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Modeling of dynamic properties thermal systems' using fuzzy approach

Abstract. *The paper presents the fuzzy model which enables to calculate temperature changes in the flat resistive wall of the furnace chamber. The rules the model is based on, are associated with the thermal state of object, more specifically with the instantaneous changes of its temperature. By using the concepts of "initial time constant", "initial gain", "saturation time constant", "saturation gain" good approximation of temperature changes in a flat wall of resistance chamber furnace has been achieved.*

Streszczenie. *W artykule przedstwiono model rozmyty pozwalający odwzorować zmiany temperatury w płaskiej ścianie rezystancyjnego pieca komorowego. Reguły, na których bazuje model, związane są ze stanem cieplnym w jakim obiekt znajduje się w danej chwili, a dokładniej z chwilową szybkością zmian jego temperatury. Poprzez wykorzystanie pojęć stałej czasowej początkowej, wzmocnienia początkowego, stałej czasowej nasycenia oraz wzmocnienia nasycenia osiągnięto dobre przybliżenie zmian temperatury zachodzących w płaskiej ścianie rezystancyjnego pieca komorowego.* ***(Modelowanie dynamicznych właściwości obiektów cieplnych z wykorzystaniem logiki rozmytej)***

Keywords: fuzzy modeling, electroheat plants
Słowa kluczowe: modelowanie rozmyte, obiekty elektroterminczne

Introduction

The article presents the possibility of using fuzzy logic and fuzzy sets to create a model describing heat exchange in a plane wall of a chamber electric furnace. There are several well known methods for formal description of the properties of such objects but they are either to complicated for implementation in control systems or they aren't accurate enough. Electroheat plants are important in various technologies and widely used in industrial practice. Therefore there is a need to develop more advanced modeling methods of such objects

The most common way to describe the phenomena of heat exchange in the object is to use of known physical laws. This leads to the use of non-linear partial differential equations whose solution is a time-consuming calculation problem. This makes difficult to use such models in the temperature control systems, especially in real-time mode.

The second solution which is often used in such cases are universal transfer function parametric models. Among such models inertia first order model with time delay plays an important role. This model is easy to interpret its parameters and to design. It is considered as a reasonable compromise between complexity and adequacy of description. However often significant differences of the actual temperature signal compared with the output signal of this model are observed.

A new approach to modeling of electroheat objects comes from possibility of including an additional information about nature of the object into a model. It is a well-known fact that the typical step input response of a chamber resistance furnace can be hardly approximated with high accuracy by the step input response of a single first-order inertia block, even if delay time is negligible. Experienced engineers often describe the state of an object using the concepts of the "initial state" and "saturation state". The behavior of the object in each of these states is different, thus the parameters of the object such as "time constant" and "gain" also are different in each state. Distinguishing this parameters has fuzzy character which makes fuzzy set theory and fuzzy logic helpful in solving this problem.

The main goal of this paper is to show the fuzzy model combining four first-order inertia transfer functions with different time constants and gains for modeling of dynamic properties of a thermal plant whose main constructional elements constitute insulation walls.

Modeling using physical laws

As insulation walls, are main constructional elements of electric resistance furnaces, the properties of the walls determine the behavior of the entire plant. The heat flow through the furnace wall can be described by partial differential equations of parabolic type with proper boundary conditions [1, 5]. Neglecting many constructional details, it can be regarded as the one-dimensional spatial model as follows:

$$\frac{d\vartheta(t,x)}{dt} = \frac{\lambda}{\rho c}\frac{\partial^2\vartheta(t,x)}{\partial x^2} \tag{1}$$

where: ϑ is temperature, λ is thermal conductivity, ρ is density, c is specific heat of the insulation material.

First order inertia model of thermal plants

Temperature changes in a resistance chamber furnace and more specifically in its plane wall can be also modeled using following first order inertia model with time delay expressed as transfer function:

$$G(s) = \frac{K}{1+sN}e^{-sL} \tag{2}$$

where: K is gain, N is time constant, L is time delay.

