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Statistical characteristics of signals at the output of a receiver with AGC

***Abstract**. An influence of the automatic gain control (AGC) system of a receiver on the statistical characteristics of the output signal in the presence of noise is investigated. The probability density functions (pdf) of the output signals with different distributions of the input signals are calculated. It is shown that energy losses which arise from AGC operation are about 0.2-5 dB depending on probability of false alarm P_{fa} and system parameters.*

Streszczenie.** W pracy rozpatrzono wpływ systemu automatycznej regulacji wzmocnienia (ARW) odbiornika na charakterystyki statystyczne sygnałów na wyjściu w obecności szumu. Obliczono gęstości rozkładów prawdopodobieństwa sygnałów wyjściowych dla różnych rozkładów prawdopodobieństwa sygnałów na wejściu. Pokazano że straty energetyczne, które pojawiają się z powodu funkcjonowania systemu ARW wynoszą wielkość rzędu 0.5-5.0 dB w zależności od prawdopodobieństwa fałszywego alarmu P_{fa} oraz parametrów systemu. **Wpływ systemu automatycznej regulacji wzmocnienia (ARW) odbiornika na charakterystyki statystyczne sygnałów na wyjściu w obecności szumu

Keywords: Automatic gain control systems (AGC), random signal pdf, non-linear transformations of random signals
Słowa kluczowe: Systemy (ARW), rozkłady prawdopodobieństwa sygnałów losowych, nieliniowe przekształcenia sygnałów losowych

Introduction

An automatic stabilization of noise power level is a necessary condition for design of measurement and detection devices in industrial, telemetric and radar applications. This task is solved by using a system of automatic gain control (AGC) [1]. Using of such a system causes an additional energy losses in detection and estimation procedures. This arises from changes of probability density functions (pdf) at the output of the receiver which is controlled by a random signal. Calculation of these pdf and corresponding losses in analog measurement systems is the main aim of the paper. A similar problem occurs in radar systems which use so called CFAR (Constant False Alarm Rate) procedures [2,3]. Such systems usually are realized by digital methods. In literature are published a lot of papers devoted to the analysis of digital methods [2, 3] but there are lack of results concerning analog procedures widely used in industrial measurement systems.

Problem formulation

Let us consider a receiver (Fig.1) with AGC system at the input of which is a random signal *x(t).* The objective of the research is to find a probability density function (pdf) of a random signal *z(t)* at the output of the receiver

A model of the AGC is supposed to be exponential [1] of the following form:

(1) $$K(u) = K_0 \exp\{-bu\}$$

where $K_0$is the receiver gain under *u*=0 and as a linear filter of the AGC system is used an integrator. Then a differential equation of the system (Fig.1) can be written as the following:

(2) $$\dot{u} = A(z - E_0) = A(xK_0 e^{-bu} - E_0)$$

From the other side from expression (1) follows thatK0

(3) $$\dot{K} = -bK\dot{u}$$

Using in equation (2) $\dot{u}$ from (3) we can obtain the following differential equation of the system:

(4) $$\dot{K} + AbxK^2 - AbE_0 K = 0$$

Equation (4) is Bernoulli Equation of which decision is:

(5) $$K[u(t)] = [\frac{e^{-t/T}}{K(u_0)} + \frac{1}{E_0}\int_0^t \frac{1}{T}\exp\{-\frac{t-\tau}{T}\}x(\tau)d\tau]^{-1}$$

where u_o is a control signal of the receiver at the moment *t*=0 and

(6) $$T = (AbE_0)^{-1}$$

is the time constant of equivalent linear filter for the random process *x(t)* [4]

