Grzegorz KOMARZYNIEC[1]

Lublin University of Technology, Institute of Electrical Engineering and Electrotechnologies (1)

doi:10.15199/48.2016.03.18

Determination of the amount of heat generated by the inrush current in the windings of a superconducting transformer

Streszczenie. *W pracy dokonano analizy ilości ciepła jaka wydziela się przy przepływie prądu włączania przez uzwojenie pierwotne transformatora nadprzewodnikowego. Wyprowadzono wzór na prąd włączania w stanie rezystywnym uzwojenia oraz na podstawie prawa Joule'a wyprowadzono wzór na ilość ciepła wydzielanego w uzwojeniu pierwotnym transformatora. Na tej podstawie wyznaczono charakterystyki zmienność ilości ciepła dla wybranych parametrów pracy transformatora. (**Wyznaczanie ilości ciepła wydzielanego przez prąd włączania w uzwojeniach transformatora nadprzewodnikowego**)*

Abstract. *The text presents an analysis of the amounts of heat generated by the inrush current flowing through the primary winding of a superconducting transformer. A formula to describe the inrush current for the winding in the resistive state is derived. Then, a formula to describe the amount of heat generated in the primary winding of the transformer is derived on the basis of Joule's first law. This formed a basis for determining the characteristics of heat amount variability for selected transformer operation parameters.*

Słowa kluczowe: prąd włączania, transformator, nadprzewodnictwo, ciepło.
Keywords: inrush current, transformer, superconductivity, heat.

Introduction

Transformers with windings made of superconducting materials constitute one of the most promising applications of high-temperature superconductors (HTS). In the case of a failure, replacement or repair of these devices is extremely expensive, due to high prices of superconducting materials. Therefore, a lot of care is taken to ensure protection of superconducting transformers, aimed at minimizing the risk of failure events.

The basic problem – both technological and related to device service – consists in maintaining the windings of superconducting transformers in the superconducting state. The superconducting state will get destroyed if current of excessive density – referred to as the critical current density J_c – flows through a winding and generates magnetic field with the intensity exceeding the critical value of H_c on the surface of the superconductor. Superconductivity also disappears and the superconducting material goes into the resistive state when the winding gets heated above the critical temperature T_c.

The phenomenon of the inrush current constitutes a serious problem. This phenomenon has been known since 1890 [1]. Yet, in spite of the fact that it has been well examined, it still keeps posing problems, both technical and related to device service. When attempting to power a superconducting transformer on, its windings go from the superconducting state to the resistive state, due to the fact that the inrush current exceeds the critical value of the superconductor current I_c. This transition is accompanied with heat effects in the windings, correlated to the inrush current by Joule's first law. The effects may lead to thermal damage of the superconducting winding.

The return of a winding to the superconducting state is conditioned by lowering the instantaneous value of the inrush current below the critical value of the superconductor current I_c, while the winding temperature is simultaneously lowered below the critical temperature T_c. Thus, the amount of heat generated in the windings conditions the rate, at which the superconductor returns to the superconducting state.

Superconducting transformer

The basic difference between conventional transformers and superconducting transformers is the material their windings are made of. Small cross-sections of superconducting tapes allow the height and diameter of windings to be reduced, compared to windings made of copper coil wires. This translates into smaller dimensions of transformer cores, smaller external dimensions and lower weight of superconducting transformers.

From the point of view of applications of superconductors, their most useful property consists in the ability to conduct high currents, accompanied by very low energy losses. Figure 1 shows a comparison of total losses of a conventional transformer (63 MVA, 21 kV/9.09 kV, 50 Hz) and a superconducting transformer with the same rated parameters [2].

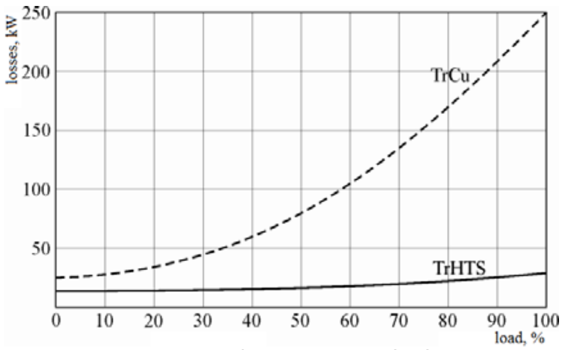

Fig. 1. Energy losses in transformers with HTS i Cu windings [2].

