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Currents' Physical Components (CPC) in Four-Wire Systems with Nonsinusoidal Symmetrical Voltage

Abstract. *Energy flow related phenomena in three-phase systems with neutral conductor and unbalanced, linear, time-invariant (LTI) loads, supplied with nonsinusoidal, but symmetrical voltage are investigated in the paper. It is demonstrated that the load current can be decomposed into Currents' Physical Components (CPC), associated with distinctive physical phenomena in the load. It is also shown how the CPC can be expressed in terms of the supply voltage and equivalent parameters of the load. This decomposition provides solid fundamentals for defining powers of such loads.*

Streszczenie. *Artykuł przedstawia wyniki badań nad zjawiskami energetycznymi w liniowych, czasowo-niezmienniczych (LTI) odbiornikach niezrównoważonych, zasilanych niesinusoidalnym lecz symetrycznym napięciem w układach trójfazowych z przewodnikiem neutralnym. Pokazano, że prąd zasilania takich odbiorników może być rozłożony na Składowe Fizyczne, jednoznacznie stowarzyszone z określonymi zjawiskami fizycznymi. Pokazano także, że prądy te mogą być określone poprzez napięcie zasilania i parametry równoważne odbiornika. Rozkład ten tworzy solidne podstawy dla definicji mocy takich odbiorników.* (***Składowe Fizyczne Prądów w obwodach trójfazowych z przewodem zerowym i niesinusoidalnym lecz symetrycznym napięciem zasilania***).

Keywords: Current decomposition, unbalanced loads, asymmetrical systems, power definitions, power theory.
Słowa kluczowe: Rozkład prądu, odbiorniki niezrównoważone, systemy niesymetryczne, definicje mocy, teoria mocy.

Introduction

The Steinmetz observation [1] in 1892 that the apparent power S in a circuit with nonsinusoidal current could be higher than the active power P, at zero reactive power Q, does not have explanation for all circuit situations even now, more than a hundred years later. The inequality $S > P$ has important technical and economic consequences: the bill for energy delivered is related to the load active power P, while the cost of this energy including the cost of its delivery and the cost of the transmission equipment is related to the apparent power S.

This inequality at nonsinusoidal voltages and currents can be explained in terms of powers for single-phase loads [10] and for three-phase, three-wire loads [12], but not for three-phase loads with a neutral conductor. As to authors' best knowledge, the power equation of three-phase loads with neutral conductor at nonsinusoidal supply voltage is not yet known.

Three-phase systems with a neutral conductor are the most common systems for the energy distribution to small and to medium power customers. Studies on description of such systems in terms of powers have therefore a long history [2-9, 11-16].

There are two main reasons for the lack of a right power equation for such systems at nonsinusoidal supply voltage. The first of them is the fact that it was not possible to develop a power equation for three-phase systems before such an equation was developed for single-phase systems. This obstacle was eventually removed in [10]. The second reason was the lack of the right definition of the apparent power S for three-phase systems. Definition of arithmetic and geometric apparent powers, introduced by the AIEE in [3], did not provide fundamentals for the power equation development. This obstacle was eventually removed in [12], where a new definition of the apparent power for three-phase systems with nonsinusoidal voltages and currents was introduced.

The Currents' Physical Components (CPC) concept is the main theoretical tool in this paper for describing three-phase linear, time-invariant (LTI) unbalanced loads in three-phase systems with neutral conductor, supplied with non-sinusoidal, but symmetrical voltage, in terms of powers.

According to the CPC concept of the power theory development, fundamental for this development is decomposition of the load current into orthogonal components associated with distinctive energy flow-related phenomena. Power definitions are secondary to this decomposition. The original Steinmetz question: ***why can the apparent power S be higher than the active power P?*** can be simplified to the question: ***why can the supply current rms value be higher than the rms value of the current needed to supply the load with power P?***

This paper is continuation of [16], where a solution of the problem was presented, but at the assumption that the supply voltage was sinusoidal.

