Andriy MALYAR

Lviv Polytechnic National University

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Study of stationary modes of sucker rod pumping unit operation

Abstract. *In the article an algorithm for calculating periodic dependencies of electrical and mechanical coordinates in the stationary operation modes of the asynchronous drive of the deep-well oil pumping unit by solving a boundary problem is proposed. Thereat, dependencies of moment of inertia and load moment of the unit on crank rotation angle as well as saturation of the magnetic path and current displacement in the bars of the driving motor rotor are taken into consideration.*

Streszczenie. *W artykule przedstawiono algorytm do obliczania zależności okresowych elektrycznych I mechanicznych w asynchronicznym napędzie jednostki pompującej. Określono zależność momentu bezwładności i obciążenia od kąta dzwigu oraz nasycenie ścieżki magnetycznej.* ***Studium pracy stacjonarnej pompy żerdziowej***

Keywords: oil pumping unit, stationary modes, asynchronous motor, boundary problem.
Słowa kluczowe: pompa olejowa, silnik asynchroniczny, pompa żerdziowa

Introduction

Oil is produced from oil wells either due to natural flow driven by reservoir energy or by using one of the mechanical methods of lifting liquids.

One of the common mechanical methods uses sucker rod pumping unit (SRPU) [1]. SRPU (Fig. 1) consists of a single-acting deep well plunger pump (1), the plunger (2) of which makes reciprocating motion; string of pipes connected by threaded collar joints (3), along which liquid is lifted to the surface; string of sucker rods (4) which are connected to the plunger in the lower part and via the polished rod (5) to the horse head (6) in the upper part. Since the horse head moves along an arch, and the string of sucker rods is supposed to move vertically, it is connected to the polished rod by suspending the sucker rods. The jack pump with the help of the walking beam (7) and the crank (8) transforms rotating motion of the gear reducer crank shaft into reciprocating motion of the string of sucker rods and the plunger of the deep-well pump, which results in lifting liquid to the surface through the string of pipes. The jack pump is set into motion by the electric motor (12), the sheave of which is connected by belt gear (11) with the sheave of the two-stage gear reducer (10). To balance the unit, both crank counterweights (9) and balancing counterweights (13) are used. Control of the electric drive of the unit is performed by the control equipment using various diagnostic and monitoring systems.

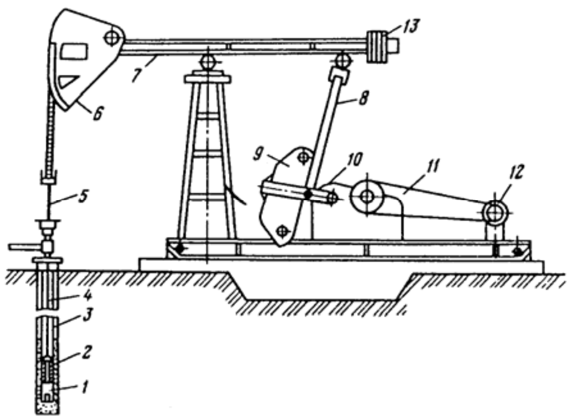

Fig. 1. Main elements of the sucker rod pumping unit

For design and research of the electric drive of SRPU, a particular practical importance is attached to stationary modes of the electric drive at different geometry of the sucker rod pump, different weights and locations of balancing weights both on the crank and on the balance beam, as well as in different load curves which determine law of variation of force at the point of rods hanging [1,2,3,4]. Actually, only analysing dependencies of electrical and mechanical coordinates of the electric drive system during a complete cycle of operation of the sucker rod pump corresponding to one revolution of the crank allows evaluating the quality of balancing and the operation status of the plunger pump.

It should be noted that for constant load torque and moment of inertia of the electric drive system in the stationary mode, speed of rotation of asynchronous motor (AM) rotor is constant, due to which the equations of electromagnetic balance can be reduced to algebraic ones by transforming the coordinates [5,6,7,8]. However, owing to saturation-driven non-linear dependence of flux linkage of AM circuits vs. current, the system of algebraic equations is non-linear. The solution to such a system of non-linear algebraic equations is a set of interrelated coordinates (current, flux linkage, motor slip, etc.) which are not time-dependent.