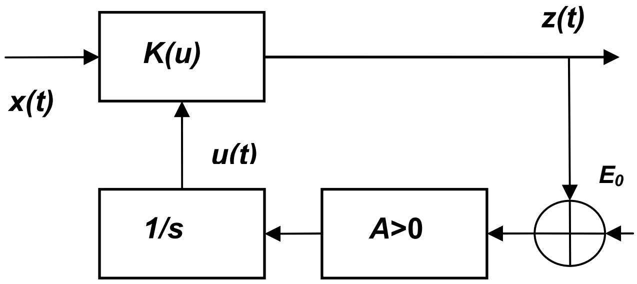

Fig.1. Block diagram of receiver with AGC

The integral in equation (5) is a convolution of the input random process *x(t)* and impulse response *h(t)* of the equivalent linear filter

$$h(t) = \{ \begin{array}{ll} 1/T\exp(-t/T), & t \geq 0 \\ 0, & otherwise \end{array}$$

Therefore processes *x(t)* and *z(t)* are connected by the following input-output relationship:

(7) $$z(t) = \frac{x(t)}{\frac{e^{-bu_0 - t/T}}{K_0} + \frac{1}{E_0}h(t) * x(t)}$$

where symbol (*) is used for convolution.

At a steady state ($t \to \infty$) the expression (7) is simplified and takes the form

(8) $$z(t) = \frac{E_0 x(t)}{h(t) * x(t)}$$

Thus the solution to the problem under consideration can be reduced to non-linear transformation (8) of the input process *x(t)*.

Main results

The analysis is carried out under assumption that filter time constant T is much in comparison with correlation time of the noise process envelope and $\Delta F_e T > 1$,where ΔF_e is the noise spectrum bandwidth. so that at the filter output the process has the normal distribution. This approximation is more accurate for small values of E_0.

We consider here three cases: a) process *x(t)* is a narrow-band Gaussian noise, b) *x(t)* is a sum of harmonic constant amplitude signal and Gaussian noise and c) *x(t)* is a sum of harmonic signal with Rayleigh amplitude fluctuations and Gaussian noise.

First case makes it possible to determine threshold levels which maintain required value of P_{fa} in conditions of changes of the output probability density functions. The last two cases permit to estimate energy losses connected with AGC operation for different signal models.

A. x(t)-narrow-band Gaussian noise

In this case an envelope of narrow-band Gaussian distributed random process *x(t)* has Rayleigh pdf [4]. Using usual procedures of non-linear transformations [4] of random processes we can obtain the following form of pdf for the output signal *z(t):*

(9)
$$w(z) = \frac{1}{\sqrt{2\pi}} e^{-\beta^2/2} \frac{\alpha^2 \beta z}{(z^2+\alpha^2)^2} + \frac{1}{2}\frac{\alpha|z|}{(z^2+\alpha^2)^{3/2}} \times$$
$$\times (1+\frac{\alpha^2\beta^2}{z^2+\alpha^2}) \exp\{-\frac{\beta^2 z^2}{2(z^2+\alpha^2)}\}\{1+\Phi[\frac{\alpha\beta \, sign\, z}{(z^2+\alpha^2)^{1/2}}]\}$$

where

(10) $$\alpha = \sigma_x / \sigma_y, \quad \beta = z_0 / \sigma_z \cong \sqrt{1.5\Delta FT}$$

and

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \int_0^x e^{-\xi^2/2} d\xi$$

In expression (9) characteristics of the input process *x(t)* and the AGC parameters are presented through the values α and β.

The probability of false alarm can be calculated from (9) in the following form

(11)
$$P_{fa}(\beta, \frac{z_t}{\sigma_x}) = \int_{z_t}^{\infty} w(z)dz =$$
$$= \frac{1}{2}\int_0^{\infty} \xi e^{-\xi^2/2}[\Phi(\beta) + \Phi(\frac{\alpha\xi}{z_t} - \beta)]d\xi$$

Data presented in Table.1 make it possible to correct values of threshold z_t / σ_x for obtaining required level of P_{fa} for different *T* and Δ*F.* For small values of β and P_{fa} assumptions about process normalization are not satisfied.

