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doi:10.15199/48.2016.02.50

Mathematical models of electric arc with variable plasma column length used for simulations of processes in gliding arc plasmatrons

***Abstract**. Differential arc models with various length of plasma column have been described. They were modified to obtain electrical processes in plasmatrons with periodic gliding arc discharge. The models were also transformed into integral forms. Macromodels of arcs have been created and simulations of electrical processes in plasmatrons have been carried out. Comparisons of results obtained with experiments known from literature have shown usefulness of the developed mathematical models to simulate gliding arc discharges.*

***Streszczenie.** Opisano modele różniczkowe łuku elektrycznego o zmiennej długości kolumny plazmowej. Dokonano ich modyfikacji w celu odwzorowania procesów elektrycznych w plazmotronach z wyładowaniem ślizgowym powtarzającym się (gliding arc). Przekształcono te modele do postaci całkowej. Utworzono makromodele łuków i wykonano symulacje procesów elektrycznych w plazmotronach. Z porównania uzyskanych wyników ze znanymi z literatury przebiegami eksperymentalnymi wynikła przydatność opracowanych modeli matematycznych do symulowania wyładowań gliding arc. (**Modele matematyczne łuku elektrycznego o zmiennej długości kolumny do symulowania procesów w plazmotronach gliding arc**).*

Keywords: gliding arc, arc model, Mayr model, Cassie model.
Słowa kluczowe: gliding arc, model łuku, model Mayra, model Cassiego.

Introduction

An electric arc in the gliding discharge is a source of thermal and non-thermal plasma. Thermal plasma is characterised by high current density, thermal power and gas temperature. Non-thermal plasma, on the other hand, is characterised by low currents and current densities, possibly leading to current decay, and low gas temperature. The sources of plasma are constructed, powered and controlled in such as way that the gliding discharge is pulsed, with the cycle lasting from over ten to a few tens miliseconds. Gliding typically occurs between two diverging electrodes (Fig. 1).

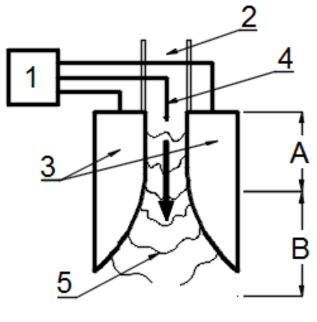

Fig. 1. Diagram of a gliding arc reactor (1 – power supply, 2 – gas supply, 3 – diverging electrodes, 4 – auxiliary electrode, 5 – discharge region, A – thermal equilibrium region, B – thermal non-equilibrium region)

It can be caused by forced gas flow, natural convection, its own or external magnetic field. The arc is struck at the shortest gap between the electrodes by an external factor, such as a high-voltage pulse, auxiliary pulse plasma generator, a mechanical system closing the electrodes for a moment, etc. GA plasma generators are supplied from a DC or AC sources of various frequencies. DC power supply systems are of simple construction and ensure stable discharges. AC-powered plasma generators, on the other hand, are cheaper to produce and simpler to be supplied from the grid. When arc spots are gliding on the electrodes, the arc is elongated and the voltage raises until the discharge becomes unstable. At the moment when the arc is disintegrated and plasma recombinates, the discharge is initiated again at the shortest gap between the electrodes. The properties of the plasma generated strongly depend on the plasma generator input power, which can range from about 100 W up to 40 kW [1]. The higher the power, the greater electromagnetic interference generated into the grid due to shorts, interruptions, transient states, nonlinearity and load asymmetry.

It is difficult to construct simple mathematical models for approximating the characteristics of gliding discharges due to the cyclic character of the discharge and arc elongation.

Modifications of the differential arc models with a variable arc length

The majority of existing arc models describe physical processes in an arc of a constant length. This is sufficient to represent the phenomena occurring in typical power devices in which the arc length is disturbed yet can be considered constant, or varies slowly as compared to the frequency of changes in the electromagnetic processes. Such models, of which the most popular ones are the Mayr and Cassie models, can be applied even for simulating processes of fast arc quenching in electrical apparatuses.