The complex cyclic character of load and variable moment of inertia of SRPU complicates analysis of stationary modes, since the processes are dynamic, and, therefore, electromagnetic processes are described by systems of differential equations (DE) which are irreducible to algebraic ones by choosing the appropriate coordinate system. In stationary modes of SRPU electric drive operation, processes are periodic, and period T m of load torque change is equal to crank rotation period. Periodic change of load torque and moment of inertia in stationary modes leads to the rise of dynamic torques and forces that are impossible to evaluate without calculating oscillations of both current and flux linkage, and rotor speed.

In contrast to static stationary modes, calculation of dynamic stationary periodic modes [9] aims at determining laws of variation of the mode coordinates during the period of the applied torque change and results in functional dependencies of status variables describing behaviour of the coordinates during the period. The task of obtaining these dependencies is much more difficult, but having them makes it possible not only to analyse operation of the unit and its elements, but also to synthesise control laws for SRPU electric drive necessary for optimal performance.

Differential Equations of Stationary Processes

As noted above, stationary processes in SRPU electric drive are periodic. To develop control systems for this electric drive and their optimization, it is necessary to know dependencies of mode coordinates vs. time on the period T_m of load change. As shown in [9], they can be obtained by solving a non-linear two-point boundary problem for a system of first-order differential equations with periodic boundary conditions using spline method [10,11], which enables getting dependencies of mode coordinates on the period without calculating the transient. To do this, the system of DE describing a stationary periodic mode should be algebraized by approximation of mode coordinates on the process repeatability period [12]. The length of this time period is determined by the time the crank makes one revolution, and since load torque changes during the period, speed of rotation of the rotor and therefore that of the crank are also variable. Thus, time value of this period is not known in advance. At the same time, irrespective of the time behaviour of the load, angular period of change of coordinates $T_{м}$ is 2π. This results from the fact that the angle of crank rotation $\alpha = 2\pi$ corresponds to the periodic dependence of load torque

$$M(\alpha) = M(\alpha + 2\pi)$$

and that of moment of inertia

$$J(\alpha) = J(\alpha + 2\pi),$$

and, therefore, of mode coordinates (flux linkage, current, speed of rotation, etc.). Hence, to solve the problem of calculating SRPU stationary mode, it is necessary to switch from derivatives with respect to time t to derivatives with respect to angle α of crank rotation. Therefore, the angle α of crank rotation is assumed to be the independent variable. To define the periodic relation of moment of inertia of all electric drive elements performing rotational motion or forward motion reduced to the motor shaft and angle α of crank rotation, we need to know angular velocity of rotating elements and speed of masses that make forward motion [13,14,15]. They are determined on the basis of moments of inertia of all motion parts of the electric drive [16].

Dynamics of SRPU electric drive is described by a system of equations of electrical equilibrium of circuits, which, in orthogonal axes x, y with division of the bars along with the cage rings into n elements, looks as follows [7]

$$\frac{d\psi_{sx}}{dt} = \omega_0\psi_{sy} - r_s i_{sx} + u_{sx};$$

$$\frac{d\psi_{sy}}{dt} = -\omega_0\psi_{sx} - r_s i_{sy} + u_{sy};$$

$$\frac{d\psi_{1x}}{dt} = \omega_0\psi_{1y} - r_1 i_{1x} - \omega\psi_{1y};$$

$$\frac{d\psi_{1y}}{dt} = -\omega_0\psi_{1x} - r_1 i_{1y} + \omega\psi_{1x};$$

(1a) $$\frac{d\psi_{1y}}{dt} = -\omega_0\psi_{1x} - r_1 i_{1y} + \omega\psi_{1x};$$

$$\vdots$$

$$\frac{d\psi_{nx}}{dt} = \omega_0\psi_{ny} - r_n i_{nx} - \omega\psi_{ny};$$

$$\frac{d\psi_{ny}}{dt} = -\omega_0\psi_{nx} - r_n i_{ny} + \omega\psi_{nx},$$

and by a system of equations of mechanical equilibrium

(1b) $$\frac{d\omega}{d\alpha} = \frac{p_0}{J}\left(\frac{3}{2}p_0\left(\psi_{sx}i_{sx} - \psi_{sy}i_{sy}\right) - M_v(\alpha)\right) - \frac{\omega^2}{2J}\frac{dJ}{d\alpha}.$$