Line-to-neutral admittances of the load

Even if the power properties of the load can be specified based on the knowledge of the load structure and its parameters, power theories attempt to specify them based on measurements of voltages and currents at the load terminals. The load should be described in terms of powers or CPC independently on the load complexity and parameters. In the case of three-phase load supplied in a three-phase system with neutral, as shown in Fig. 1, three line-to-neutral conductor voltages $u_R(t)$, $u_S(t)$, $u_T(t)$ and three line currents $i_R(t)$, $i_S(t)$, $i_T(t)$ are the input data.

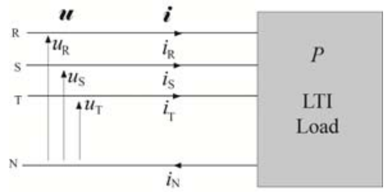

Fig.1. Three-phase LTI load supplied in a system with neutral conductor.

We assume that all these quantities, denoted generally by $x(t)$, are periodic and can be expressed in terms of their harmonics $x_n(t)$

$$x(t) = \sum_{n\in N} x_n(t) \; = X_0 + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{X}_n\, e^{jn\omega_1 t} \tag{1}$$

where N denotes the set of harmonic orders n and

$$\boldsymbol{X}_n = X_n\, e^{j\alpha_n} = \frac{\sqrt{2}}{T}\int_0^T x(t)\, e^{-jn\omega_1 t} dt \tag{2}$$

is the complex rms (crms) value of the n^{th} order harmonic. Modern measurement instruments calculate these values digitally. Instead of using (2), they process samples of voltages or currents x_k, provided by voltage or current sensors and analog to digital (A/D) converters. Such digital instruments can calculate the crms values $\boldsymbol{X}_n$ using Discrete Fourier Transform (DFT), namely

$$\boldsymbol{X}_n = \frac{\sqrt{2}}{K}\sum_{k=0}^{k=K-1} x_k\, e^{-j\frac{2\pi n}{K}k} \tag{3}$$

where K denotes the number of samples in one period T of the supply voltage variability.

Symbols $\boldsymbol{u}$ and $\boldsymbol{i}$ in Fig. 1 denote three-phase vectors of line-to-neutral voltages and line currents, namely

$$\boldsymbol{u}(t) \overset{\text{df}}{=} \begin{bmatrix} u_{\text{R}}(t) \\ u_{\text{S}}(t) \\ u_{\text{T}}(t)\end{bmatrix} = \sum_{n\in N}\boldsymbol{u}_n(t) = \sqrt{2}\,\text{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{U}_{\text{R}n} \\ \boldsymbol{U}_{\text{S}n} \\ \boldsymbol{U}_{\text{T}n}\end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\text{Re}\sum_{n\in N}\boldsymbol{U}_n e^{jn\omega_1 t} \tag{4}$$

$$\boldsymbol{i}(t) \overset{\text{df}}{=} \begin{bmatrix} i_{\text{R}}(t) \\ i_{\text{S}}(t) \\ i_{\text{T}}(t)\end{bmatrix} = \sum_{n\in N}\boldsymbol{i}_n(t) = \sqrt{2}\,\text{Re}\sum_{n\in N}\begin{bmatrix}\boldsymbol{I}_{\text{R}n} \\ \boldsymbol{I}_{\text{S}n} \\ \boldsymbol{I}_{\text{T}n}\end{bmatrix} e^{jn\omega_1 t} = \sqrt{2}\,\text{Re}\sum_{n\in N}\boldsymbol{I}_n e^{jn\omega_1 t}\,. \tag{5}$$

Having crms values of the load voltage and current harmonics, $\boldsymbol{U}_{\text{R}n}$, $\boldsymbol{U}_{\text{S}n}$, $\boldsymbol{U}_{\text{T}n}$, $\boldsymbol{I}_{\text{R}n}$, $\boldsymbol{I}_{\text{S}n}$, and $\boldsymbol{I}_{\text{T}n}$, equivalent line-to-neutral admittances can be calculated

$$\boldsymbol{Y}_{\text{R}n} = G_{\text{R}n} + jB_{\text{R}n} = \frac{\boldsymbol{I}_{\text{R}n}}{\boldsymbol{U}_{\text{R}n}} \tag{6}$$

$$\boldsymbol{Y}_{\text{S}n} = G_{\text{S}n} + jB_{\text{S}n} = \frac{\boldsymbol{I}_{\text{S}n}}{\boldsymbol{U}_{\text{S}n}} \tag{7}$$

$$\boldsymbol{Y}_{\text{T}n} = G_{\text{T}n} + jB_{\text{T}n} = \frac{\boldsymbol{I}_{\text{T}n}}{\boldsymbol{U}_{\text{T}n}}\,. \tag{8}$$

In such a way, an equivalent circuit of the load for each order harmonic, as it is shown in Fig. 2, can be found.