Figure 2 presents an electric diagram of the HTS transformer in an idle state. Resistance values R_1 and R_2 represent the resistance of the primary and the secondary superconducting winding.

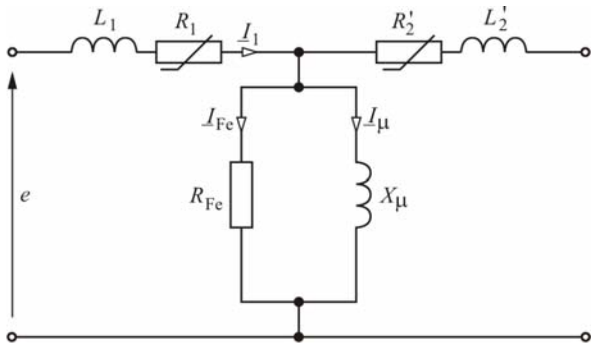

Fig. 2. The equivalent circuit of the superconducting transformer in an idle state

Winding material

Second generation (2G) superconducting wires used to form windings of superconducting transformers are manufactured in the form of tapes with laminar structure (fig. 3, table 1) [3]. The cross-sectional area of a superconductor generally does not exceed 5% of the total cross-sectional area of the tape. A layer of the YBCO or ReBCO superconductor is deposited on a buffer layer. This layer allows the anisotropy of the superconductor to be properly oriented and separates the superconductor chemically from the substrate. The structure of the layer varies, depending on the wire type and its manufacturer. The substrate ensures the tape has got appropriate mechanical and strength parameters. The substrate is made of stainless steel, Hastelloy C-276 or NiW. A layer of silver is deposited above the YBCO layer and under the substrate. This layer ensures good electric and thermal conductivity at temperatures above the critical temperature. The layer of silver may be shunted with a layer of copper. This enhances the ability of the tape to conduct current in the resistive state of the superconductor.

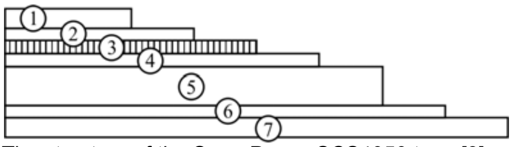

Fig. 3. The structure of the SuperPower SCS4050 tape [3]

Table 1. The structure of the SCS4050 tape [3]

	Material	Thickness	Resistivity at 20°C	Temperature coefficient of resistance
1	Copper	20 μm	$1.72{\cdot}10^{-8}\ \Omega{\cdot}\mathrm{m}$	$3.9{\cdot}10^{-3}\ \mathrm{K}^{-1}$
2	Silver	2 μm	$1.59{\cdot}10^{-8}\ \Omega{\cdot}\mathrm{m}$	$4.1{\cdot}10^{-3}\ \mathrm{K}^{-1}$
3	(RE)BCO	1 μm	–	–
4	Buffer layer	1 μm	–	–
5	Hastelloy C-276	50 μm	$1.26{\cdot}10^{-6}\ \Omega{\cdot}\mathrm{m}$	$1.3{\cdot}10^{-4}\ \mathrm{K}^{-1}$
6	Silver	1.8 μm	$1.59{\cdot}10^{-8}\ \Omega{\cdot}\mathrm{m}$	$4.1{\cdot}10^{-3}\ \mathrm{K}^{-1}$
7	Copper	20 μm	$1.72{\cdot}10^{-8}\ \Omega{\cdot}\mathrm{m}$	$3.9{\cdot}10^{-3}\ \mathrm{K}^{-1}$

An analysis of such a structurally complex wire leads to complicated equivalent circuits that show elements representing resistance and inductance of each layer and describe their interlinking (fig. 4) [4]. In these circuits, the parameters of the superconductor are described with non-linear superconductor resistance R_s, superconductor self-inductance L_{se} and inductance that represents losses L_{si} due to hysteresis in the superconductor. The metallic layer is described with resistance R_b and self-inductance L_b. Parameter M is the measure of the magnetic coupling between the superconductor and the metallic layer.

