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Modeling of core losses of switched reluctance motor

Abstract. *The paper deals with a mathematical model of switched reluctance motor, with stator of on induction motor. The parameters of this model were determined experimentally. The motor winding were supplied from different frequencies AC voltage source. Based on the measurements of current and voltage equivalent resistance and inductance of motor windings as a function of frequency were determined using the least squares method. The measurements for different rotation angle of the rotor were carried out. The measured resistance value was significantly higher than the value of the winding resistance because of "iron losses". The losses are modeled as the losses of a single turn wound on the rotor core, representing the eddy currents circuit. The losses are dependent on the angle of the rotor rotation relative to the stator. Based on the relationship between the equivalent inductance and resistance of the motor measured and specified of the model, the parameters were determined.*

Streszczenie*. W pracy przedstawiano model matematyczny przełączalnego silnika reluktancyjnego wykorzystującego stojan trójfazowego silnika indukcyjnego. Parametry tego modelu określono eksperymentalnie. Uzwojenie jednej z faz silnika zasilano napięciem przemiennym o różnych częstotliwościach. Na podstawie pomiarów prądu i napięć wyznaczano zastępczą rezystancję i indukcyjność poszczególnych uzwojeń silnika wykorzystując metodę najmniejszych kwadratów dla różnych częstotliwości. Pomiary prowadzono w funkcji kąta obrotu wirnika. Otrzymana rezystancja była istotnie większa od wartości stało-prądowej rezystancji uzwojeń. Dlatego dodatkowo do modelu wprowadzono „straty w żelazie". Przedstawiono je jako straty w pojedynczym zwartym zwoju, nawiniętym na rdzeniu wirnika, reprezentującym prądy wirowe. Straty te są zależne od kąta obrotu wirnika względem stojana. Na podstawie relacji między indukcyjnością i rezystancją zastępczą silnika, zmierzoną i określoną dla przyjętego modelu wyznaczono parametry tego modelu. Przedstawiono parametry dodatkowego uzwojenia reprezentującego straty w żelazie.* ***(Modelowanie strat w rdzeniu przełączalnego silnika reluktancyjnego****).*

Keywords: switched reluctance motor, iron losses, identification.
Słowa kluczowe: silnik reluktancyjny, straty w żelazie, identyfikacja.

Introduction

The energy losses in the motor are divided into power losses: in the copper, magnetic in core, and the mechanical. In [1-3] also introduced additional losses that are related to skin effect and high frequency. However, they have little importance during the operation of the electric motor. Their estimation is difficult using on direct measurements or calculations.

The first attempts to describe magnetic losses have focused on formulating a mathematical description of losses in the core. The method of calculating the losses proposed Steinmetz in 1891 [4]. These losses are attributable to 1 cm3 volume of the material and determined by the equation in the form [5]:

$$P_F = \varepsilon \cdot f^2 \cdot B_m^2 \quad (1)$$

where: ε – coefficient of eddy current, f – frequency, B_m – peak value of the magnetic induction.

These losses are proportional to the peak value of induction and frequency. In order to more accurately determine the magnetic loss over the years the equations describing the losses in the core, which were a modification of the Steinmetza equations [6-7] were formed. The equation allowed to calculate the power losses in the motor, as a constants dependent of magnetic induction. Other methods are focused on the development of new materials testing procedures. They facilitate accurate description of the magnetic material properties to describe the core losses in the equations of motion. However, the exact determination of the core losses remains difficult for many reasons [6].

In summary, the equations found in the literature describes the average parameters which do not reflect properly phenomena in the magnetic core. On the basis of the relationships and analysis of the phenomena the iron losses a new model has been proposed in [8]. The model contains the equivalent circuit of the iron losses for instantaneous values. On the basis iron losses model of a three-phase switched reluctance motor was worked out. Identification of the model parameters was carried out using two method. A comparison of the measured directly equivalent parameters with the parameters determined from frequency characteristics of the model is presented.

Modeling of losses in reluctance motor

Mathematical models of electric motors consist of equations describing electric circuits and mechanical system. The equations in the literature describing the switched reluctance motor does not explicitly take into account the losses in the mathematical model. They are hidden in the resistance of the circuit.

The equivalent circuit of three-phase reluctance motor taking into account the eddy current losses has been proposed in [8] - figure 1.