In many cases the main source of time delay L in (2) is a temperature sensor [2]. Sometimes it can be assumed that temperature inside the furnace is measured by sensor with negligibly small inertia. This gives the possibility of taking L = 0 and then formula (2) reduces to the form:

$$G(s) = \frac{K}{1+sN}e \tag{3}$$

Models (2) and (3) are often used in practice because they are simple and easy to implemented. However in many technologies the accuracy of such models comparing to experimental results becomes unsatisfactory.

Identification of temporary dynamic parameters of the furnace

Instantaneous values of dynamic parameters of the furnace can be obtained through the approximation of its answer by the equation:

$$\begin{aligned} &{\vartheta_k}^m = w_k\theta \\ &\theta = [K(1-e^{-\frac{\Delta}{N}}), e^{-\frac{\Delta}{N}}]^T \\ &w_k = [P_k, \vartheta_{k-1}] \end{aligned} \tag{4}$$

where: ${\vartheta_k}^m$ is temperature in time step k, θ is vector of model parameters, w_k is model input vector, Δ is time interval.

Having set pairs of values $\{\vartheta_i, P_i\}_r$, $i = r, \ldots, r+n$ it is possible to determine the vector of model parameters θ_r. This vector will provide the best representation of the real signals using model (4) in the r-th time interval comprising n samples. Static gain K_r and time constant N_r of the object obtained for an appropriately small value of n can be regarded as instantaneous dynamic parameters of the object. The values of parameters K_r and N_r vary depending on what r and n was adopted.

For the considered class of electrothermal objects there exist both in the literature and in practice a descriptive terms: "initial time constant", "initial gain","saturation time constant", "saturation gain". These terms are associated with variable degree of saturation of the thermal components of the object. This terms encourage to develop the model in which the parameters N and K depends on the conditions under which the object operates.

Fuzzy modeling of dynamic properties of the furnace

Changes of the parameters N and K, depending on the thermal state of the object, are not crisp, especially if described in terms of linguistic terms given above. This is a good basis for the use of fuzzy set theory and fuzzy logic to incorporate these terms into a model. It can be done by combination of four inertia first order blocks. The behavior of an object is different in an initial state and a saturated state. It is also influenced by heating phase or cooling phase. Applied fuzzy inference machine assures smooth transition from one block to another when the thermal state of the plant gradually changes. Such a model has been proposed in [2, 3]. To determine the state of the object th temperature change in the following time steps is observed. When temperature change ($d\vartheta$) is big it means that object is in "initial state", while small changes are typical for"saturation sate". When $d\vartheta$ is negative object is in cooling phase and when is positive object is in heating phase.

In the space of temperature changes two fuzzy sets are defined: small and big change in temperature. These sets are described by the formulas (5) and are shown in Fig. 1.

(5)

$$\mu_{small}(d\vartheta) = \begin{cases} 1 & for \quad d\vartheta \geq b \quad and \quad d\vartheta \leq c \\ \frac{d\vartheta-b}{b-a} & for \quad d\vartheta > a \quad and \quad d\vartheta < b \\ \frac{d-d\vartheta}{d-c} & for \quad d\vartheta > c \quad and \quad d\vartheta < d \\ 0 & for \quad d\vartheta \leq a \quad or \quad d\vartheta \geq d \end{cases}$$

$$\mu_{big}(d\vartheta) = \begin{cases} 0 & for \quad d\vartheta \geq b \quad and \quad d\vartheta \leq c \\ \frac{b-d\vartheta}{b-a} & for \quad d\vartheta > a \quad and \quad d\vartheta < b \\ \frac{d\vartheta-c}{d-c} & for \quad d\vartheta > c \quad and \quad d\vartheta < d \\ 1 & for \quad d\vartheta \leq a \quad or \quad d\vartheta \geq d \end{cases}$$

where: $\mu_{small}(d\vartheta)$ is membership function of the fuzzy set "small change in temperature", $\mu_{big}(d\vartheta)$ is membership function of the fuzzy set "big change in temperature", $d\vartheta$ is temperature difference.