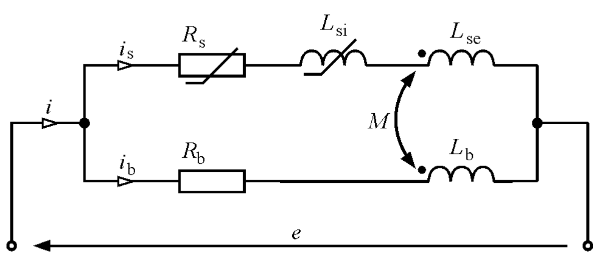

Fig. 4. The equivalent circuit of the superconducting tape [4]

In this article, the parameters of the superconducting tape have been reduced to the equivalent non-linear resistance R of the entire winding of the transformer. This resistance may be easily determined for a transformer by technical methods, for the winding both in the superconducting state and in the resistive state. Superconductor resistance is a non-linear function of current density J, magnetic field intensity H and temperature T. When the circuit is analysed in terms of current values, the transition characteristics of the superconducting winding can be simplified to assume the form presented in figure 5.

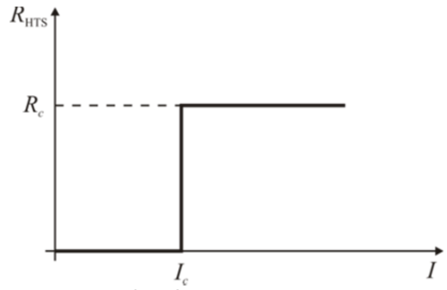

Fig. 5. Characteristics of HTS tape transition

Inrush current

The transient state in an electric circuit coupled with a magnetic circuit is the source of the transformer inrush current. In certain conditions, this state can lead to transformer core saturation and appearance of a unidirectional component of the magnetizing current. At the moment of swathing the transformer on, when the instantaneous value of the power-supply mains voltage is equal to zero and the residual magnetism values are high, the unidirectional current may exceed many times the value of the rated current of the transformer.

At the state of core saturation, the equivalent circuit of the transformer in an idle state is reduced to a serial connection of the resistance R of the primary winding and the inductance L of this winding. Thus, the circuit presented in figure 2 is described with the following formula:

$$e = -\sqrt{2}E\sin\omega t = Ri + L\frac{di}{dt} \qquad (1)$$

Formula (1) is true for the circuit presented in figure (2) on assumption that the active component of the current is passed over to cover the losses in iron, i.e. $R_{Fe}=\infty$, then $R=R_1$ and that $L=L_\mu+L_1$. The initial conditions need to be determined, for formula (1) to be solved and transformed into a formula that yields the current value.

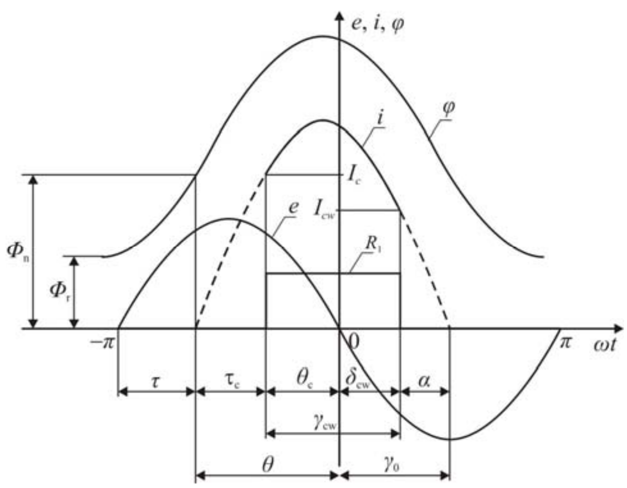

Fig. 6. Inrush current impulse

If it is assumed that the transformer is switched on, when the angle is $-\pi$, then – according to the labels in figure 6 – the unidirectional current appears, when the angle is θ and the core becomes saturated. For angle values within the range from θ to θ_c, the primary winding of the transformer is in the superconducting state.