Equation (1b) includes derivatives with respect to time and angle α. For analysis of stationary mode of SRPU operation by solving a boundary problem we need to substitute in the DE system (1) the differentiation operator with respect to time t with the differentiation operator with respect to crank rotation angle α according to the equation

$$\frac{d}{dt} = \frac{\omega}{p_0 k_i}\frac{d}{d\alpha}.$$

The result is

$$\frac{\omega}{p_0 k_i}\frac{d\psi_{sx}}{d\alpha} = \omega_0\psi_{sy} - r_s i_{sx} + u_{sx};$$

$$\frac{\omega}{p_0 k_i}\frac{d\psi_{sy}}{d\alpha} = -\omega_0\psi_{sx} - r_s i_{sy} + u_{sy};$$

$$\frac{\omega}{p_0 k_i}\frac{d\psi_{1x}}{d\alpha} = \omega_0\psi_{1y} - r_1 i_{1x} - \omega\psi_{1y};$$

(2) $$\frac{\omega}{p_0 k_i}\frac{d\psi_{1y}}{d\alpha} = -\omega_0\psi_{1x} - r_1 i_{1y} + \omega\psi_{1x};$$

$$\vdots$$

$$\frac{\omega}{p_0 k_i}\frac{d\psi_{nx}}{d\alpha} = \omega_0\psi_{ny} - r_n i_{nx} - \omega\psi_{ny};$$

$$\frac{\omega}{p_0 k_i}\frac{d\psi_{ny}}{d\alpha} = -\omega_0\psi_{nx} - r_n i_{ny} + \omega\psi_{nx};$$

$$\frac{\omega}{p_0 k_i}\frac{d\omega}{d\alpha} = \frac{p_0}{J}\left(\frac{3}{2}p_0\left(\psi_{sx}i_{sx} - \psi_{sy}i_{sy}\right) - M_v(\alpha)\right) - \frac{\omega^2}{2J}\frac{dJ}{d\alpha},$$

or, in Cauchy form,

$$\frac{d\psi_{sx}}{d\alpha} = \frac{p_0 k_i\left(\omega_0\psi_{sy} - r_s i_{sx} + u_{sx}\right)}{\omega};$$

$$\frac{d\psi_{sy}}{d\alpha} = \frac{p_0 k_i\left(-\omega_0\psi_{sx} - r_s i_{sy} + u_{sy}\right)}{\omega};$$

$$\frac{d\psi_{1x}}{d\alpha} = \frac{p_0 k_i\left(\omega_0\psi_{1y} - r_1 i_{1x}\right)}{\omega} - p_0 k_i\psi_{1y};$$

$$\frac{d\psi_{1y}}{d\alpha} = \frac{p_0 k_i\left(-\omega_0\psi_{1x} - r_1 i_{1y}\right)}{\omega} + p_0 k_i\psi_{1x};$$

$$\vdots$$

(3) $$\frac{d\psi_{nx}}{d\alpha} = \frac{p_0 k_i\left(\omega_0\psi_{ny} - r_n i_{nx}\right)}{\omega} - p_0 k_i\psi_{ny};$$

$$\frac{d\psi_{ny}}{d\alpha} = \frac{p_0 k_i\left(-\omega_0\psi_{nx} - r_n i_{ny}\right)}{\omega} + p_0 k_i\psi_{nx};$$

$$\frac{d\omega}{d\alpha} = \frac{p_0^2 k_i}{\omega J}\left(\frac{3}{2}p_0\left(\psi_{sx}i_{sx} - \psi_{sy}i_{sy}\right) - M_v(\alpha)\right) - \frac{p_0 k_i\omega}{2J}\frac{dJ}{d\alpha}.$$