It is then possible to formalize the qualitative description of thermal plant using a fuzzy model in the form of Takagi-Sugeno-Kanga system [4]. Fuzzy model diagram is shown in Fig. 2

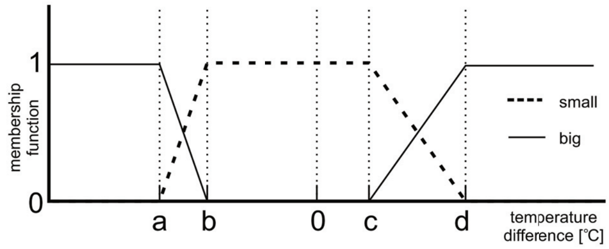

Fig. 1. Fuzzy sets: small and big change in temperature

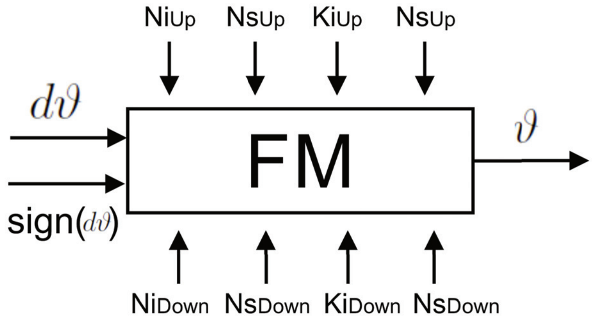

Fig. 2. Fuzzy model diagram

where:
N_{iUp} is "initial time constant" for heating phase,
N_{sUp} is "saturation time constant" for heating phase,
K_{iUp} is "initial gain" for heating phase,
K_{sUp} is "saturation gain" for heating phase,
N_{iDown} is "initial time constant" for cooling phase,
N_{sDown} is "saturation time constant" for cooling phase,
K_{iDown} is "initial gain" for cooling phase,
K_{sDown} is "saturation gain" for cooling phase,

Taking into account this four time constants and four gains: the initial: N_{iUp}, N_{iDown}, K_{iUp}, K_{iDown} and saturation: N_{sUp}, N_{sDown}, K_{sUp}, K_{sDown} and using (4) the following rules can be created:

IF $d\vartheta$ is *big* AND $d\vartheta \geq 0$ THEN $\vartheta_k = w_k\theta_{iUp}$
IF $d\vartheta$ is *small* AND $d\vartheta \geq 0$ THEN $\vartheta_k = w_k\theta_{sUp}$
IF $d\vartheta$ is *big* AND $d\vartheta < 0$ THEN $\vartheta_k = w_k\theta_{iDown}$
IF $d\vartheta$ is *small* AND $d\vartheta < 0$ THEN $\vartheta_k = w_k\theta_{sDown}$

where:

$$\theta_{iUp} = [K_{iUp}(1 - e^{-\frac{\Delta}{N_{iUp}}}), e^{-\frac{\Delta}{N_{iUp}}}]^T$$
$$\theta_{sUp} = [K_{sUp}(1 - e^{-\frac{\Delta}{N_{sUp}}}), e^{-\frac{\Delta}{N_{sUp}}}]^T$$
$$\theta_{iDown} = [K_{iDown}(1 - e^{-\frac{\Delta}{N_{iDown}}}), e^{-\frac{\Delta}{N_{iDown}}}]^T$$
$$\theta_{sDown} = [K_{sDown}(1 - e^{-\frac{\Delta}{N_{sDown}}}), e^{-\frac{\Delta}{N_{sDown}}}]^T$$

Implementing typical forms of logical operations [4] the final version of the model can be expressed as :