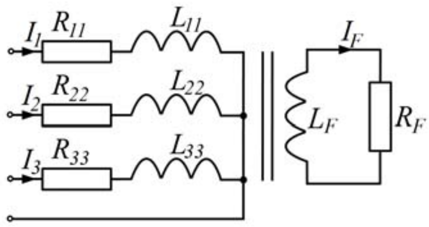

Fig.1. The analytical model of eddy current losses of three-phase reluctance motor 3/2

The model presented in figure 1 shows the equivalent circuit of a three-phase reluctance motor with a winding resistances R_{nn} and inductances L_{nn} which have z number of turns of the transformer. Secondary winding is representing the iron losses, having a resistance R_F and inductance L_F. It was assumed that winding representing eddy currents has a single turn. The parameters describing the model are functions of the rotation angle. Winding of the iron losses is magnetically coupled with inductances of the three-phases of the stator. Winding of the iron losses is loaded by resistance R_F. The resistance depends on the construction of the core and the properties of the material from which it is made.

The model presented in figure 1 is described in matrix form as follows:

$$(2)\quad \begin{bmatrix} L_{11} & M_{12} & M_{13} & M_{F1} \\ M_{21} & L_{22} & M_{23} & M_{F2} \\ M_{31} & M_{32} & L_{33} & M_{F3} \\ M_{1F} & M_{2F} & M_{3F} & L_F \end{bmatrix} \cdot \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_F \end{bmatrix} + \begin{bmatrix} R_{11} & 0 & 0 & 0 \\ 0 & R_{22} & 0 & 0 \\ 0 & 0 & R_{33} & 0 \\ 0 & M_{2F} & 0 & R_F \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_F \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ 0 \end{bmatrix}$$

where: L_{nn} – self-inductances of motor windings as a function of rotation angle, M_{nm}– mutual inductances of the stator as a function of rotation angle, M_{Fn}– mutual inductance losses as a function of rotation angle, R_{nn} – motor winding resistance as a function of rotation angle, n,m– 1, 2, 3.

The currents I_1, I_2, I_3 flowing through the motor windings are currents which may be measured easily. However direct measuring of the current I_F flowing in motor core is impossible. For the mathematical model the motor current I_F will occur in each equation of phase circuit of the motor. If only the first phase of the motor is supplied by voltage source, the equations (2) may be presented in the form:

$$(3)\quad \begin{aligned} L_{11}\dot{I}_1 + M_{F1}\dot{I}_F + R_{11}I_1 &= U_1 \\ M_{21}\dot{I}_1 + M_{F2}\dot{I}_F &= U_{12} \\ M_{31}\dot{I}_1 + M_{F3}\dot{I}_F &= U_{13} \\ M_{1F}\dot{I}_1 + L_F\dot{I}_F + R_F I_F &= 0 \end{aligned}$$

The voltage U_{12} and U_{13} are only observed. The system described by equation (3) has 9 parameters. Some of them cannot be measured due to the inability to direct observation of currents and voltages of the coil which model the iron losses circuit. Therefore, the system of equations (3) was converted, so as to eliminate from the equation describing the core losses current. First, Laplace transform was applied and the I_F current from the last equation was determined.

Quantity ($j\omega$) denotes Fourier transform. The equations of the motor with one winding powered after substitution the I_F variable into equations and by using Fourier transform are as follows:

$$(4)\quad \left(j\omega\left(L_{11} - \frac{L_F M_{1F}^2 \omega^2}{R_F^2 + \omega^2 L_F^2} \right) + R_{11} + \frac{R_F M_{1F}^2 \omega^2}{R_F^2 + \omega^2 L_F^2} \right) I_{1(j\omega)} = U_{1(j\omega)}$$

$$(5)\quad \left(j\omega\left(M_{21} - \frac{L_F \omega^2 M_{F2} M_{1F}}{R_F^2 + \omega^2 L_F^2} \right) + \frac{R_F \omega^2 M_{F2} M_{1F}}{R_F^2 + \omega^2 L_F^2} \right) I_{1(j\omega)} = U_{12(j\omega)}$$

$$(6)\quad \left(j\omega\left(M_{31} - \frac{L_F \omega^2 M_{F3} M_{1F}}{R_F^2 + \omega^2 L_F^2} \right) + \frac{R_F \omega^2 M_{F3} M_{1F}}{R_F^2 + \omega^2 L_F^2} \right) I_{1(j\omega)} = U_{13(j\omega)}$$

where: U_1 is the voltage supplied on the winding, U_{12} and U_{13} are induced voltages of the unpowered windings.

