Maciej PAŃCZYK[1], Jan SIKORA[1,2]

Lublin University of Technology (1), Electrotechnical Institute (2)

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Boundary element method application in wall dampness tomography

***Abstract**. The boundary element method implementation used to find tomographic image of damped wall humidity distribution is presented. Damped wall represents non-homogeneous media where humidity has a spatial distribution that varies with two co-ordinates. Real measurements were taken and function coefficients where found using boundary element method, verified again by finite element method.*

***Streszczenie.** Artykuł prezentuje opis matematyczny implementacji środowisk niejednorodnych w metodzie elementów brzegowych zastosowanej do poszukiwania rozkładu zawilgocenia wnętrza muru. Wyniki obliczeń porównane są z realnymi pomiarami zawilgocenia zbudowanego w warunkach laboratoryjnych muru, wykonanymi z użyciem 26 elektrodowego tomografu impedancyjnego (**Zastosowanie metody elementów brzegowych dla środowisk niejednorodnych na przykładzie badania zawilgocenia murów**).*

Keywords: Boundary Element Method, Functionally Graded Materials, Impedance Tomography.
Słowa kluczowe: Metoda elementów brzegowych, środowiska niejednorodne, tomografia impedancyjna.

Introduction

Serious problem form many brick masonry historic buildings are their excessive moisture. Too much moisture causes a reduction in the compressive strength of both brick and mortar and to reduce the durability of the walls. Water, usually containing aggressive chemicals (chlorides, sulfates and nitrates), is transmitted to higher altitudes of the wall. High humidity of the walls encourages growth of fungi and mold, which have a negative impact on the human health. Humidity distribution, the type and concentration of salt in the wall are helpful in assessing the causes of damage and should be the basis for selecting the appropriate security methods related to excessive humidity and salinity.

Typical method used to find humidity distribution performed by drilling, sampling and checking samples moisture cannot be applied for continuous monitoring of dehumidification process. Relatively inexpensive method based on Electrical Impedance Tomography (EIT) [1-6] can be implemented. The wall conductivity is changing with humidity of the wall and its salinity, so object properties – conductivity - varies smoothly with one ore more co-ordinates. Boundary Element Method (BEM) [7, 8] was used to solve the EIT forward problem. BEM implementation requites to use either multiregional model or Functionally Graded Materials (FGM) to find spatial distribution of wall humidity.

Laboratory walls with 26 electrode impedance tomograph were investigated [1, 2].

Some theoretical aspects of BEM and FGM usage in wall dampness investigations [3], are within the scope of this paper.

Laboratory model and problem formulation

Tested wall parameters, measurements and theoretical aspects of EIT will be presented.

Real, prepared for the experiment brick wall presented in figure 1 was 1m height, 1m long and 0.51m thick. It was flooded up to half height by water. The measurements were made a day after the water was drained.

Location of electrodes (marked by e1, e2, e3 … e26) and bore-holes for drying-weighing verification (marked 1, 2 and 3) are presented in figure 2.

Projections angles and measurements corresponds to the voltage of 7V with 1kHz frequency applied between pairs of electrodes 1-26, 2-25, 3-24, ... 12-15, 13-14 like presented in figure 3.

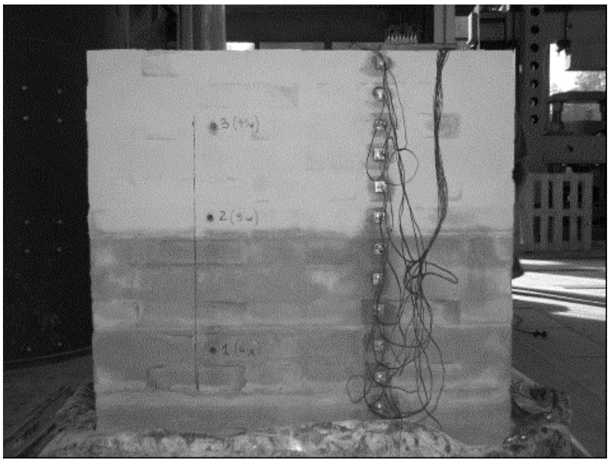

Fig.1. Investigated dumped wall [1]

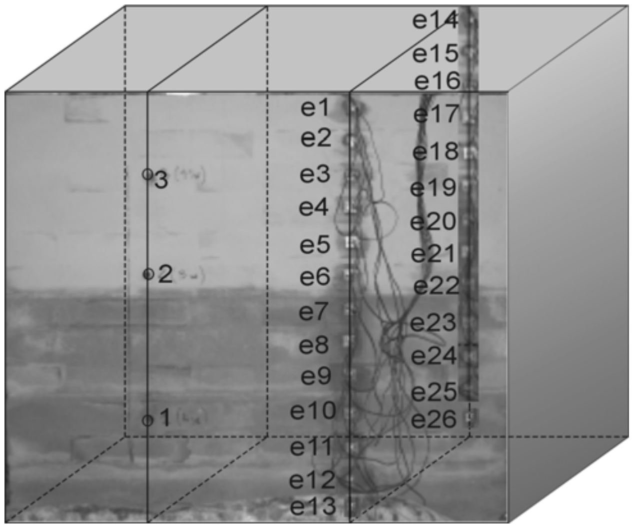

Fig.2. Investigated dumped wall with 26 electrodes for Electrical Impedance Tomography and 3 bore-holes for drying-weighing verification method

