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Influence of significant radar system parameters on resolution of image in Doppler Radar Tomography

***Abstract**. Doppler radar tomography is a method of creation of a target image from Doppler profiles of a rotating target. Electromagnetic backscattering from rotating objects generates time-varying Doppler spectra which can be a base of tomographic projections in the time-frequency approach. In this paper the influence of projections and significant radar system parameters on the final image resolution is considered. The results of simulations of the proposed imaging method are also presented.*

Streszczenie.** Dopplerowska tomografia radarowa jest metodą formowania obrazów obracających się obiektów radarowych z profili Dopplerowskich. Rozpraszanie fali elektromagnetycznej przez rotujacy obiekt powoduje powstanie czasowo zmiennych charakterystyk częstotliwościowych, które są bazą rzutów tomograficznych w podejściu czas-częstotliwość. Rozważono wpływ jakości rzutów tomograficznych oraz istotnych dla zobrazowania parametrów systemu radarowego na rozdzielczość tworzonego obrazu. Przedstawiono również rezultaty symulacji.* ***Wpływ parametrów radaru na rozdzielczość obrazu Dopplerowskiej Tomografii Radarowej

Keywords: Radar tomography, radar signal processing, time-frequency transform.
Słowa kluczowe: Tomografia radarowa, przetwarzanie sygnałów radarowych, transformacje czas-częstotliwość.

Introduction

Electromagnetic (EM) backscattering from moving objects is subjected to different modulations of Doppler spectra [2, 3]. These characteristics of the spectra carry on a lot of useful information about the object used in many areas, such as missile defence, space security, target recognition, and so on. Imaging of moving targets using radar has been a major challenge. The imaging methods depend on signals which are used. The well known imaging methods Synthetic Aperture Radar (SAR) and Inverse Synthetic Aperture Radar (ISAR) mainly use wideband waveforms. The wideband imaging approach is sensitive to translational motions, which may distort the image seriously [2, 3, 4]. On the other hand narrow-band radar with a small signal bandwidth insures high system sensitivity, and its maximum detectable distance is larger than that of wide-band radar for a constant minimum detectable signal-to noise ratio. In this paper only a narrow-band signal in a Doppler radar tomography of a rotating one-point object about the center of rotation with constant angular velocity is addressed. It is assumed that translational motions have been previously removed.

The work demonstrates the reassigned spectrograms as projections used in the back projection algorithm for improving image resolution [5, 6]. An important factor of the image quality is its resolution. Radar range resolution defines the ability of resolving two point-targets within the same antenna beam close together in the range domain. Doppler resolution is the ability of resolving two targets in the radial velocity. The Doppler resolution Δf_D is connected to the observation time T of a signal. The finite coherent processing interval (CPI) required to formulate a tomographic projection introduces a finite frequency spread for each point scatterer. The longer the CPI, the finer the Doppler filter width would become, but the smearing due to cross-range motion appears. The analysis of image resolution also requires the analysis of the sampling rate of the radar system. The sampling rate (or pulse repetition frequency (PRF) in narrow-band pulse Doppler radars which collects only one sample per transmitted pulse) needs to be sufficient to cover the entire Doppler frequency extent of the target. Selected aspects connected with an image resolution in Doppler radar tomography will be considered in detail. Finally, simulation results demonstrate the correctness of the theoretic analysis of the presented method.

Range-Doppler Imaging of a rotating Target

A rotating target might be positioned on a turntable, or it might be an airborne target. Herein it is assumed that an object is a one-point target, where only the rotational motion is considered. A model of a rotating object is shown in the figure 1, where a Cartesian coordinate system is defined. The target center is set to be the origin. The line of sight (LOS) angle is assumed to be constant. The rotating speed is denoted by ω. A rotating object causes Doppler shifts in radar returns.

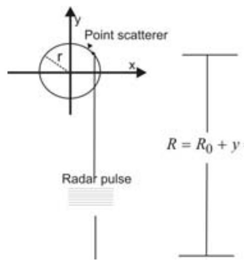

Fig. 1. The geometry for locating a rotating point target

The origin of coordinates coincides with the centre of the rotation. It is also assumed that this centre of rotation is a large distance R from the antenna. Because the range R is large it means that all the horizontal points are (approximately) at the same range from radar.