For angle θ_c, the instantaneous value of the unidirectional current impulse i reaches the value I_c of the critical current of the superconductor and the primary winding of the transformer goes into the resistive state, according to the characteristics presented in figure 5. The winding returns to the superconducting state again at the moment, when the instantaneous value of the inrush current impulse drops down below the critical value I_{cw}, which occurs for angle δ_{cw}.

The current value I_{cw} for which the primary winding of the transformer returns to the superconducting state depends on the heat phenomena occurring in the superconductor. If heat phenomena are disregarded or the efficiency of superconductor cooling is sufficiently high, the value of the critical current I_{cw} equals the current value I_c.

According to Joule's first law, a superconductor in the resistive state is heated up by the impulse of the unidirectional current. The superconductor will not return to the superconducting state, in spite of the fact that the instantaneous value of the unidirectional current is lower than the critical current I_c, of the superconductor, until its temperature becomes lower than critical temperature T_c. Figure 6 presents the value of current I_{cw} for which the superconductor returns to the superconducting state is lower than the value of the critical current I_c for which the superconductor went into the resistive state.

When the cooling system efficiency is low or the thermal capacity of the transformer winding is high, the superconductor may regain the superconducting state only upon waning of the unidirectional current impulse or, in extreme cases, it will not regain this state before the appearance of the next impulse, as shown in figure 7.

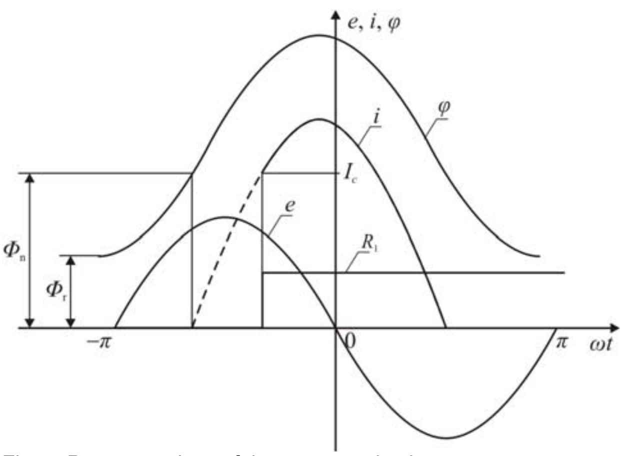

Fig. 7. Permanent loss of the superconducting state

Heat generated in windings

According to Joule's first law, the amount of heat generated in a transformer winding by the unidirectional current may be calculated with the following formula:

$$Q = R\int_{-\theta_c/\omega}^{\delta_{cw}/\omega} i^2 dt \tag{2}$$

where R is the resistance of the primary winding, while i is the inrush current.

In the superconducting state, the resistance R of the primary winding of the transformer equals zero. Therefore, it follows from Joule's first law that heat is not generated. This is the case for the leading edge of the current impulse, within the angle range from θ to θ_c and for the trailing edge of the current, within the angle range from δ_{cw} to γ_0 (fig. 6). Within the angle range from θ_c to δ_{cw}, the winding is in the resistive state. Its resistance is greater than zero. Therefore, it follows from Joule's first law that heat is generated.

In order determine the amount of heat generated in the primary winding due to the flow of the inrush current from Joule's second law presented in formula (2), one should determine the formula that describes the value i of the current impulse within the angle range from θ_c to δ_{cw}. According to the assumptions presented in figure 6, the moment, when the instantaneous value of the unidirectional current assumes the value of the critical current I_c corresponds to angle θ_c. When the general formula of the circuit presented in figure 2, given as formula (1), is solved for the initial conditions $i=I_c$ and $\omega t=-\theta_c$, one gets the formula that describes the unidirectional current in the circuit, when the primary winding of the transformer leaves the superconducting state.