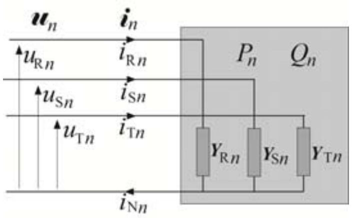

Fig. 2. Equivalent circuit of the load for the n^{th} order harmonic.

The load current decomposition

With respect to the active power P at the voltage $\boldsymbol{u}$, the load is equivalent to a purely resistive balanced load of conductance

$$G_{\text{e}} = \frac{P}{\|\boldsymbol{u}\|^2} \tag{9}$$

referred to as an ***equivalent conductance*** of the load. The equivalent load is shown in Fig. 3. Such an equivalent resistive and balanced load draws the current referred to as ***an active current***.

It is the current proportional to the supply voltage

$$\boldsymbol{i}_{\text{a}}(t) \overset{\text{df}}{=} \begin{bmatrix} i_{\text{Ra}}(t) \\ i_{\text{Sa}}(t) \\ i_{\text{Ta}}(t)\end{bmatrix} = G_{\text{e}}\,\boldsymbol{u}(t) = \sqrt{2}\,\text{Re}\sum_{n\in N} G_{\text{e}}\,\boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{10}$$

and the minimum three-phase rms value needed to supply the load with the active power P at the supply voltage $\boldsymbol{u}$.

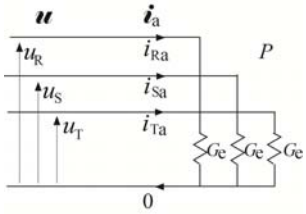

Fig. 3. Resistive balanced load equivalent to the original load with respect to its active power P.

This rms value is equal to

$$\|\boldsymbol{i}_{\text{a}}\| = G_{\text{e}}\|\boldsymbol{u}\| = \frac{P}{\|\boldsymbol{u}\|}\,. \tag{11}$$

With respect to the active power of the n^{th} order harmonic P_n the load in Fig. 2 is equivalent to a balanced resistive load, shown in Fig. 4,

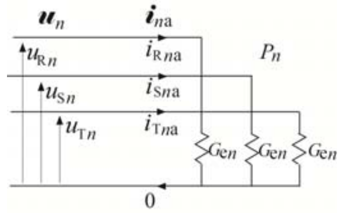

Fig. 4. Resistive load equivalent to the original load with respect to its active power P_n of the n^{th} order harmonic.

of conductance $G_{\text{e}n}$ equal to

$$G_{\text{e}n} = \frac{P_n}{\|\boldsymbol{u}_n\|^2} = \frac{1}{3}(G_{\text{R}n} + G_{\text{S}n} + G_{\text{T}n})\,. \tag{12}$$

Such a circuit draws the current

$$\boldsymbol{i}_{\text{a}n}(t) = G_{\text{e}n}\,\boldsymbol{u}_n(t) = \sqrt{2}\,\text{Re}\{G_{\text{e}n}\,\boldsymbol{U}_n\, e^{jn\omega_1 t}\}\,. \tag{13}$$

With respect to the reactive power of the n^{th} order harmonic Q_n the load in Fig. 2 is equivalent to a balanced reactive load, shown in Fig. 5,

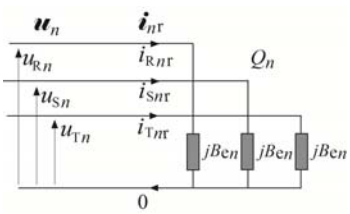

Fig. 5. Reactive load equivalent to the original load with respect to its reactive power Q_n of the n^{th} order harmonic.