In classical radar imaging the image of the object is created in a Doppler-range plane [7]. In this plane a range and Doppler shifts of objects can be estimated A radar system can measure the range $R+y$ and the range rate $\dot{R}=x\dot{\theta}$. In particular, the radar measures the time delay τ and the Doppler shift f_D. It means that localisation of the object in a new imaging plane (X, Y) with coordinates Cartesian (x, y) called range and cross-range respectively, can be determined as

$$(x,y)=\left(\frac{-f_D}{f_0}\frac{c}{2\dot{\theta}},\quad \frac{c\tau}{2}-R_0\right) \tag{1}$$

An important factor of the image quality is its resolution. The imaging defined by the equation (1) is based on measurements of f_D and τ. Because the delay-time τ of a signal returned from an object is related to the range R, the resolution in range is directly related to the resolution in delay-time and is a function of the bandwidth B of the transmitted signal. The Doppler resolution Δf_D is connected to the coherent integration time T.

$$\Delta r_r = \frac{c}{2B} \quad \Delta f_D = \frac{1}{T} \tag{2}$$

Doppler resolution is connected with the cross-range resolution. The cross-range resolution Δr_{cr} is determined by the angle extent of aperture during the coherent integration time

$$\Delta r_{cr} = \frac{c}{2\omega f_0 T} = \left(\frac{c}{2\omega f_0}\right)\Delta f_D \tag{3}$$

where f_0 is the frequency of the transmitted signal and ω is a rotation rate of the object. Cross-range resolution is proportional to the Doppler resolution with a scaling factor.

Doppler Radar Tomography

Another method of radar imaging is imaging according to principles of tomography originated from X-ray tomography in diagnostic medicine, where an image is built from 1D projections [1, 2, 3]. In radar tomography, projections are developed from range profiles or Doppler profiles (cross range profiles). For radars of high bandwidth, mainly range profiles are used. For narrow-band radars, the range profiles are necessarily cross-range profiles which can be obtained through coherent Doppler processing. The backscatter from an object generates Doppler shift of the scatterer giving time-varying frequency characteristics, from which projections are created used in Doppler radar tomographic imaging.

A number of time-frequency distribution techniques have been proposed for analysing time-varying spectrum to obtain high frequency and time resolutions simultaneously. The STFT (Short Time Fourier Transform) has been applied due to its simplicity and linearity. The resolution of a tomographic image depends on the resolution of the input projections. A reassigned spectrogram is assumed as cross-range projections in Doppler radar tomography, which shows better resolution than a classical spectrogram [8]. The spectrogram $SP(f, t_k)$ is the squared modulus of the STFT. We can interpret the spectrogram as a measure of the energy of the signal in the time-frequency domain centered on the point (t, f).

$$SP(f, t_k) = \left| \int_{t_k}^{t_k+T_{CPI}} h(t-t_k)s(t,t_k)e^{-j2\pi ft}dt \right|^2 \tag{4}$$

where $h(t)$ is an analysis window for the STFT, which controls the resolution of this transformation.
The reassignment method moves each value of the spectrogram computed at any point (t, f) to another point $(\hat{t}, \hat{f})$, which is the centre of gravity of the signal energy distribution around (t, f)

$$SPR\ (t', f', h) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} SP\ (t, f, h)\delta(t'-\hat{t}(x;t,f))\delta(f'-\hat{f}(x;t,f))dtdf \tag{5}$$

A reassigned spectrogram is the time-frequency transformation whose value at any point (t', f') is the sum of all the spectrogram values reassigned to this point.

Requirements for sampling rate and coherent processing interval

The achievable resolution of an image in Doppler radar tomography depends on several factors. These factors include the finite sampling rate (pulse repetition frequency (PRF)) of the radar system, the coherent processing interval (CPI) used for projection, as well as the rotation rate ω and the cross-range dimension of the object. Some limitations result from the fundamental limit due to available bandwidth expressed by the point spread function (PSF). The analysis of the resolution resulting from the PSF function is omitted in this paper. The imaging resolution also depends on the resolution of the projections used in tomographic processing.

The sampling rate (or pulse repetition frequency (PRF)) in narrow-band pulse Doppler radars collects only one sample per transmitted pulse and has to be sufficient to cover the entire Doppler frequency extent of the target. Sampling rate without aliasing (or pulse repetition frequency PRF) has to fulfil the assumption of the Nyquist–Shannon sampling theorem. This criterion imposes the inequality

$$PRF_{\min} > \frac{4R_{\max}\omega}{\lambda} \tag{6}$$

where R_{max} is a maximal rotation radius of a scatterer on the object, λ is a length of a transmitted wave, and ω is the rotation rate.

