial \|\mathbf{y} - \mathbf{H}\mathbf{x}\|}{\partial \mathbf{x}} = 0 \quad &\Leftrightarrow \quad 0 = -2\mathbf{H}^T\mathbf{y} + 2\mathbf{H}^T\mathbf{H}\mathbf{x} \\ &\Leftrightarrow \quad \mathbf{H}^T\mathbf{y} = \mathbf{H}^T\mathbf{H}\mathbf{x} \\ &\Leftrightarrow \quad \mathbf{z} = \mathbf{W}\mathbf{x}. \end{aligned}$$

Thus, $\mathbf{H}$ is replaced into the $\mathbf{W}$ group in the distribution of eigenvalues according to the β factor, as depicted in Fig. 1.

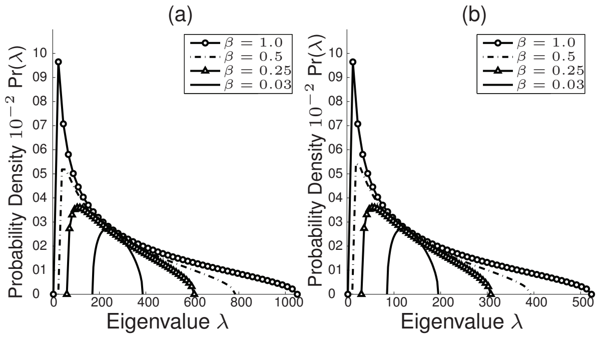

Fig. 1. Distribution of eigenvalues for the Marchenko-Pastur theory; (a) the distribution for 256 antennas and (b) the distribution for 128 antennas.

As shown in (5), the term $\mathbf{W} = \mathbf{H}^T\mathbf{H}$, which directly influences the eigenvalues λ of the channel matrix, has a direct relationship with the number of antennas. This influence, known as the *Marchenko-Pastur* [26] effect, exposes the channel hardening phenomenon, which provides a gradual reduction in the mutual information variance with an increasing number of antennas [27, 28]. One interesting aspect of the relation $\mathbf{W}$, which becomes more influential with increasing array order, is the empirical distribution that converges to a non-random limit of eigenvalues, as denoted by

$$\text{(6)}\quad \lambda_{min} = N_t\sigma^2(1 - \sqrt{\beta})^2,$$

$$\text{(7)}\quad \lambda_{max} = N_t\sigma^2(1 + \sqrt{\beta})^2.$$

This characteristic provides a faster convergence of the method because $\mathbf{H}^T\mathbf{H} \longrightarrow N_t\mathbf{I}$ when N_t,$N_r \longrightarrow \infty$ with $\beta = N_t/N_r$.

Also, the Richardson method needs stability to ensure convergence of the method [29, Theorem 7.2.2]. In order to provide the stability necessary for the convergence, the spectral radius $\rho(\mathbf{D})$ should be defined by

$$\text{(8)}\quad \rho(\mathbf{D}) = \max_{1 \le n \le N_t} |\mu_n(\mathbf{D})| < 1.$$

Hence, we included the hardening effect and the relaxation factor ω in the $\mathbf{y}$ and $\mathbf{H}$ terms based on (4) as follows:

$$\text{(9)}\quad \tilde{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_i + \mathbf{I}^{-1}(\omega\mathbf{z} - \omega\mathbf{W}\tilde{\mathbf{x}}_i).$$

Then the Richardson method (9) can be rewritten as

$$\tilde{\mathbf{x}}_{i+1} = (\mathbf{I} - \omega\mathbf{W})\tilde{\mathbf{x}}_i + (\omega\mathbf{z}), \tag{10}$$

where $\mathbf{D} = \mathbf{I} - \omega\mathbf{W}$ is responsible for guaranteeing the convergence of the method. Hence to determine the rate, which ensures the convergence of the method, the relaxation factor is optimal when $\mathbf{D}$ satisfies (8); thus, we have $0 < \omega < 2/\lambda$, where λ is the largest eigenvalue [20]. Finally, considering the influence of the hardening channel (7) around a certain eigenvalues group (β = 0.5), it is possible to observe the evolution of the error, as depicted in Fig. 2

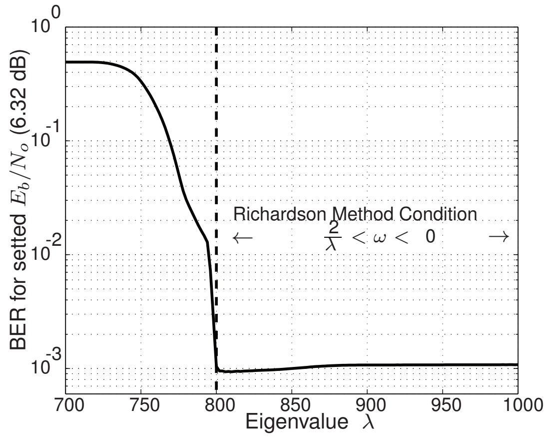

Fig. 2. BER performance analysis based on a comparison of the relaxation parameter for 256×128 ($\beta = 0.5$) Richardson detection.