(6)
$$\begin{aligned} & IF d\vartheta \geq 0 \\ & \vartheta_k = w_k[\mu_{big}(d\vartheta_k) \cdot \theta_{iUp} + \mu_{small}(d\vartheta_k) \cdot \theta_{sUp}] \\ & ELSE \\ & \vartheta_k = w_k[\mu_{big}(d\vartheta_k) \cdot \theta_{iDown} + \mu_{small}(d\vartheta_k) \cdot \theta_{iDown}] \end{aligned}$$

Optimization of model parameters

Estimation of the parameters of this model using the least sum of squares requires solving the following optimization problems:

For heating phase:

(7) $$\{c, d, N_{iUp}, N_{sUp}, K_{iUp}, K_{sUp}\} = arg\min \sum_{k=1}^{n} (\vartheta_k - w_k[\mu_{big}(d\vartheta_k) \cdot \theta_{iUp} + \mu_{small}(d\vartheta_k) \cdot \theta_{sUp}])^2$$

For cooling phase:

(8) $$\{a, b, N_{iDown}, N_{sDown}, K_{iDown}, K_{sDown}\} = arg\min \sum_{k=1}^{n} (\vartheta_k - w_k[\mu_{big}(d\vartheta_k) \cdot \theta_{iDown} + \mu_{small}(d\vartheta_k) \cdot \theta_{sDown}])^2$$

Since results of solution of (8) and (7) using Matlab lsqnonlin function were not satisfactory genetic optimization methods were adopted.

Results

In order to assess the adequacy of the proposed fuzzy model of thermal objects its results have been compared to real furnace temperature as well as to a single first-order inertia transfer function. Measurements were performed on the industrial furnace of rating power of 10kW with insulation fiber.

Chosen examples of the step input responses of analyzed models are shown in Fig. 3.

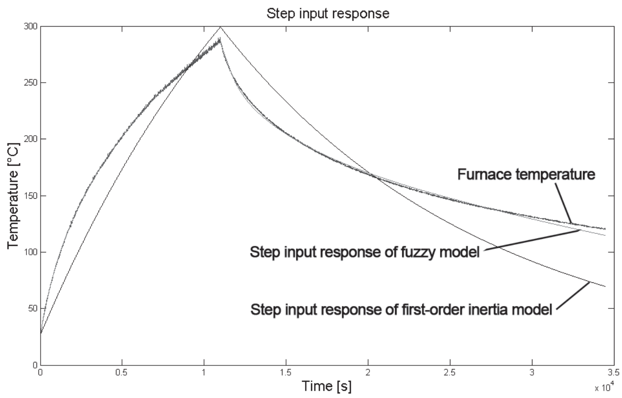

Fig. 3. Comparison of temperature in real furnace, with inertia first order model and fuzzy model

The set of best values of the parameters:
a = -0.18863, b = -0.02024, c = 0.064065, d = 0.30448
N_{iUp} = 340.30, N_{sUp} = 1036.78
K_{iUp} = 17.07, K_{sUp} = 33.05
N_{iDown} = 1359.51, N_{sDown} = 7503.54
K_{iDown} = 18.64, K_{sDown} = 41.27

The parameters of inertia first order model:
N = 3221.85, K = 57.75

It is noticeable that the graphs of furnace temperature and those obtained from the fuzzy model (6) almost overlap whereas first order transfer function model gives unsatisfactory results. It proves that proposed fuzzy modeling can be a very promising approach to describe the properties of thermal devices with distributed parameters.

Conclusions

Fuzzy approach to modeling of dynamic properties of thermal systems has been proposed. In particular, by this approach distributed parameters of such a system and their influence on its dynamics can be taken into account. The proposed method allows the qualitative description of thermal plant to be formalized using Takagi-Sugeno-Kang fuzzy structure. The antecedents of IF-THEN rules describe different thermal states of insulation walls of the plant while their consequences realize first order inertia dynamics including both ”initial” and ”final” state.

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