For simplification, system (2) is transformed into the vector form

(4) $$\frac{d\vec{y}(\vec{x},\alpha)}{d\alpha} = \vec{z}\left(\vec{y},\vec{x},\vec{u}\right),$$

where the respective vectors are defined as:

$$\vec{y} = (\vec{\psi},\omega)^{T};$$

$$\vec{x} = (\vec{i},\omega)^{T};$$

$$\vec{u} = \left(u_{sx}, u_{sy}, 0, 0, \ldots, 0\right)^{T};$$

$$\vec{\psi} = \left(\psi_{sx}, \psi_{sy}, \psi_{1x}, \psi_{1y}, \ldots, \psi_{nx}, \psi_{ny}\right)^{T};$$

$$\vec{i} = \left(i_{sx}, i_{sy}, i_{1x}, i_{1y}, \ldots, i_{nx}, i_{ny}\right)^{T};$$

$$\vec{z}=\begin{vmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ \vdots \\ z_{2n+1} \\ z_{2n+2} \\ z_m \end{vmatrix} = \begin{vmatrix} \dfrac{p_0k_i\left(\omega_0\psi_{sy}-r_si_{sx}+u_{sx}\right)}{\omega} \\ \dfrac{p_0k_i\left(-\omega_0\psi_{sx}-r_si_{sy}+u_{sy}\right)}{\omega} \\ \dfrac{p_0k_i\left(\omega_0\psi_{1y}-r_1i_{1x}\right)}{\omega}-p_0k_i\psi_{1y} \\ \dfrac{p_0k_i\left(-\omega_0\psi_{1x}-r_1i_{1y}\right)}{\omega}+p_0k_i\psi_{1x} \\ \vdots \\ \dfrac{p_0k_i\left(\omega_0\psi_{ny}-r_ni_{nx}\right)}{\omega}-p_0k_i\psi_{ny} \\ \dfrac{p_0k_i\left(-\omega_0\psi_{nx}-r_ni_{ny}\right)}{\omega}+p_0k_i\psi_{nx} \\ \dfrac{p_0^2k_i}{\omega J}\left(\dfrac{3}{2}p_0\left(\psi_{sx}i_{sx}-\psi_{sy}i_{sy}\right)-M_v(\alpha)\right)-\dfrac{p_0k_i\omega}{2J}\dfrac{dJ}{d\alpha} \end{vmatrix}$$

In the stationary mode all the variable in DE system (3) are periodic functions of angle α. The solution to DE system (4) is $T = 2\pi$, which is a periodic dependence of vector $\vec{x}$ consisting of values of current in AM circuits and speed of rotation of the rotor ω, on crank rotation angle α . This allows considering their calculation as a boundary problem for a system of first order differential equations with periodic boundary conditions [9,11]. To obtain periodic dependency relations of the coordinates by solving a boundary problem, system (4) of DE is algebraized by approximation of status variables on a grid of N+1 nodes of the period, the first and the last node located at the beginning and at the end of the period, respectively, using cubic splines [17]

(4) $\vec{y}(\alpha)=\vec{a}_j+\vec{b}_j(\alpha_j-\alpha)+\vec{c}_j(\alpha_j-\alpha)^2+\vec{d}_j(\alpha_j-\alpha)^3$,

where $j=\left(\overline{1,N}\right)$ is the number of a section of the period defined by the number of the right-hand node of the section; $\vec{a}_j,\vec{b}_j,\vec{c}_j,\vec{d}_j$ are vectors of spline coefficients; α_j is the value of angular coordinate (angle of crank rotation) at a j-th node.

Using (5) for a j-th node, we determine

(6a,b) $\vec{y}(\alpha)=\vec{y}_j=\vec{a}_j$; $\left.\dfrac{d\vec{y}}{d\alpha}\right|_{\alpha=\alpha_j}=-\vec{b}_j$.