of susceptance $B_{\text{e}n}$ equal to

$$B_{\text{e}n} = -\frac{Q_n}{\|\boldsymbol{u}_n\|^2} = \frac{1}{3}(B_{\text{R}n} + B_{\text{S}n} + B_{\text{T}n})\,. \tag{14}$$

Such a circuit draws the current

$$\boldsymbol{i}_{\text{r}n}(t) = B_{\text{e}n}\frac{d}{d(n\omega_1 t)}\boldsymbol{u}_n(t) = \sqrt{2}\,\text{Re}\{jB_{\text{e}n}\,\boldsymbol{U}_n\, e^{jn\omega_1 t}\}\,. \tag{15}$$

When the load is unbalanced, then the n^{th} order current harmonic contains moreover an unbalanced current

(16) $$\boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}_n - \boldsymbol{i}_{\mathrm{a}n} - \boldsymbol{i}_{\mathrm{r}n}\,.$$

Since the n^{th} order current harmonic of the equivalent circuit in Fig. 2 can be expressed in terms of equivalent admittances as follows

(17) $$\boldsymbol{i}_n(t) = \begin{bmatrix} i_{\mathrm{R}n}(t) \\ i_{\mathrm{S}n}(t) \\ i_{\mathrm{T}n}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\{\begin{bmatrix} \boldsymbol{Y}_{\mathrm{R}n}\boldsymbol{U}_{\mathrm{R}n} \\ \boldsymbol{Y}_{\mathrm{S}n}\boldsymbol{U}_{\mathrm{S}n} \\ \boldsymbol{Y}_{\mathrm{T}n}\boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}\}$$

the unbalanced current of the n^{th} order harmonic can be expressed in terms of the load equivalent parameters as follows

(18) $$\boldsymbol{i}_{\mathrm{u}n} = \begin{bmatrix} i_{\mathrm{Ru}n}(t) \\ i_{\mathrm{Su}n}(t) \\ i_{\mathrm{Tu}n}(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\{\begin{bmatrix} \boldsymbol{I}_{\mathrm{Ru}n} \\ \boldsymbol{I}_{\mathrm{Su}n} \\ \boldsymbol{I}_{\mathrm{Tu}n} \end{bmatrix} e^{jn\omega_1 t}\} = \sqrt{2}\,\mathrm{Re}\{\begin{bmatrix} (\boldsymbol{Y}_{\mathrm{R}n} - G_{\mathrm{e}n} - jB_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{R}n} \\ (\boldsymbol{Y}_{\mathrm{S}n} - G_{\mathrm{e}n} - jB_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{S}n} \\ (\boldsymbol{Y}_{\mathrm{T}n} - G_{\mathrm{e}n} - jB_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} e^{jn\omega_1 t}\}.$$

This current can be asymmetrical, so that it can be decomposed into symmetrical components of the positive, negative and the zero sequences, namely

(19) $$\boldsymbol{i}_{\mathrm{u}n} = \boldsymbol{i}^{\mathrm{p}}_{\mathrm{u}n} + \boldsymbol{i}^{\mathrm{n}}_{\mathrm{u}n} + \boldsymbol{i}^{\mathrm{z}}_{\mathrm{u}n}\,.$$

The crms values of these components are equal to

(20) $$\begin{bmatrix} \boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_{\mathrm{Ru}n} \\ \boldsymbol{I}_{\mathrm{Su}n} \\ \boldsymbol{I}_{\mathrm{Tu}n} \end{bmatrix} \stackrel{\mathrm{df}}{=} \mathrm{S}\begin{bmatrix} \boldsymbol{I}_{\mathrm{Ru}n} \\ \boldsymbol{I}_{\mathrm{Su}n} \\ \boldsymbol{I}_{\mathrm{Tu}n} \end{bmatrix}$$

where

$$\alpha = 1e^{j2\pi/3}, \quad \alpha^* = 1e^{-j2\pi/3}$$

are coefficients of rotation on the complex plane.