General Log-Likelihood Ratio Expression (GLLRE)

The soft-bit technique is widely used in mobile communication systems, such as 3G and 4G. This decoding method, when concatenated with QAM detection, can provide significant improvements, but it requires posterior probability calculations for a bit metric, which rapidly becomes prohibitive.

An alternative method for improving the existing transmission rate is to use the GLLRE, which does not require tedious calculations of average and variance values. Moreover, the possibility of parallel execution by a GLLRE [30] in cooperation with a Richardson detector forms a solid foundation contributing to a complete parallelization of the detection methods. Thus, the test of the terms involved in communication, when considering a stationary channel, is presented by the following equation:

$$LLR(b_k) \approx 1/4\Big[\min_{B\in\{s:b_k=0\}} |\hat{x} - B|^2 - \min_{A\in\{s:b_k=1\}} |\hat{x} - A|^2\Big]$$
$$= 1/4\Big[\min_{B\in\{s:b_k=0\}} (B^2 - 2B\hat{x}) - \min_{A\in\{s:b_k=1\}} (A^2 - 2A\hat{x})\Big]. \tag{11}$$

To illustrate the process, let us consider the last bit quadrature "b_{Q5}", belonging to 64-QAM set given by $\{b_{Q5}, b_{Q4}, b_{Q3}, b_{I2}, b_{I1}, b_{I0}\}$. Conventionally, the minimum values that satisfy each region are defined by the appropriate set of values of A and B (11). In this case, using the general expression, a minimum function is given for each hypothesis of $A = \{1, 3, 5, 7\}$ and $B = \{-1, -3, -5, -7\}$ as shown below.

$$\Lambda(b_{Q5}) = \begin{cases} -\hat{x} + 6 & \hat{x} > 4 \\ \hat{x} - 2 & 0 < \hat{x} \le 4 \\ -\hat{x} - 2 & -4 \le \hat{x} < 0 \\ \hat{x} + 6 & \hat{x} < -4. \end{cases} \tag{12}$$

In this process, it is possible to determine the minimum condition in the hypothesis through an analysis of four possible elements in sets A and B for each region. Thereby, the general expression obtained in the analysis allows for the comparison to be simplified to obtain the soft-bit. This process may appear irrelevant, but this approach becomes valuable when subjected to parallel flow execution.

Simulation Results

In this section, we use numerical simulation to analyze the complexity and performance results of the modified Richardson iteration method with parallel soft-output. Over a Rayleigh fading channel, the information bits were modulated with 64-QAM, in conjunction with an encoding rate of $1/2$. The arrangement used during the simulations was determined by the antenna ratio that achieved the best performance in terms of throughput and error rate, as depicted in Fig. 3(a) and 3(b).

The proposed detection method was analyzed by comparing the different gains in each antenna arrangement. Aiming to optimize the traffic data, the analysis considered the most significant result achieved when the number of BSs was twice the number of users and when the number of antennas was greater than 150, as depicted in Fig. 3 (a) and (b), respectively.

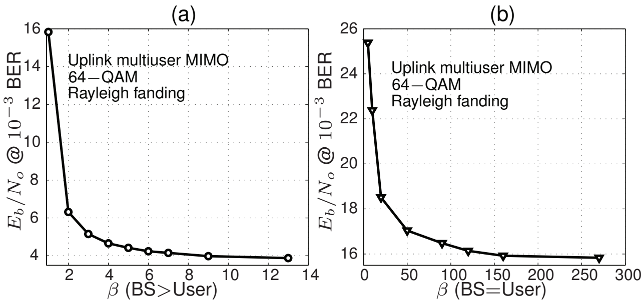

Fig. 3. Bit error rate (BER) analysis for antenna arrays: (a) the number of base stations (BSs) is greater than the number of users and (b) the number of base stations (BSs) is equal to the number of users.

The convergence of the method was determined for each relaxation factor, where a certain eigenvalues group is ensured. Fig. 4(a) presents an analysis of the influence of the iterations in each distribution. In the analysis, the variation of the relaxation factor ω did not lead to a faster convergence. A higher relaxation factor can extend the execution beyond previous values, as indicated in area 2 (Fig 4(b)). Thus, an optimized relaxation factor may occur when the distribution requires fewer iterations and relaxation factor, as indicated in area 1 by $\omega = 860$.

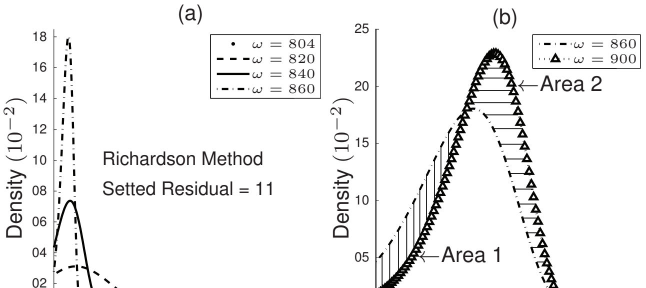

Fig. 4. Iteration distribution according to the relaxation parameter; all values were achieved through extensive simulations, which used the average of a transmitted package with 1536 bits.