On the basis of (4) equivalent self-inductance L_{z11} and equivalent resistance R_{z11} of powered circuit are derived in the form:

$$(7)\quad R_{z11} = R_{11} + \frac{R_F M_{1F}^2 \omega^2}{R_F^2 + \omega^2 L_F^2}$$

$$(8)\quad L_{z11} = L_{11} - \frac{L_F M_{1F}^2 \omega^2}{R_F^2 + \omega^2 L_F^2}$$

The above quantities may be directly described using current I_1, current derivation dI_1/dt and voltages U_1 measurements with the least square method from the equation:

$$(9)\quad L_{z11}\dot{I}_1 + R_{z11}I_1 = U_1$$

From the above relations, it follows that the equivalent inductance with respect to the motor winding inductance is reduced by losses in the core. However the equivalent resistance is sum of the winding DC resistance and resistance of losses in the core. Equivalent mutual inductances M_{z21} and M_{z31} and equivalent resistances R_{z21} and R_{z31} may be measured in the unpowered windings are as follows and after pi/6 shift angle.

$$(10)\quad M_{z21} = M_{21} - \frac{L_F \omega^2 M_{F2} M_{1F}}{R_F^2 + \omega^2 L_F^2} \qquad M_{z31} = M_{31} - \frac{L_F \omega^2 M_{F3} M_{1F}}{R_F^2 + \omega^2 L_F^2}$$

$$(11)\quad R_{z21} = \frac{R_F \omega^2 M_{F2} M_{1F}}{R_F^2 + \omega^2 L_F^2} \qquad R_{z31} = \frac{R_F \omega^2 M_{F3} M_{1F}}{R_F^2 + \omega^2 L_F^2}$$

Measurements of the instantaneous values voltages on the other phase windings of the U_{12} and U_{13} from the equation (12) mutual inductance were determined.

$$(12)\quad M_{z12}\dot{I}_1 + R_{z12}I_1 = U_{12} \qquad M_{z13}\dot{I}_1 + R_{z13}I_1 = U_{13}$$

It should be noted that the core losses resistance is observable in the voltages of unpowered windings. It results of current flow in the motor magnetic core.

Identification of electrical parameters in three-phase reluctance motor

To identify the parameters the test bench with the three-phase reluctance motor was constructed. The stator of the motor had three latent stator poles and a rotor only one pair of poles with an angle of 120 degrees is shown in figure 2.

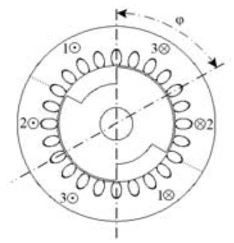

Fig. 2. View of reluctance motor cross section

In order to specify the inductance and stator resistance as a function of rotation angle of the rotor three-phase reluctance motor in a static state ($d\varphi/dt$ = 0), identification of the parameters as a function of the rotor rotation angle was carried out. The motor winding was supplied by from AC voltage source with frequencies in the range of 40 Hz to 100 Hz - figure 3..

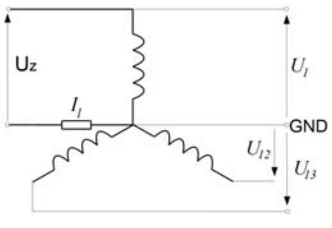

Fig. 3. Connection diagram of motor during measuring of single winding parameters

A block diagram of the system for determination the motor parameters is presented in figure 4. In order to calculate motor various parameters the instantaneous values of voltage U_1 and current I_1 flowing in the powered winding were used. The values were digitized using NI 9225 board. In order to determine the rotation angle of the rotor used the board NI 6216. The boards are serviced by a LabView program installed on the computer where the data are recorded in a text data file. The data files are loaded into MATLAB and processed using Golay- Savitzky filter to eliminate noise and calculate derivatives [9].

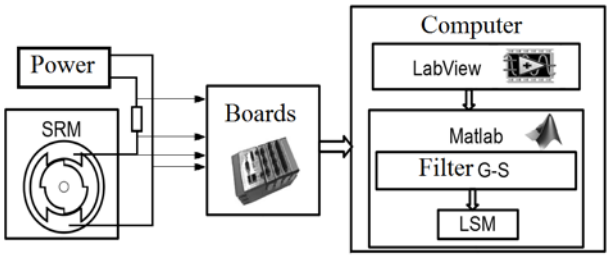

Fig. 4. The block diagram of the reluctance motor parameter measuring system

The current and voltage on the motor windings was measured with a step 5 degrees. Additionally by calculating the derivative of the current and using equation (9), characteristics of equivalent inductance as a function of rotation angle are presented in figure 5. Analogously characteristics of equivalent resistance as a function of rotation angle are presented in figure 6. The characteristics of equivalent inductance and resistance for the frequencies in the range of 40 Hz to 100Hz are shown.