When harmonics of line R, S and T voltages are symmetrical, then they have the positive, negative or the zero sequence. The crms values of the voltage harmonics of lines S and T can be expressed in terms of the crms value $\boldsymbol{U}_{\mathrm{R}n}$ of the voltage harmonic on line R and α coefficient.

For harmonics of the positive sequence order, n = 1, 4, 7, 10…, in general, $n = 3k+1$, k = 0, 1, 2…

$$\boldsymbol{U}_{\mathrm{S}n} = \alpha^*\, \boldsymbol{U}_{\mathrm{R}n}, \quad \boldsymbol{U}_{\mathrm{T}n} = \alpha\, \boldsymbol{U}_{\mathrm{R}n}.$$

For harmonics of the negative sequence order, n = 2, 5, 8, 11…., in general, $n = 3k-1$

$$\boldsymbol{U}_{\mathrm{S}n} = \alpha\, \boldsymbol{U}_{\mathrm{R}n}, \quad \boldsymbol{U}_{\mathrm{T}n} = \alpha^*\, \boldsymbol{U}_{\mathrm{R}n}.$$

For harmonics of the zero sequence order, n = 0, 3, 6, 9, 12…., in general, $n = 3k$

$$\boldsymbol{U}_{\mathrm{S}n} = \boldsymbol{U}_{\mathrm{R}n}, \quad \boldsymbol{U}_{\mathrm{T}n} = \boldsymbol{U}_{\mathrm{R}n}.$$

These three sets of relations for crms values of the positive, negative and the zero sequence symmetrical harmonics can be superseded by a single one, using rotation coefficient defined as follows

(21) $$\beta \stackrel{\mathrm{df}}{=} (\alpha^*)^n = \begin{cases} 1, & \text{for} \quad n = 3k \\ \alpha^*, & \text{for} \quad n = 3k+1 \\ \alpha, & \text{for} \quad n = 3k-1 \end{cases}.$$

With this coefficient, independently on the harmonic order

(22) $$\boldsymbol{U}_{\mathrm{S}n} = \beta\, \boldsymbol{U}_{\mathrm{R}n}, \qquad \boldsymbol{U}_{\mathrm{T}n} = \beta^*\, \boldsymbol{U}_{\mathrm{R}n}$$

With (22), formula (20) for crms values of symmetrical components of the unbalanced current of the n^{th} order harmonic can be rearranged to the form

(23) $$\begin{bmatrix} \boldsymbol{I}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{I}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{I}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \mathrm{S}\begin{bmatrix} \boldsymbol{I}_{\mathrm{Ru}n} \\ \boldsymbol{I}_{\mathrm{Su}n} \\ \boldsymbol{I}_{\mathrm{Tu}n} \end{bmatrix} = \mathrm{S}\begin{bmatrix} (\boldsymbol{Y}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{R}n} \\ (\boldsymbol{Y}_{\mathrm{S}n} - \boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{S}n} \\ (\boldsymbol{Y}_{\mathrm{T}n} - \boldsymbol{Y}_{\mathrm{e}n})\boldsymbol{U}_{\mathrm{T}n} \end{bmatrix} = \mathrm{S}\begin{bmatrix} (\boldsymbol{Y}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{e}n}) \\ (\boldsymbol{Y}_{\mathrm{S}n} - \boldsymbol{Y}_{\mathrm{e}n})\beta \\ (\boldsymbol{Y}_{\mathrm{T}n} - \boldsymbol{Y}_{\mathrm{e}n})\beta^* \end{bmatrix}\boldsymbol{U}_{\mathrm{R}n} = \begin{bmatrix} \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix}\boldsymbol{U}_{\mathrm{R}n}.$$

Symbols $\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}$, $\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}$ and $\boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}$ on the right side of (23) denote unbalance admittances of the load of the zero, positive and negative sequence for the n^{th} order harmonic. These admittances have the following general forms