In Doppler radar tomography based on the Fourier transform the length of a coherent processing interval (CPI) is limited by the rotational motion of a point scatterer on the object. Too long CPI causes that Doppler spread will be larger than one Doppler frequency filter width and Doppler spectrum will occupy more than one cross-range bin. This phenomenon is called 'walk-off'. Imaging with too long CPI results in a blurred image. Cross range is given by $x = r\cdot\cos\theta$, hence the differential change in direction x is given by $dx = -r\cdot\omega\cdot\sin\theta\cdot dt$. Maximum cross-range walk appears when θ is odd $n\cdot\pi/2$. If dt represents a CPI time, denoted by T_{CPI}, then the requirement for no cross-range walk-off during a CPI translates to the relation $dx = r\cdot\omega\cdot T_{CPI}$ [1]. T_{CPI} can be represented by the number of samples N_s used in the STFT algorithm

$$N_s < \frac{PRF}{\omega}\left(\frac{\lambda}{2R_{\max}}\right)^{0.5} \tag{7}$$

Fulfilment of inequalities (6) and (7) guarantees that smear-free projection can be achieved. The cross-range resolution limit for methods based on the STFT transform is given by

$$dx_{\lim} = \left(\frac{\lambda R_{\max}}{2}\right)^{0.5} \tag{8}$$

Comparing (3) and (8) it is clearly seen that the resolution in imaging improves in methods based on Fourier transform.

Model of radar returns

For narrow-band radar, echoes of the k-th scatterer in the slow time domain after frequencies mixing satisfy

$$s(t_k) = \sigma\exp(-j4\pi\frac{R(t_k)}{\lambda}),\ R(\theta) = x\sin\theta + y\cos\theta \tag{9}$$

where t_k is the slow time, σ is the backscattering coefficient and $R(t_k)$ denotes the instantaneous distance between the scatterer and the radar [3].

The equation (9) shows that the received signal characterises a time-varying phase in sinusoidal manner. The amplitude and the phase depend on the relative position of that scatterer with regard to the radar. The synthetic signal derived in equation (9) is used to generate projections.

Results of numerical experiments

In the experiment the scatterer is located in (20, 20) in the (X, Y) plane. The reassigned spectrogram and the filtered back-projection method are proposed to estimate the scatterer position. According to the simplification in (6) the following values are taken in experiments: ω = 0.0028 Hz, PRF = 1 Hz, $\lambda = 4\pi$ m, 360 samples of the phase are available for one cycle of a rotation in 360 s. T_{CPI} for smear-free projections is less than 180 *s.* Those values result from equations (6), (7) and (9) and are taken for simulations simplicity. The use of real life parameters requires scaling of proper equations of the imaging algorithm. The filtered back projection is calculated according to the formula (10) for obtaining a spatial position of a scatterer $q(x, y)$ in the plane (X, Y).

$$(10) \qquad q(x,y)=\int_{0}^{\pi} F^{-1}\{|f_r|P_{\theta}(f_r)\}_{(x\cos\theta+y\sin\theta)}\,d\theta$$

where the term $P_{\theta}(f_r)$ stands for the spatial Fourier transform of the projection in polar coordinates. The subscript $(x\cos\theta + y\sin\theta)$ means that the inverse Fourier transform, denoted by (F^{-1}) is calculated for this subscript. The Cartesian coordinate system is put in the middle of the matrix. The point (200, 200) in the matrix is located exactly in (20, 20) point in the coordinate system (X, Y).

The resolution of a tomographic image depends on the resolution of the input projections. In this paper projections are calculated as reassigned spectrograms. For comparison task the classical spectrogram in the figure 2 and the reassigned spectrogram in the figure 3 are presented.

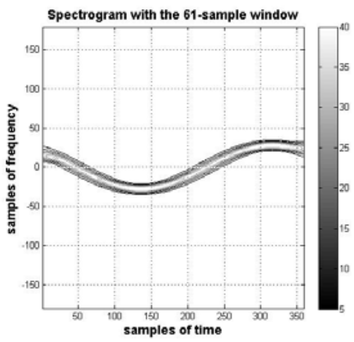

Fig. 2. Spectrogram with the 61-sample window as tomographic projections with bad resolution but satisfies yet the condition of smear-free projections

The much better resolution of projections for the reassigned spectrogram what can be seen in the figure 3.