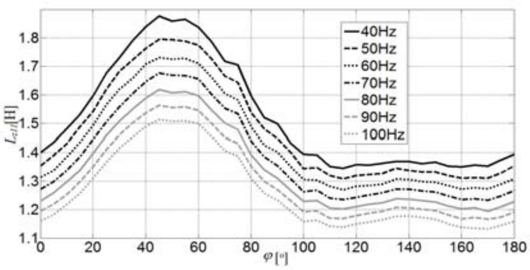

Fig. 5. Equivalent self-inductance L_{z11}

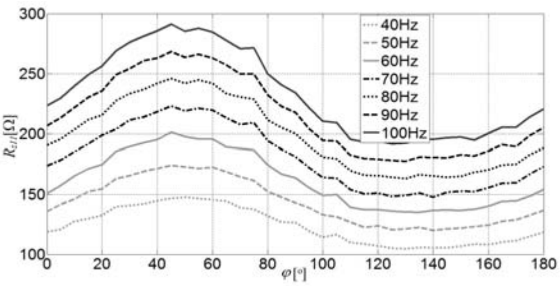

Fig. 6. Equivalent resistance R_{z11}

Supplying the motor winding DC voltage, resistance of the coil R_{11} was determined. Its value is 62,5 Ω. It can be seen, that the measured the motor windings resistances are several times larger than the DC winding resistance. The resistance value increases with increasing frequency. While the inductance value decreases with frequency increase. Rewriting the (7) yields the form:

$$\frac{1}{R_{z11}-R_{11}}=\frac{\nu^2+\omega^2}{\nu\,\xi\omega^2}=\frac{\nu}{\xi}\cdot\frac{1}{\omega^2}+\frac{1}{\nu\,\xi} \tag{13}$$

where: $\xi = M_{1F}^2 / L_F$, $\nu = R_F / L_F$.

Basing on the relation of resistance R_{z11} on frequency „ω” the parameters ν/ξ and $1/(\nu\cdot\xi)$ may be determined. The square root of the coefficients product determines the ξ value. Similarly, the ratio of these values defines the coefficient ν. In figures 7 and 8 characteristics ξ and ν as a function of the rotation angle are presented.

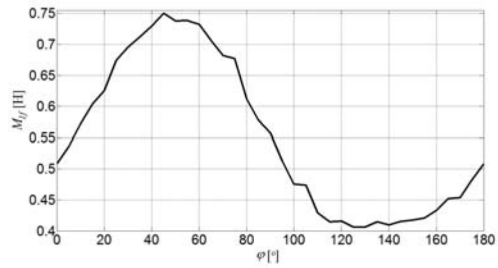

Fig.7. Characteristics ξ as a function of the rotation angle

The coefficient ν has a high value. Taking into account the coefficient ν in the equations of (7-8) and (10-11), will be occur in the denominator in a square. For this reason values of the coefficient ν should be as large as possible. When, the core inductance L_F should be as small as possible.

Coefficient ν may be taken as a constant, independent of the rotation angle. In the figure 9 is presented the equivalent resistance R_{z11} as a function of the rotation angle obtained from the direct measurement, calculated from the determined for 50Hz parameters and for the constant ν. The correlation coefficient between measured and calculated resistance equals 0.999 in whole frequency range. For other frequencies, the value of the correlation coefficient changes in fourth place after the decimal point.

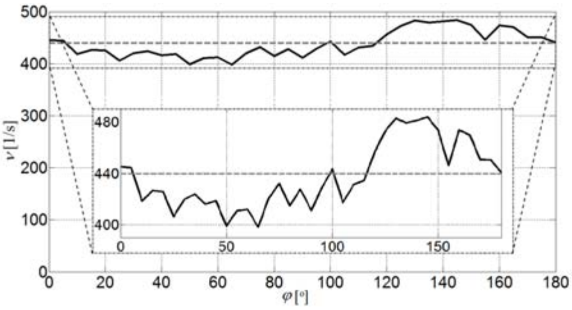

Fig.8. Characteristics ν as a function of the rotation angle

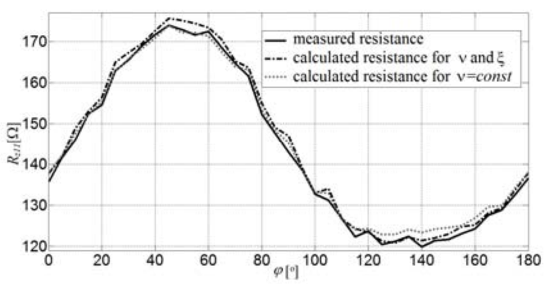

Fig.9. Equivalent resistance for a frequency 50Hz

At low frequencies the inductance is constant. Therefore, the inductance L_{11} will not depend on the frequency but only on the angle of rotation. Transforming of equations (7) and (8) and using a constant value of ν coefficient of the self inductance L_{11} was determined. Using the last square method, the value of the self inductance L_{11} may be determined from the relationship:

(14) $$L_{11} = \frac{1}{7}\sum_{n=1}^{7}\left(L_{z11n} + \frac{1}{\nu}\left(R_{z11n} - R_{11}\right)\right)$$

where: n – number of components vectors R_{z11} and L_{z11} in range from 1 to 7 for the frequency [40, 50, 60, 70, 80, 90, 100]Hz.