(24) $$\begin{bmatrix} \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^* \\ 1 & \alpha^* & \alpha \end{bmatrix}\begin{bmatrix} (\boldsymbol{Y}_{\mathrm{R}n} - \boldsymbol{Y}_{\mathrm{e}n}) \\ (\boldsymbol{Y}_{\mathrm{S}n} - \boldsymbol{Y}_{\mathrm{e}n})\beta \\ (\boldsymbol{Y}_{\mathrm{T}n} - \boldsymbol{Y}_{\mathrm{e}n})\beta^* \end{bmatrix} = \frac{1}{3}\begin{bmatrix} (\boldsymbol{Y}_{\mathrm{R}n} + \beta\boldsymbol{Y}_{\mathrm{S}n} + \beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\beta+\beta^*) \\ (\boldsymbol{Y}_{\mathrm{R}n} + \alpha\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\alpha\beta+\alpha^*\beta^*) \\ (\boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\beta\boldsymbol{Y}_{\mathrm{S}n} + \alpha\beta^*\boldsymbol{Y}_{\mathrm{T}n}) - \boldsymbol{Y}_{\mathrm{e}n}(1+\alpha^*\beta+\alpha\beta^*) \end{bmatrix}.$$

Since the coefficient β depends, according to its definition (21), on the harmonic sequence, these admittances depend on the sequence as well. Namely: for harmonics of the zero sequence

(25) $$\begin{bmatrix} \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\boldsymbol{Y}_{\mathrm{T}n} \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\boldsymbol{Y}_{\mathrm{S}n} + \alpha\boldsymbol{Y}_{\mathrm{T}n} \end{bmatrix};$$

for harmonics of the positive sequence

(26) $$\begin{bmatrix} \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\boldsymbol{Y}_{\mathrm{S}n} + \alpha\boldsymbol{Y}_{\mathrm{T}n} \\ 0 \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\boldsymbol{Y}_{\mathrm{T}n} \end{bmatrix};$$

for harmonics of the negative sequence

(27) $$\begin{bmatrix} \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} \\ \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} \boldsymbol{Y}_{\mathrm{R}n} + \alpha\boldsymbol{Y}_{\mathrm{S}n} + \alpha^*\boldsymbol{Y}_{\mathrm{T}n} \\ \boldsymbol{Y}_{\mathrm{R}n} + \alpha^*\boldsymbol{Y}_{\mathrm{S}n} + \alpha\boldsymbol{Y}_{\mathrm{T}n} \\ 0 \end{bmatrix}.$$

Observe that harmonics of specified sequence have zero unbalanced admittance for just this sequence. For harmonics of the zero sequence $\boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n} = 0$, for positive sequence $\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n} = 0$ and for the negative sequence $\boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n} = 0$.

To describe symmetrical components of the unbalanced current $\boldsymbol{i}_{\mathrm{u}n}$ of the n^{th} order harmonic, given by (19), in more explicit form related to the supply voltage harmonics, let us introduce ***symmetrical three-phase unit vectors***:

(28) $$\begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{p}}, \qquad \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{n}}, \qquad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \stackrel{\mathrm{df}}{=} 1^{\mathrm{z}}.$$

With these vectors,

(29) $$\boldsymbol{i}_{\mathrm{u}n} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{Y}^{\mathrm{p}}_{\mathrm{u}n}1^{\mathrm{p}} + \boldsymbol{Y}^{\mathrm{n}}_{\mathrm{u}n}1^{\mathrm{n}} + \boldsymbol{Y}^{\mathrm{z}}_{\mathrm{u}n}1^{\mathrm{z}})\boldsymbol{U}_{\mathrm{R}n}e^{jn\omega_1 t}\}.$$

Symmetrical components of different sequence are mutually orthogonal, so that the three-phase rms values of the components of the unbalanced current $\boldsymbol{i}_{\mathrm{u}n}$ have to satisfy the relation

$$\|\boldsymbol{i}_{\mathrm{u}n}\|^2 = \|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}}\|^2 . \tag{30}$$

Three-phase rms values of symmetrical quantities are by root of three higher than the line rms values, hence

$$\|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}}\| = \sqrt{3}\,Y_{\mathrm{u}n}^{\mathrm{p}} U_{\mathrm{R}n} \tag{31}$$

$$\|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}}\| = \sqrt{3}\,Y_{\mathrm{u}n}^{\mathrm{n}} U_{\mathrm{R}n} \tag{32}$$

$$\|\boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}}\| = \sqrt{3}\,Y_{\mathrm{u}n}^{\mathrm{z}} U_{\mathrm{R}n} . \tag{33}$$