According to continuity conditions of spline (5), as well as its first and second derivatives on the whole period T ,

(6c) $$\vec{b}_j=\frac{\vec{a}_{j-1}-\vec{a}_j}{h_j}-\frac{h_j}{3}\left(\vec{c}_{j-1}+2\vec{c}_j\right),$$

(6d) $$\frac{3}{h_j}\vec{a}_{j-1}-\left(\frac{3}{h_j}+\frac{3}{h_{j+1}}\right)\vec{a}_j+\frac{3}{h_{j+1}}\vec{a}_{j+1}= =h_j\vec{c}_{j-1}+2\left(h_j+h_{j+1}\right)\vec{c}_j+h_{j+1}\vec{c}_{j+1} ,$$

where $h_j=\alpha_j-\alpha_{j-1}$ is a j-th step of the grid of nodes on the period, which will further be uniform.

As seen from equations (6), all spline coefficients can be determined using nodal values of the respective curves. Formulating equation (4) for each j-th node of the period results in the system of discrete equations

(7) $$\left.\frac{d\vec{y}(\vec{x}_1)}{d\alpha}\right|_{\alpha=\alpha_1}-\vec{z}_1\left(\vec{y}_1,\vec{x}_1,\vec{u}_1\right)=0\ ; \quad \vdots \quad \left.\frac{d\vec{y}(\vec{x}_N)}{d\alpha}\right|_{\alpha=\alpha_N}-\vec{z}_N\left(\vec{y}_N,\vec{x}_N,\vec{u}_N\right)=0\ .$$

Having switched from continuous change of coordinates to their nodal values according to (7) and taking into consideration continuity conditions for cubic spline (5), its first and second derivatives, as well as periodic boundary conditions

$$\vec{z}_j=\vec{z}_{j+N},\quad \vec{y}_j=\vec{y}_{j+N},\quad \vec{x}_j=\vec{x}_{j+N}\ ,$$

we obtain the algebraic counterpart of DE system (4).

Taking into account expressions (6) – (7) according to [7], system (4) algebraized by spline-approximation on the grid of N+1 nodes of the period is written as vector equation

(8) $$H\vec{Y}-\vec{Z}=0\ ,$$

Where $H=diag(H_0,...,H_0)$ is a diagonal matrix mN of transition from continuous change of coordinates to their nodal values on the basis of approximation of variables by 3rd order splines consisting of N identical units (sub-matrices H_0), each of which measures m×m, and whose elements are defined only by a distance between nodes of the period [9];

$\vec{Y}=\left(\vec{y}_1,...,\vec{y}_N\right)^{\mathrm{T}}$ is a vector made up of vectors with the length N of nodal values of the corresponding variables, i.e. $\vec{y}_j=\left(\vec{\psi}_j,\omega_j\right)^{\mathrm{T}}$;

$\vec{Z}=\left(\vec{z}_1,...,\vec{z}_{2n+2},\vec{z}_m\right)^{\mathrm{T}}$ is the vector whose components are vectors of nodal values of the right-hand parts of DE system (4).

Thus, instead of the non-linear system of m=(3+2n)-th order DE (4) we obtained system (8) of mN-th order algebraic equations, which is its discrete counterpart. The indeterminates in the obtained system are values of vector $\vec{x}$ in N nodes of the period, which form the vector

$$\vec{X}=\left(\vec{x}_1,...,\vec{x}_N\right)^{\mathrm{T}}\ ,$$

where $\vec{x}_j=\left(\vec{i}_j,\omega_j\right)^{\mathrm{T}}$, $j=\left(\overline{1,N}\right)$.

Therefore, vector $\vec{X}$ is a discrete counterpart of periodic solution of the process subject to approximation of the coordinate by splines, which makes it possible to build functional dependency relations for all the variables on the period using its obtained nodal values.

The developed mathematical model for calculating stationary modes underlies a computer model which allows studying behavior of SRPU electric drive system for different laws of variation of the load on the polished rod at the point of hanging the pump rods and various systems of balancing the sucker rod pump. The simulation results in a matrix of mode coordinates. The algorithm of the program allows the user to set the number of nodal points.