$$i = -\frac{\sqrt{2}EX}{Z^2}\left(\begin{array}{l}\frac{R}{X}\sin\omega t - \cos\omega t + \\ \left(\frac{R}{X}\sin\theta_c + \cos\theta_c\right)e^{-\frac{R}{X}(\omega t+\theta_c)}\end{array}\right) + I_c e^{-\frac{R}{X}(\omega t+\theta_c)} \tag{3}$$

When formula (3) is substituted in formula (2), one can calculate the amount of heat generated in the primary winding of the transformer during the flow of the impulse of the unidirectional current.

$$Q = 2R\frac{E^2X^2}{\omega Z^4}\left(\begin{array}{l}K_c + M_c + N_c \\ +\frac{Z^2}{\sqrt{2}EX}I_c(Q_c + P_c) \\ +\frac{Z^4}{4E^2XR}I_c^2 S_C\end{array}\right) \tag{4}$$

where parameters K_c, M_c, N_c, O_c, P_c i S_c are given with the following formulas:

$$K_c = \frac{Z^2}{X^2}\frac{\delta_{cw}+\theta_c}{2} + \frac{X^2-R^2}{4X^2}\left(\begin{array}{l}\sin(2\delta_{cw}) \\ +\sin(2\theta_c)\end{array}\right) + \frac{R}{2X}\left(\cos(2\delta_{cw}) - \cos(2\theta_c)\right) \tag{5}$$

$$M_c = \frac{X}{2R}\left(\frac{R}{X}\sin\theta_c + \cos\theta_c\right)^2\left(1 - e^{-2\frac{R}{X}(\delta_{cw}+\theta_c)}\right) \tag{6}$$

$$N_c = -2\left(\frac{R}{X}\sin\theta_c + \cos\theta_c\right)\left(\sin\theta_c + e^{-\frac{R}{X}(\delta_{cw}+\theta_c)}\sin\delta_{cw}\right) \tag{7}$$

(8) $$O_c = 2e^{-\frac{R}{X}(\delta_{cw}+\theta_c)} \sin\delta_{cw} + 2\sin\theta_c$$

(9) $$P_c = -\left(\sin\theta_c + \frac{X}{R}\cos\theta_c\right)\left(1 - e^{-2\frac{R}{X}(\delta_{cw}+\theta_c)}\right)$$

(10) $$S_c = 1 - e^{-2\frac{R}{X}(\delta_{cw}+\theta_c)}$$

It follows from formula (4) that the amount of heat generated in the primary winding in the resistive state of the superconductor depends on the value of the R/X ratio, the value of angles θ_c and δ_{cw} and the value I_c of the critical current. The diagrams of heat amount variability as a function of angles θ_c and δ_{cw} are shown in figures 8 and 9. These diagrams are prepared for a hypothetical transformer, on assumption that the R/X ratio equals 0.5 in the resistive state of the primary winding and that the values I_c and I_{cw} of critical currents equal 100 A.

It follows from the presented diagrams that angle θ (fig. 6) – absent from formula (4) – influences significantly the amount of heat generated in the primary winding of an HTS transformer in the resistive state. The value of the R/X ratio for the primary winding in the resistive state cannot be disregarded either. Figure 10 presents the impact of the values of angle θ on the amount of heat generated in the winding for different values of the R/X ratio.