To evaluate the results, we compared the performance of the proposed detection algorithm for various antenna arrangements. In Fig. 5, it is clear that the increasing influence of channel hardening promotes a small difference of 0.25dB E_b/N_o between the arrangements for 128 and 256 antennas. Moreover, we can also observe that there is a correlation between the proposed algorithm for an arrangement of 256 antennas and the exact inversion method.

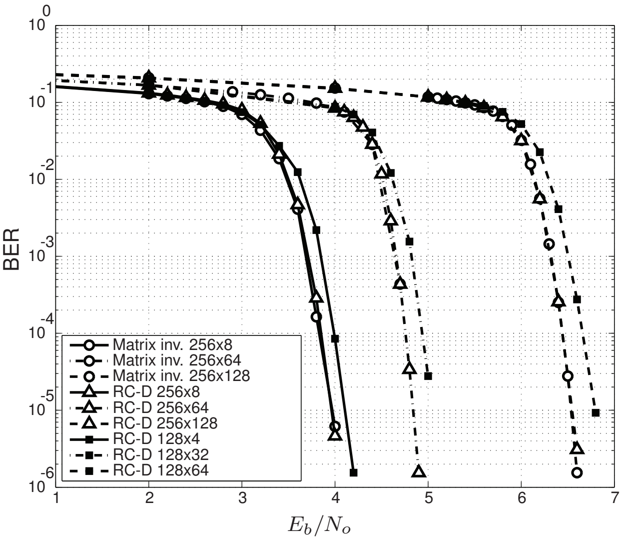

Fig. 5. BER performance comparison of a massive MIMO for a soft-output MMSE detector and the modified Richardson soft-output detector (RC-D); each point in the curve is based on 1.6Mb with a rate of 1/2 in LDPC during the simulation.

As shown in the Table 2, it can be noticed that the proposed RC-D method reduces considerably the computational complexity when compared with the MPD [27], MMSE [31] and SUMIS [32] methods. In the analysis, all of the algorithms were run with 20 iterations. Based on this approach, the proposed detection is less complex than that of the other methods, and the number of users N_t is correlated with the complexity of matrix operations in the **Algorithm 1** can promote the proportionally accelerated execution by parallel execution of the algorithm in GPGPU or in dedicated FPGA, achieving a throughput improvement in 5G technology.

Table 2. A computational complexity comparison between MMSE, MPD, SUMIS and the proposed RC-D methods.

Complexity in number of real operation x 10^6				
N_t	N_r = 128			
	MPD[27]	MMSE[31]	SUMIS[32]	RC-D(prop.)
16	0.179	0.177	0.483	0.0052
32	0.749	0.748	1.737	0.0205
64	3.200	0.3593	7.538	0.0820
96	7.208	9.584	19.368	0.1845
128	12.814	19.770	39.194	-
	N_r = 256			
16	0.296	0.333	0.917	0.0052
32	1.190	1.321	3.130	0.0205
64	4.773	5.789	12.420	0.0820
96	10.748	14.450	29.837	0.1845
128	19.116	28.355	57.347	0.3279
256	76.505	157.373	307.633	-

Algorithm 1 Richardson Method with relaxation factor

Input: $\mathbf{W}, \mathbf{z}, \omega, \mathbf{x}_0, tol$
Output: $\mathbf{x}_i$
Initialization
$\mathbf{D} \leftarrow I - \omega\mathbf{W}$;
$\mathbf{c} \leftarrow \omega\mathbf{z}$;
LOOP Process
while $\|\mathbf{r}_i\|_2 > tol$ **do**
  $\mathbf{x}_{i+1} \leftarrow \mathbf{D}\mathbf{x}_i + \mathbf{c}$;
  $i \leftarrow i + 1$;
  $r_i \leftarrow \mathbf{z} - \mathbf{W} * \mathbf{x}_i$;
end while
return

Conclusions

In this work, we present a new approach for the scalable soft detection method for a massive MIMO system based on the Richardson algorithm. This proposal aims to combine the advantage of the channel hardening effect, a low-complexity linear solution and the parallel generation of soft-values. The channel hardening gain enables to achieve a reduced number of iterations in the soft Richardson detector. One of the strong achievements of the proposed method is that it presents a considerable reduction in the computational complexity when compared to other methods (as shown in Table 2) and an extensive matrix multiplication. Based on this approach, the proposed detection is very suitable for GPU application when the problem needs run in parallel. Another implication of the method and a possible trend for future work, which increases the throughput in massive MIMO receivers, includes making the linear solution method highly parallelizable to solve high-dimensional linear systems.

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