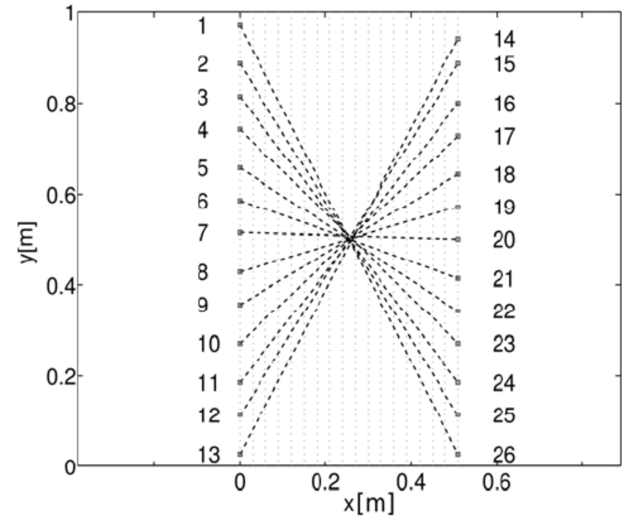

Fig.3. Applied projection angles: 1-26, 2-25, 3-24, ... 12-15, 13-14

Two-dimensional model with 174 (70×17) second order boundary elements of the damped wall was used. Conductivity distribution was interpolated by quadratic function [8]:

(1) $$\gamma(x,y)=\gamma_0\left[a_{00}+a_{10}x+a_{01}y+a_2\left(x^2-y^2\right)\right]^2$$

where coordinate x corresponds to wall thickness and y to wall height.

Coefficients a_{00}, a_{10}, a_{01}, a_{11} and a_2 where calculated in inverse problem solution in accordance with data measured in Impedance Tomography.

In a simple BEM usage fundamental solutions are known and tabulated. For Functionally Graded Materials it is necessary to find the adequate Green's function [9-14].

Green's function in two dimensions is derived for graded material represented by damped wall. The corresponding boundary integral equation formulations for this problem is derived, and the two-dimensional case is solved numerically.

Theoretical considerations

Boundary Integral Equation (BIE) [7, 8] corresponding to the problem is:

(2) $$c(\mathbf{r})\Phi(\mathbf{r})+\int_\Gamma \gamma(\mathbf{r'})\frac{\partial G^*(\mathbf{r},\mathbf{r'})}{\partial \mathrm{n}(\mathbf{r'})}\Phi(\mathbf{r'})\mathrm{d}\Gamma(\mathbf{r'})= =\int_\Gamma \gamma(\mathbf{r'})G^*(\mathbf{r},\mathbf{r'})\frac{\partial \mathbf{\Phi}(\mathbf{r'})}{\partial \mathrm{n}(\mathbf{r'})}\mathrm{d}\Gamma(\mathbf{r'})$$

Comparing the above equation to the similar BIE for homogenous materials [7, 8] two new elements can be found. At first $\gamma(\boldsymbol{r})$ in equation (2) represents conductivity spatial distribution and secondly G^* stands for so called modified Green function for non-homogenous materials. Curve Γ corresponds to the wall boundaries.

Modified Green function G* is not a Laplace equation fundamental solution. The differential equation for a potential function Φ defined on a surface S bounded by a wall boundary curve Γ, with an outward normal n, can be written as:

(3) $$\nabla\cdot\left[\gamma(\mathbf{r})\nabla\mathbf{\Phi}\right]=0$$

The kernels are based on the Green's function G^* defined as:

(4) $$\nabla\cdot\left[\gamma(\mathbf{r})\nabla G^*(\mathbf{r},\mathbf{r'})\right]=-\delta(\mathbf{r},\mathbf{r'})$$

where: $\boldsymbol{r}$ and $\boldsymbol{r'}$ are load and source points respectively.

Following E. Kurgan [9] or A. Sutradhar and G. H. Paulino [11-14] and assuming that conductivity will fulfil a condition:

(5) $$\nabla^2\left(\sqrt{\gamma(\mathbf{r})}\right)=0$$

we will receive applicable solution:

(6) $$G^*(\mathbf{r},\mathbf{r'})=\frac{G(\mathbf{r},\mathbf{r'})}{\sqrt{\gamma(\mathbf{r})\gamma(\mathbf{r'})}}$$

where G is a typical Green's function for Laplace equation [7, 8].