In figure 10 characteristics of the self-inductance L_{11} as determined from equation (14). Characteristics of the equivalent self-inductance L_{z11} as function of the rotation angle were obtained from direct measurements and based on the determined parameters for the frequency of 50 Hz are presented. The correlation coefficient between measured and calculated inductance for the each frequency is equal 0.999.

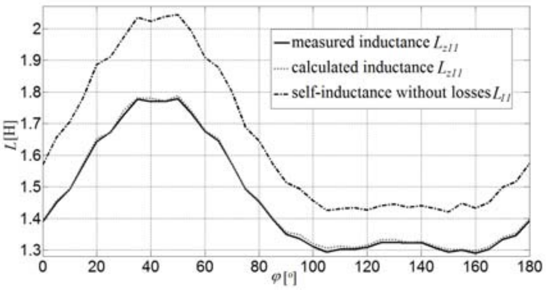

Fig.10. Self-inductance L_{11} and equivalent inductance L_{z11} for a frequency 50Hz

Transforming equations (10) and (11) similarly as the equations (7) and (8) to the mutual inductance is obtained as in form:

(15) $$M_{12} = \frac{1}{7}\sum_{n=1}^{7}\left(M_{z12n} + \frac{1}{\nu} R_{z12n} \right)$$

The magnetic flux generated by current flow in one coil is coupled with the second coil windings. The degree of inductive coupling of the two coils of the magnetic circuit is characterized by a magnetic coupling coefficient.

In figure 11 characteristics of the equivalent mutual inductance M_{z12} as function of the rotation angle obtained from the direct measurements and calculated from the determined parameters for 50Hz are presented. The correlation coefficient between measured and calculated mutual inductance for the each frequency is equal 0.999.

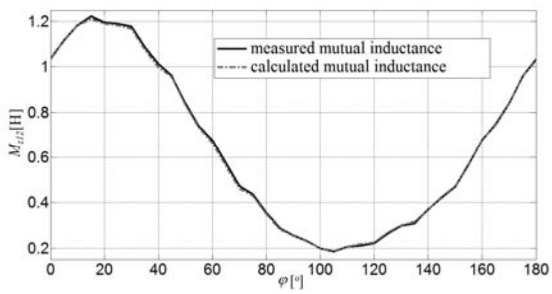

Fig.11. Equivalent mutual inductance M_{z12} for a frequency 50Hz

In figure 12 characteristics of an equivalent resistance R_{z12} visible on unpowered windings as function of the rotation angle obtained from the measurements and calculated from the determined parameters for 50Hz are presented. The correlation coefficient between measured and calculated resistance for the each frequency is equal 0.996.

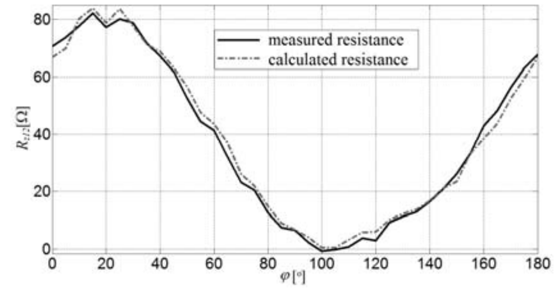

Fig.12. Additional equivalent resistance R_{z12} for a frequency 50Hz

Summary

The paper presents a simple model of the magnetic losses and method of it determination for SRM, on the base of instantaneous values of the current and voltages on the motor windings.

A model of a three-phase switched reluctance motor which takes into account the losses in the iron was analyzed. Core losses model is carried out by additional equivalent single-turn winding. The single-turn supplies the resistance modeling eddy currents in magnetic core. The resistance depends on the electric parameters of the core. The equivalent resistance is proportional to the equivalent inductance. Hence the resistance is also a periodic function of the rotor rotation angle.

The motor parameters were obtained using two methods. The results of the methods were compared. The results are highly correlated. On the basis of the results the proposed mathematical model well represents the magnetic losses in the core of SRM..

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