The reassigned spectrogram and the classical spectrogram are used as tomographic projections in tomographic imaging. The tomographic processing of reassigned spectrogram via the filtered back projection shown in the figure 4 gives the image of the object with much better resolution comparing with the image from the spectrogram show in the figure 5.

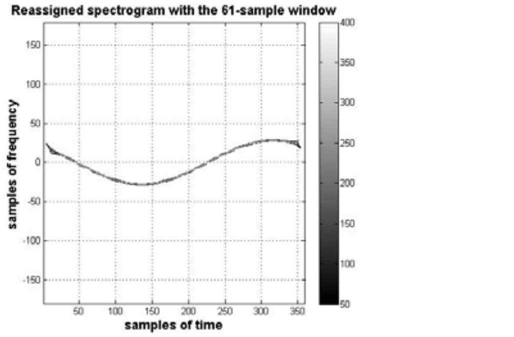

Fig. 3. Reassigned spectrogram with the 61-sample window with excellent resolution with condition of smear-free projections

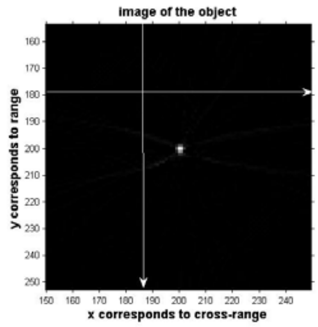

Fig. 4. Tomographic zoomed image from the reassigned spectrogram with the 61-sample window with proper location of the object in (200, 200) point

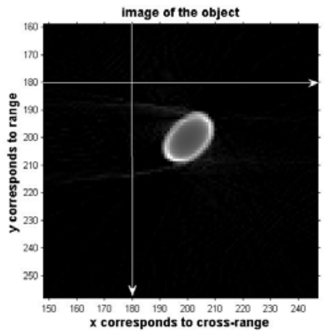

Fig. 5. Tomographic zoomed image from the spectrogram with the 61-sample window

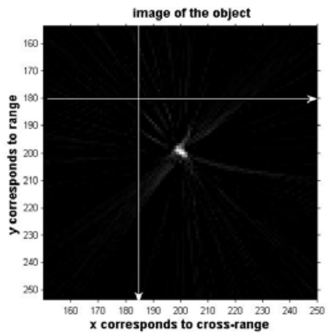

Fig. 6. Tomographic zoomed image from the reassigned spectrogram with the 215-sample window without condition of smear-free projections

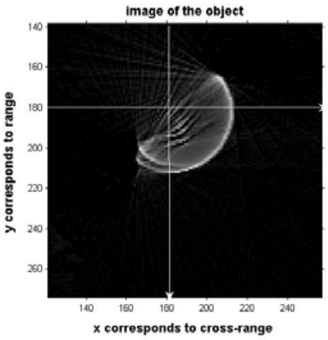

Fig. 7. Tomographic zoomed image from the spectrogram with the 215-sample window without condition of smear-free projections

The figures 4 and 5 show images from smear-free projections, where the condition for free-smear projections is given in equation (7).

Next experiments are performed with the higher number of samples N_s than the limit results from the equation (7). It is obvious that the too big number of samples used in projections gives the smeared projections and consistently smeared images.

Again, low resolution of the projection based on the STFT gives the large spread of the tomographic image comparing with the imaging based on the reassigned spectrogram. Some conclusion can be drawn analysing results of these experiments. Imaging based on the reassigned spectrogram is less sensitive to exceed the CPI parameter.

Conclusions

In this paper the imaging approach with the Doppler-processed cross-range profiles as projections represented by the spectrogram and the reassigned spectrogram in the tomographic processing is considered. A lot of factors determine the achievable resolution of the image. The length of the CPI and the value of the PRF have been found as very important in the imaging process to achieve a high resolution of an image. The limitations of these parameters have been showed. If the projections are 'cleaner' and 'sharper', resulting tomographic images with better resolution will be obtained. The reassigned spectrogram turned out to be better than the classical spectrogram.

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