The Mayr model satisfies the energy balance equation and is expressed by

$$\theta_M \frac{dg}{dt} + g = \frac{i^2}{P_M} \tag{1}$$

where: g – arc electrical conductance, i – momentary value of the forcing current; θ_M – Mayr model time constant, P_M – Mayr model constant power. The Cassie model likewise satisfies the energy balance equation and can be expressed as

$$\theta_C \frac{dg^2}{dt} + g^2 = \frac{i^2}{U_C^2} \tag{2}$$

where: θ_C – Cassie model time constant; U_C – Cassie model constant voltage.

There are various methods of modifying the Mayr and Cassie models discussed in the literature [2-4]. Such modifications aim to allow for the external influences on the arc and often require adopting some simplifying assumptions. Consequently upon that, in the physical conditions diverging from those originally stipulated there may be further discrepancies between the actual processes and the predictions of the mathematical model.

Since the heat dissipation processes are relatively insensitive to external disturbance, the loss power can be roughly assumed to be determined by the static characteristics [3] $P_{dys}(t) \approx P_{stat}(i(t))$. Modifying the Mayr model leads to

$$\theta_{NS}\frac{dg}{dt}+g=\frac{i^2}{P_{stat}(i)} \tag{3}$$

where: θ_{NS} – time constant. The function of the loss power can be approximated by means of the modified static voltage-current characteristics $U_{stat}(I)$, and then (4), where $G_{stat}(I) = I/U_{stat}(I)$ – value of static conductance. With

$$P_{dys}(t)=U_{stat}(i)\cdot i=\frac{i^2}{G_{stat}(i)} \tag{4}$$

substituted into (3) and $P_{dys} \approx P_{stat}$, the modified Novikov-Shellhase equation can be obtained

$$\theta_{NS}\frac{dg}{dt}+g=\frac{i}{U_{stat}(i)} \tag{5}$$

Owing to the simplifying assumptions adopted in the model, it does not strictly satisfy the power balance equation, which can allow for arc length variation

$$\theta_{NS}\frac{dg}{dt}+g=\frac{i}{E_{stat}(i)\cdot l} \tag{6}$$

where: $E_{stat}(I) = U_{stat}(I)/l$ – static characteristics of the electric field intensity in the arc; l – arc length. The static voltage-current characteristics of the arc can be approximated by the Ayrton equation

$$U_{stat}(I,l)=A+Bl+\frac{C+Dl}{I} \tag{7}$$

Adopting the Novikov-Shellhase assumption

$$U_{stat}(i,l)=(A+Bl)\cdot \mathrm{sgn}(i)+\frac{C+Dl}{i} \tag{8}$$

where: A, B, C, D – constant approximation factors. Hence the electric field intensity is defined by

$$E_{stat}(i)=\frac{\partial U}{\partial l}=B\cdot \mathrm{sgn}(i)+\frac{D}{i} \tag{9}$$

The Berger-modified arc equation with the Cassie variable voltage $U_C(l)=U_C(l(t))$ leads to the conductance form

$$\theta_{CB}\frac{dg^2}{dt}+g^2=\frac{i^2}{u_C^2(l)} \tag{10}$$

where: θ_{CB} – time constant. Since the arc voltage increases with the increase in arc length, in [5] a method of determining the component of the voltage square in the Cassie-Berger model was offered

$$u_C^2(l)=al \tag{11}$$

with the parameter a [V^2/m] being almost constant in the wide interval of current i variation.

As indicated by experimental results [6, 7], the damping factor function is strongly nonlinear and depends on a number of parameters. For instance, it falls with an increase in current or in gas flow. Theoretical analyses of some models [8] in turn demonstrate that increase in current causes increase in the arc cross-section and because of that the function should rise too. The real shape of the damping function is in fact affected by phenomena associated with nonlinear gas characteristics, such as viscosity and heat transfer coefficient [9].