The input data for the developed model is certificate details of sucker rod pumps and AM, which are available in reference books [18, 19]. There were used saturation curves for electrical steel.

Digital Simulation and Results

Figure 2 offers simulation curves of relative values of load torque on AM shaft, force P_0 at the point of hanging the rods and *cosφ* of the motor 4AP160S4У3 which drives the sucker rod pump 7СК8-3,5-4000 in the stationary mode of SRPU.

Figure 3 presents periodic dependencies of stator current and torque on the shaft of the motor 4AP160S4У3 driving the sucker rod pump 7СК8-3,5-4000, which were calculated by the above-discussed algorithm in the stationary mode of SRPU operation according to the law of variation of force P_0 at the point of hanging the rods illustrated in Figure 4 (load curve).

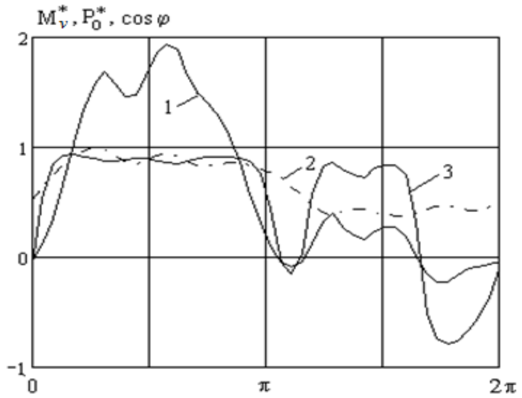

Fig.2. Load torque on AM shaft (1), force P_0 (2) at the point of hanging the rod and cos*φ* (3) vs. angle of crank rotation

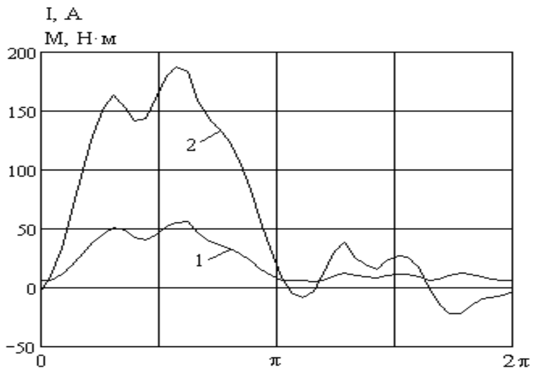

Fig.3. Dependencies of stator current (1) and AM load torque (2) for the law of variation of force P_0 at the point of rods hanging illustrated in Fig.4

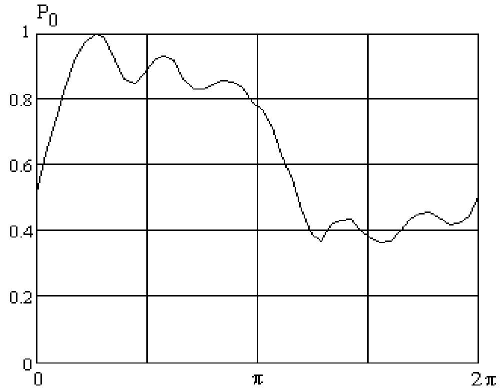

Fig. 4. Periodic dependence of the relative value of force $P_0 = P/P_{max}$ at the point of rods hanging

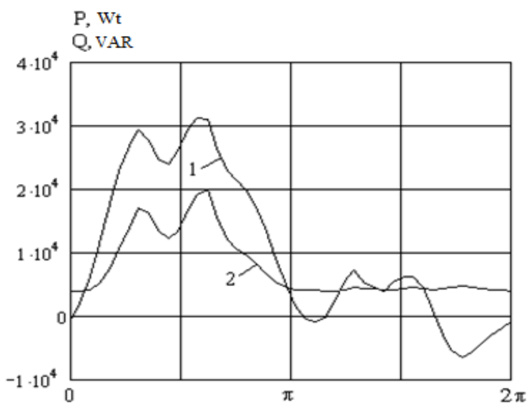

Fig. 5. Dependence of active (1) and reactive (2) power for AM load torque and law of variation of force P_0 at the point of rods hanging as shown in Fig. 2 and Fig.4