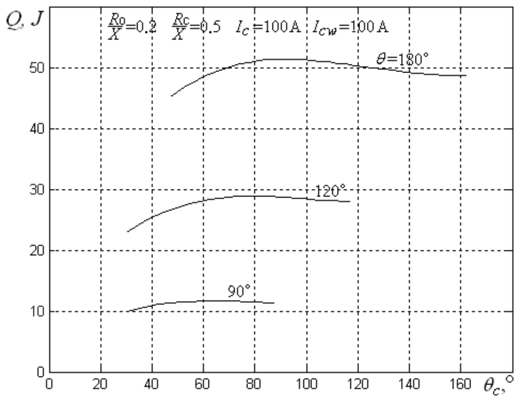

Fig. 8. Amount Q of heat as a function of angle θ_c

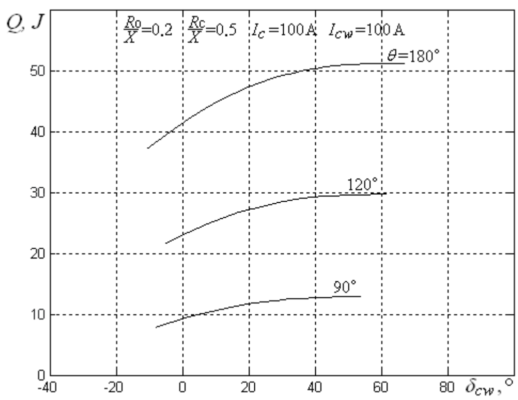

Fig. 9. Amount Q of heat as a function of angle δ_{cw}

Summary

The derived formula (4) allows one to calculate the amount of heat generated in the primary windings of a superconducting transformer by an impulse of the unidirectional current. This interdependence is true, when the windings are in the resistive state. To use the above formula/interdependence, it is necessary to know the value of the R/X ratio for the primary winding of the transformer in the resistive state.

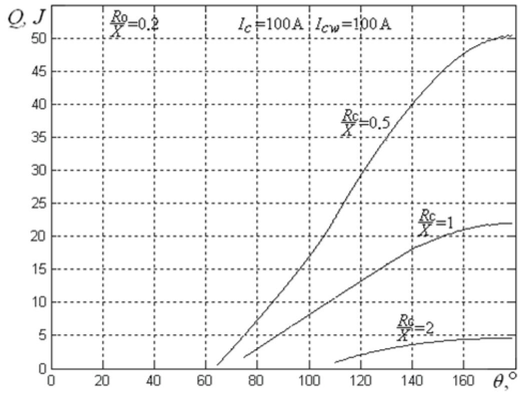

Fig. 10. Amount Q of heat as a function of angle θ for selected values of the R/X ratio

This value may be easily determined by technical methods. It is also indispensable to know angles θ_c and δ_{cw} that are correlated, respectively, with the value I_c of the critical current, at which the winding goes into the superconducting state, and the current I_{cw}, at which the winding returns to the superconducting state. In the conditions, when the winding does not return to the superconducting state, in spite of the fact that the current impulse waned (fig. 7), one should assume the current I_{cw} equals zero in formula (4). To determine the amount of heat generated in the winding from the moment of appearance of the first impulse of the inrush current until its waning, it is required to calculate – on the basis of formula (4) – separately the amounts of heat generated by each current impulse.

The research was realized within the project entitled ""Analysis of inrush current phenomenon and the phenomena related in superconducting transformers"". The project was financed from the means of the National Science Centre, granted pursuant to decision DEC-2012/05/D/ST8/02384.

dr inż. Grzegorz Komarzyniec,
Lublin University of Technology,
Institute of Electrical Engineering and Electrotechnologies,
ul. Nadbystrzycka 38A,
20-618 Lublin
E-mail: g.komarzyniec@pollub.pl

REFERENCES

[1] R. A. Turner, K. S. Smith, Resonance Excited by Transformer Inrush Current in Inter-connected Offshore Power Systems, *IEEE Industry Applications Society Annual Meeting*, Edmonton, Canada, October 2008

[2] A. Berger, S. Cherevatskiy, M. Noe, T. Leibfried, Comparison of the efficiency of superconducting and conventional transformers, *Journal of Physics*, *Conference Series*, vol. 234, part 3, 032004, 2010

[3] S. Lee, V. Petrykin, A. Molodyk, S. Samoilenkov, A. Kaul, A. Vavilov, V. Vysotsky, S. Fetisov, Development and production of second generation high Tc superconducting tapes at SuperOx and first tests of model cables, *Superconductor Science and Technology*, vol. 27, no. 4, 18 March 2014

[4] M. Sjöström, B. Dutoit, and J. Duron, "Equivalent Circuit Model for Superconductors," *IEEE Transactions on Applied Superconductivity*, vol. 13, no. 2, June 2003

[5] G. Komarzyniec, P. Surdacki, J. Kozieł, The calculation of the inrush current peak value of superconducting transformers, *Wybrane Zagadnienia Elektrotechniki i Elektroniki WZEE'2015*, Kielce 2015, p. 61-64