When the arc is supplied from a nearly ideal current source, the arc cross-section does not depend on its length. The Cassie model time constant $\theta(i, l) = \theta(i)$ should not depend on it, either. If, on the other hand, the arc is supplied from a real current source (close to a real voltage source), variation in arc length is accompanied by variation in current and arc diameter. Because of this, the dependence $\theta(d(i(l)))) = \theta(l)$ holds and the quantity θ referred to as the time constant is often assumed to be dependent on the arc length [2]

$$\frac{\theta}{\theta_0}=\left(\frac{l}{l_0}\right)^{\gamma} \tag{12}$$

where: γ - constant coefficient (in short circuit arcs γ = 0.4); θ_0 – initial value of the time constant corresponding to l_0; l_0 – initial arc length. In the Schwarz-Avdonin models [10], the values of the parameters are assumed to be dependent on the arc conductance ($P_M(g)$ and $\theta_M(g)$ or $U_C(g)$ and $\theta_C(g)$), which in turn can be a function of arc length.

In publications [1, 11-13] a modification of the Mayr and Cassie models was put forward, consisting in adding a discharge ignition function. An analogous effect can be obtained by including an additional term including voltage forcing in the input equations (6) and (10) to decrease conductance. In the Novikov-Shellhase model, this will be

$$\frac{1}{g}\frac{dg}{dt}=\frac{1}{\theta_{NS}}\left(\frac{u}{E_{stat}(i)\cdot l}-1\right)-\frac{\zeta(t_l)}{\theta_{NS}}\left(\frac{u_z}{E_{stat}(i)\cdot l}-1\right) \tag{13}$$

where

$$\zeta(t_l)=\begin{cases}1, & \text{if}\quad t_z<t_l\le t_1\\ 0, & \text{if}\quad t_1<t_l<t_2\end{cases} \tag{14}$$

where: t_l – local time of a single discharge cycle; t_z – moment of ignition, t_z=0s; t_1 – duration of an ignition pulse; t_2 – duration of a cycle defined by the condition ($t_2=t_l \leftrightarrow g(t_l)=g_{\min}$); $g_{\min}$ – minimal prescribed value of conductance just before the arc quenching; u_z – voltage of the auxiliary ignition source.

The modified Cassie-Berger model is as follows

$$\frac{1}{g^2}\frac{dg^2}{dt}=\frac{1}{\theta_{CB}}\left(\frac{u^2}{u_C^2(l)}-1\right)-\frac{\zeta(t_l)}{\theta_{CB}}\left(\frac{u_z^2}{u_C^2(l)}-1\right) \tag{15}$$

In periodic gliding discharges a small value of g can signal a step change in the arc length and initiation of a new discharge.

Directly after each initiation, the arc is the shortest, the voltage the lowest and the current the highest. If the mass of the gas flux is relatively low, the environment temperature is high, time constant also high and the static characteristic correspond to Cassie arc model, due to reduction of the breakdown voltage of the gas. When the arc is the longest, the voltage the highest and the current the lowest, the arc properties can be adequately represented by the Mayr model. The arc is typically cooled most intensively in the initial section of the discharge channel where it is narrowed. The metal electrodes are also cooled intensively and prone to ionisation metal vapours are constantly removed from the plasma generator chamber. Because of that, the voltage-current characteristics tend to fall steeply and the Mayr model is preferred as an account of the gliding arc discharge.

The approximations applied to the accounts of the physical phenomena occurring in electric arcs can cause discrepancies in the assessment of discharge conditions. This is especially important in modelling extensively elongated arcs. Arc quenching corresponds to the conductance approaching zero, which causes a peculiarity in numerical solutions, breaks the computations and the simulation program.

Modifications of the integral models of the elongated arc

As can be seen on the basis of Eqs (1) and (2), the differential Mayr and Cassie equations are linear with respect to the variables g and g^2. It is difficult to assess the stability of solutions of mathematical models of strongly nonlinear electric circuits with excitations and arc discharges. Besides, in GA devices it is necessary to take into account arc quenching and electric disturbances applied to induce new ignitions and a step change in the discharge location. A method for simplifying the analysis and simulation offered in [11-13] involves modifications of the analytic solutions of the Schwarz-Avdonin models [10], and a subsequent utilisation of macromodels in the form of controlled two-clamp elements in the electric circuits.