Matrices elements ***A*** and ***B*** [7, 8] can be written as follows:

(7) $$A_{i,j}(\mathbf{r},\mathbf{r'})=\int_\Gamma \gamma(\mathbf{r})\frac{\partial G^*(|\,\mathbf{r}-\mathbf{r'}|)}{\partial n}\mathrm{d}\Gamma$$
$$B_{i,j}(\mathbf{r},\mathbf{r'})=\int_\Gamma \gamma(\mathbf{r})G^*(|\,\mathbf{r}-\mathbf{r'}|)\mathrm{d}\Gamma$$

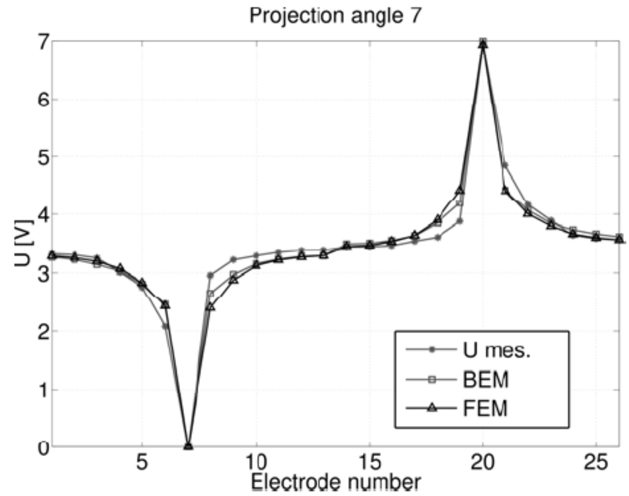

Fig.4. Successive EIT measurements, BEM 2D calculation and FEM 2D calculation results corresponding to the voltage of 7V with 1kHz frequency applied between pairs of electrodes 7-20

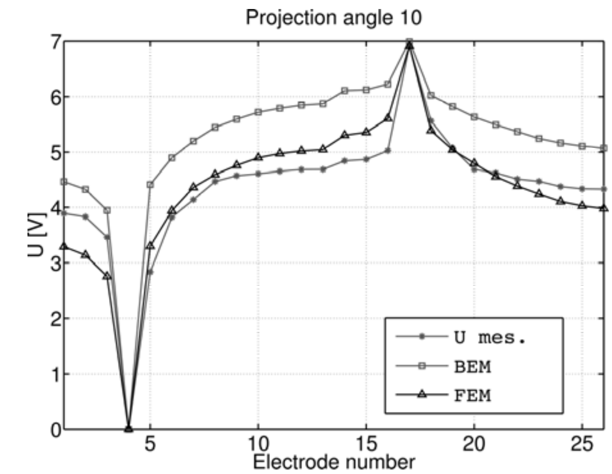

Fig.5. Successive EIT measurements, BEM 2D calculation and FEM 2D calculation results corresponding to the voltage of 7V with 1kHz frequency applied between pairs of electrodes 10-17

Results

Measured potential values were compared to these calculated with BEM (used to solve forward problem) and additionally with calculation results taken from FEM.

Inverse problem solution required 7 iterations to find interpolation coefficients related to investigated conductivity function (1).

Potentials - measured and calculated – for selected projection angles: 7 (electrodes 7-20) and 10 (electrodes10-17), as defined in figure (3), are presented in figures 4 and 5. Projection 7-20 presents the best achieved results and projection 10-17 the worst.

Remarks and conclusion

Application of Functionally Graded Materials theory into Boundary Element Method used to find spatial distribution of humidity and salinity into damped walls seems to be better idea than using multiregional BEM due to simpler mesh and boundary conditions.

Presented considerations are related with preparations to the NCBR project “Tomograf hybrydowy do badania zawilgocenia i stanu budynków” (Hybrid CT scanner to examine buildings moisture and condition) number POIR.01.01.01-00-0167/15. Described method will be used to find thomographic image of dumped wall.

Most of articles discusses objects where FGM changes their properties only in one direction [10, 12-14].

Damped wall presented above represents the graded material where humidity and salinity varies in at least two directions (corners of the buildings will require 3 dimensional model).

Achieved results are quite promising although are worse than ones received during theoretical calculations based on flat capacitor model with known analytical solution or dumped wall model calculated by BEM and FEM where material properties varied only in one direction.

Possible reason of low accuracy in some projection angles (see Fig. 5) can be caused by the insufficient data associated with too few angle projections. Possessed measurements are more suitable for the model where conductivity distribution varies only in one direction corresponding to the thickness of the wall and not its height.

Next stage will be to calculate theoretical model of the wall where material properties varies in two directions x and y (along wall thickness and height respectively) and compare FEM and BEM results.

We believe that new measurements which will include all possible projections angles allows us to receive results with much better accuracy.

Final model will also include infinite boundary elements to avoid an unknown height of wall humidity and possible related errors. BEM is known to be well suited to handle open boundary models containing infinite elements.

***Authors**: dr inż. Maciej Pańczyk, Politechnika Lubelska, Instytut Informatyki, ul. Nadbystrzycka 36B, 20-618 Lublin, E-mail: m.panczyk@pollub.pl; prof. dr hab. inż. Jan Sikora, Politechnika Lubelska, Instytut Elektroniki i Technik Informacyjnych, ul. Nadbystrzycka 38A, 20-618 Lublin oraz Instytut Elektrotechniki, ul. Pożaryskiego 28, 04-703 Warszawa, E-mail: sik59@wp.pl*

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