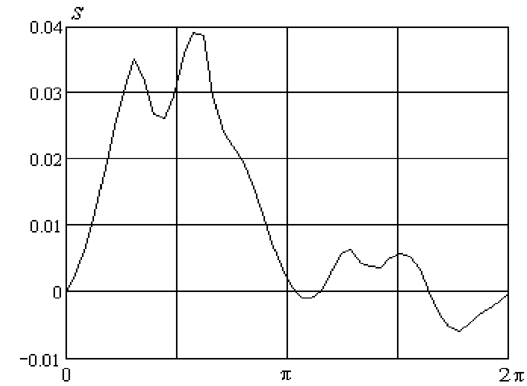

Fig.6. Slip of AM rotor vs. angle of crank rotation in SRPU stationary mode

Periodic dependencies of active and reactive power presented in Figure 5 and periodic dependence of rotor slip shown in Figure 6. correspond to the laws of variation of load torque on the AM shaft and force P_0 at the point of rods hanging shown in Fig.3 and Fig.4.

Adequacy of the proposed method of calculating stationary modes was verified by comparing the measured and calculated values. Calculation and experimental data for the filled-up (a) and partially filled (b) pump are presented in Fig. 7.

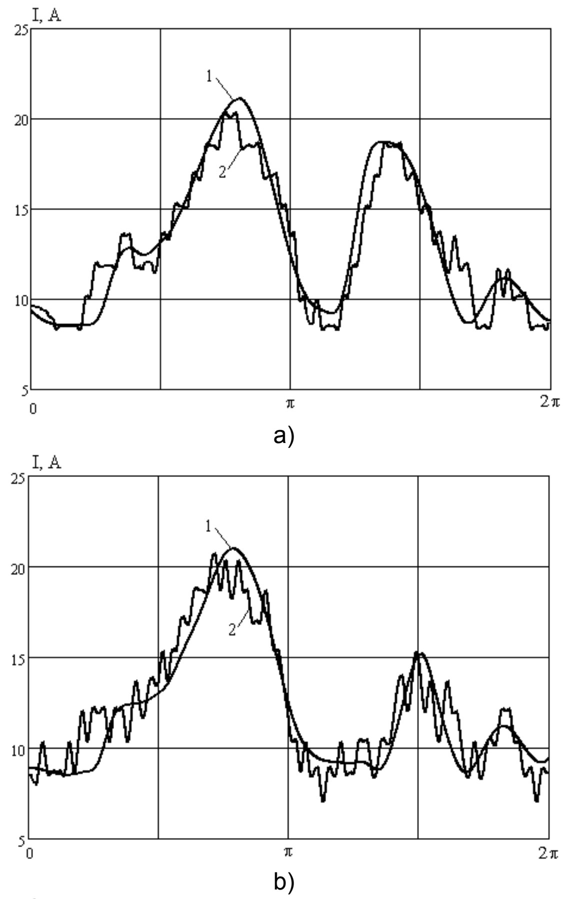

Fig.7. Calculation (1) and experimental (2) current vs. time curves for the stator of the induction motor

Conclusions

The developed mathematical model and programmed created on its basis make it possible:

– to calculate stationary modes of SRPU electric drive operation effectively and accurately without calculating transients;

– to carry out mathematical experiments studying the impact of various factors on the character of change of electrical and mechanical coordinates in stationary modes;
– to calculate characteristics of stationary modes as dependency of a whole set of mode coordinates on the period on one variable;
– to study and select laws of regulating the electric drive of SRPU with a view to optimizing its operation;
– to evaluate boundary values of periodic load of SRPU electric drive with the control system taken into account;
– to perform optimizing calculation both of the electric part of the unit, and of its mechanism.

***Authors**: Prof, Dr. of Sc., Andriy Malyar, Lviv Polytechnic National University, Institute of Power Engineering and Control Systems, 12 Bandera Str., Lviv, 79013, Ukraine, E-mail: svandrew@i.ua.*

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