The solution to the Mayr equation, expressed as (1), is the formula for momentary conductance

$$g = g_0 \exp\left(\frac{1}{\theta_M} \int \left(\frac{ui}{P_M} - 1 \right) dt \right) \tag{16}$$

where g_0 – initial value of the conductance. If the same set of assumptions is adopted as was for the Novikov-Shellhase model (6), then will be obtained

$$g = g_0 \exp\left(\frac{1}{\theta_{NS}} \int \left(\frac{u}{E_{st}(i) \cdot l} - 1 \right) dt \right) \tag{17}$$

The solution to the Cassie equation, expressed as (2), is a formula for momentary conductance

$$g^2 = g_0^2 \exp\left(\frac{1}{\theta_C} \int \left(\frac{u^2}{U_C^2} - 1 \right) dt \right) \tag{18}$$

If the same assumptions hold as for the Berger model (10), then

$$g^2 = g_0^2 \exp\left(\frac{1}{\theta_{CB}} \int \left(\frac{u^2}{u_C^2(l)} - 1 \right) dt \right) \tag{19}$$

In publications [1, 11-13] it was suggested that Eqs (16) and (18) could be modified by taking into account an additional function which represents the discharge ignition. It can also be applied as an additional interfering component, corresponding to the character of the ignition in solutions (17) and (19). The new solutions will be:

- Novikov-Shellhase model

$$g = g_0 \exp\left[\int \left(\frac{1}{\theta_{NS}} \left(\frac{u}{E_{st}(i) \cdot l} - 1 \right) - \frac{\zeta(t_l)}{\theta_{NS}} \left(\frac{u_z}{E_{st}(i) \cdot l} - 1 \right) \right) dt \right] \tag{20}$$

- Cassie-Berger model

$$g^2 = g_0^2 \exp\left[\int \left(\frac{1}{\theta_{CB}} \left(\frac{u^2}{u_C^2(l)} - 1 \right) - \frac{\zeta(t_l)}{\theta_{CB}} \left(\frac{u_z^2}{u_C^2(l)} - 1 \right) \right) dt \right] \tag{21}$$

Changes that the arc length undergoes in time can be described by a formula analogous to [13]

$$l(t_l) = l_0 + k\alpha v_g t_l \tag{22}$$

where: l_0 – gap between the electrodes at the location where the arc is struck; α - divergence angle between the electrodes; v_g – velocity of the gas flow forcing the discharge motion; k - correction factor. The speed of the arc motion can be also changed by applying an external magnetic field. In the models (15) and (21), the Cassie voltage is given by [10]

$$u_C^2(l) = a \cdot (l_0 + k\alpha v_g t_l) \tag{23}$$

Computer simulations of the electric quantities in circuits with various arc models in GA plasma generators

The processes occurring in the GA plasma generator were simulated by means of the MATLAB-Simulink programme. First, a macromodel was created of a simple supply circuit with a real sinusoid voltage source, connected in series with a macromodel of a variable-length electric arc. As in the majority of other analyses [3, 4], it was assumed constant damping factors (time constants). Small and constant values of θ correspond to very intensive cooling of the arc.

In the first case under scrutiny, a modified differential Novikov-Shellhase arc model was considered (13), with the following parameters: B = 2 V/m; D = 26 W/m; θ_{NS} = 0.8·10^{-3} s. From the assumptions concerning the construction and operation of the plasma generator it followed that l_0 = 3 mm; α = 25°; v_g = 10 m/s, k = 1, t_z = 3·10^{-3} s, g_{min} = 1·10^{-4} S. This apparatus was supplied from a source of the following parameters: U_m = 1500 V, R_w = 250 Ω, L = 50mH, f = 50 Hz, U_z = 2500 V. The results of the simulation are shown in Fig. 2. Comparing them to the simulation and experimental data discussed in [11-13], we can see that the model offered meets the requirements concerning the accuracy of approximating the voltage and current waveforms in GA plasma generators, especially of low power.