Table 1. Values of threshold level z_t/σ_x

β→	5	10	20	∞
$P_{fa}=10^{-2}$	3.75	3.25	3.08	3.00
$P_{fa}=10^{-4}$	7.25	4.75	4.39	4.25
$P_{fa}=10^{-6}$	--	6.01	5.31	5.12

B. x(t)-is a sum of harmonic constant amplitude signal and Gaussian noise

In this case pdf of Rayleigh-Rice can be approximated by function [4]

$$w(x) = \frac{1}{\sqrt{2\pi}\sigma_x}\sqrt{\frac{x}{x_0}} \exp\{-\frac{(x-x_0)^2}{2\sigma_x^2}\}, \; x \ge 0$$

and corresponding pdf of the output signal takes the following form:

(12)
$$w(z) = \frac{\alpha\sqrt{z}}{\sqrt{\gamma}(z^2+\alpha^2)^{5/4}} \exp\{\frac{(\alpha\beta+\gamma z)^2}{(z^2+\alpha^2)} - \frac{2(\alpha^2+\beta^2)+z^2}{4} \times$$
$$\times [F_1^1(\frac{5}{4};\frac{1}{2};\frac{\alpha\beta+\gamma z}{2(z^2+\alpha^2)})\, 0{,}117 - zF_1^1(\frac{7}{4};\frac{3}{2};\frac{\alpha\beta+\gamma z}{2(z^2+\alpha^2)})0{,}167]$$

where F_1^1 is a hypergeometrical function and $\gamma = x_0 / \sigma_x$.

Thus the probability of signal detection P_d can be found as:

(13) $$P_d(\beta,\alpha,z_t) = \int_{z_t}^{\infty} w(z)dz = \frac{1}{2\sqrt{2\pi}}\int_0^{\infty}\sqrt{\frac{\xi}{\gamma}} e^{-\frac{(\xi-\gamma)^2}{2}} \psi(\beta,\alpha)d\xi$$

where

$$\psi(\beta,\alpha) = \Phi(\beta) + \Phi(\frac{\alpha\xi}{z_t} - \beta)$$

The expression (13) allows to evaluate energy losses due to influence of AGC system on detection procedure. In Tabl.2 these losses in dB are presented for different values of P_{fa} and probability of detection P_d=0.9

Table 2. Energy losses in dB for constant amplitude signal

β→	5	10	20
$P_{fa}= 10^{-1}$	0.5	0.3	0.2
$P_{fa}= 10^{-2}$	1.7	0.4	0.2
$P_{fa}= 10^{-3}$	2.9	0.6	0.2
$P_{fa}= 10^{-4}$	5.0	0.8	0.2
$P_{fa}= 10^{-5}$	--	1.0	0.2

As it follows from data presented in Tabl.2 under small values of P_{fa} energy losses are essential when Δ*FT*≈100. The losses increase for small values of Δ*FT.*

C. x(t)-is a sum of harmonic Rayleigh distributed amplitude signal and Gaussian noise

In the same manner can be evaluated the energy losses when the input signal amplitude is fluctuating according to Rayleigh pdf. In this case pdf of *z(t)* and P_d can be calculated using expressions (9) and (11) where the value α should be change to

$$\alpha\sqrt{1+q} \qquad q = \sigma_s^2 / \sigma_n^2$$

The corresponding losses in dB are presented in Tabl.3 for P_d =0.7 and different values of P_{fa} and β

Table 3. Energy losses in dB for fluctuating signal

β→	5	10	20
$P_{fa}= 10^{-1}$	1.4	0.4	0.1
$P_{fa}= 10^{-2}$	3.5	0.5	0.3
$P_{fa}= 10^{-3}$	--	1.0	0.5
$P_{fa}= 10^{-4}$	--	1.3	1.0

Conclusion

When the AGC system is used in receivers for measurement or detection signals an additional energy losses appear. This arises from changes of probability density functions (pdf) at the output of the receiver which is controlled by random signals. It is shown that losses depend on the level of false alarm probability and AGC filter parameters. The values of these losses are about 0.5-5dB.

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