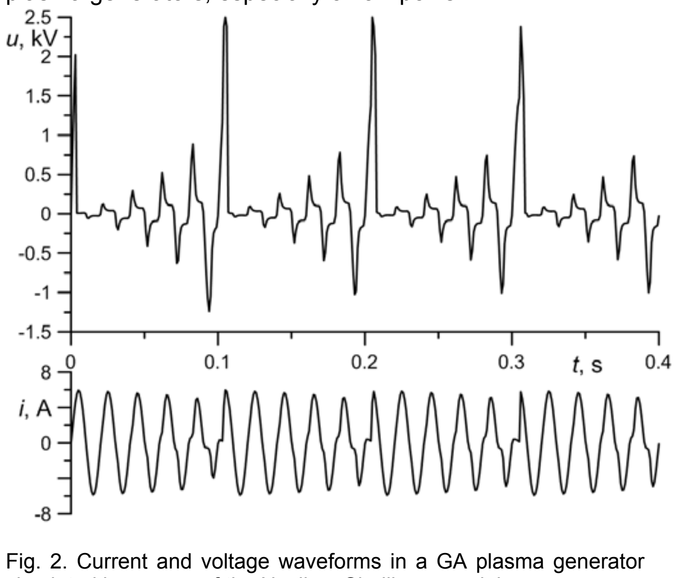

Fig. 2. Current and voltage waveforms in a GA plasma generator simulated by means of the Novikov-Shellhase model.

In the second case, a modified differential Cassie-Berger arc model (15) was considered, with the following parameters: a = 400 V^2/m; θ_{CB} = 1.1·10^{-4} s. The assumptions concerning the construction and operation of the plasma generator gave rise to the following: l_0 = 3 mm;

α = 25°; v_g = 10 m/s, k = 1, t_z = 3·10^{-3} s, $g_{\min}$ = 1·10^{-3} S. The apparatus can be supplied from a source with: U_m = 500 V, R_w = 7 Ω, f = 50 Hz, U_z = 500 V. The results of the simulation are presented in Fig. 3. Comparing them to the simulation and experimental data discussed in [11-13] we can see again that the model also meets the requirements concerning the accuracy of approximating the current and voltage waveforms in GA plasma generators, especially of high power.

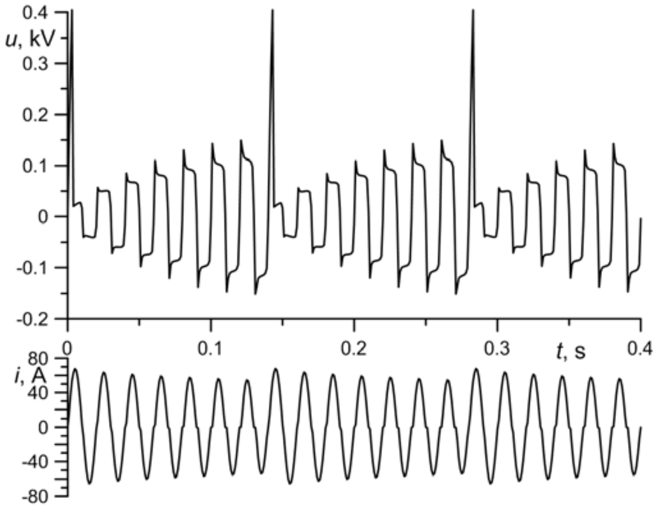

Fig. 3. Current and voltage waveforms in a GA plasma generator simulated by means of the Cassie-Berger model.

Simulations of the GA plasma generator were also performed with the use of the Novikov-Shellhase (20) and Cassie-Berger (21) integral models. Assuming the same set of parameters as it was done in the differential models, we obtain identical results as those presented in Figs 2 and 3.

Conclusions

1. The differential models of GA offer an intuitive approach to simulating the processes, with a relatively small number of parameters. They are also easy to interpret in physical terms.
2. The integral models of the GA discharge match the differential ones in their functionality. They also provide an alternative way of a computer implementation but do not in fact offer any advantages over the differential ones.
3. The mathematical models of the GA discharge presented are valid for plasma generators of low and high power, which results in their universality and potentially broad spectrum of applications.

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