When (16) and (19) are combined, the following decomposition of the load n^{th} order current harmonic is obtained

$$\boldsymbol{i}_n = \boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}} \tag{34}$$

and decomposition of the load current

$$\boldsymbol{i} = \sum_{n\in N} \boldsymbol{i}_n = \sum_{n\in N} (\boldsymbol{i}_{\mathrm{a}n} + \boldsymbol{i}_{\mathrm{r}n} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}}) . \tag{35}$$

Observe that the first term on the right side of (35) is not the active current $\boldsymbol{i}_{\mathrm{a}}$ defined by (11), however. It can be rearranged to the form

$$\begin{aligned}\sum_{n\in N} \boldsymbol{i}_{\mathrm{a}n} &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_{\mathrm{e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} = \\ &= \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_{\mathrm{e}} \boldsymbol{U}_n e^{jn\omega_1 t} + \sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})\, \boldsymbol{U}_n e^{jn\omega_1 t} .\end{aligned} \tag{36}$$

The first term on the right side of (36) is the active current of the load as defined by (10). The second term,

$$\sqrt{2}\,\mathrm{Re}\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})\, \boldsymbol{U}_n e^{jn\omega_1 t} \stackrel{\mathrm{df}}{=} \boldsymbol{i}_{\mathrm{s}} \tag{37}$$

which occurs when equivalent conductances of the load for harmonic frequencies $G_{\mathrm{e}n}$ differ from the load equivalent conductance G_{e}. It will be called the scattered current. The presence of such a current was revealed [10] for the first time in single-phase circuits with LTI loads and nonsinusoidal supply voltage.

Combining (36) and (37) with (35), the following decomposition of the current of three-phase unbalanced loads is obtained

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{s}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} \tag{38}$$

where

$$\boldsymbol{i}_{\mathrm{r}} \stackrel{\mathrm{df}}{=} \sum_{n\in N} \boldsymbol{i}_{\mathrm{r}n} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_{\mathrm{e}n}\, \boldsymbol{U}_n\, e^{jn\omega_1 t} \tag{39}$$

is the reactive current of the load,

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} \stackrel{\mathrm{df}}{=} \sum_{n\in N} \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{p}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}} 1^{\mathrm{p}} \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{40}$$

is the unbalanced current of the positive sequence,

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \sum_{n\in N} \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{n}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}} 1^{\mathrm{n}} \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{41}$$

is the unbalanced current of the negative sequence

$$\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}} \stackrel{\mathrm{df}}{=} \sum_{n\in N} \boldsymbol{i}_{\mathrm{u}n}^{\mathrm{z}} = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}} 1^{\mathrm{z}} \boldsymbol{U}_{\mathrm{R}n}\, e^{jn\omega_1 t} \tag{42}$$

is the unbalanced current of the zero sequence.

The load current components in decomposition (38) are associated with distinctive phenomena in the circuit. The active current $\boldsymbol{i}_{\mathrm{a}}$ is associated with permanent flow of energy to the load; the scattered current $\boldsymbol{i}_{\mathrm{s}}$ is associated with the change of the load equivalent conductance $G_{\mathrm{e}n}$ with harmonic order n; the reactive current is associated with the phase shift between the voltage and current harmonics. Unbalanced currents $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$ are associated with asymmetry of the load current harmonics and consequently, with the presence of the positive, negative and zero sequence components of the unbalanced current.

Orthogonality

The three-phase rms value of the active current is given by (11), while for the remaining components, defined as a sum of terms, harmonic-by-harmonic, of the order n from the set N, the three-phase rms value, can be calculated directly, because harmonics of different orders are mutually orthogonal. All these components are symmetrical, thus, to calculate their three-phase rms value it is enough to multiply the root of the sum of squares of their line rms values by the root of three, namely

$$\|\boldsymbol{i}_{\mathrm{s}}\| = \sqrt{3}\sqrt{\sum_{n\in N} (G_{\mathrm{e}n} - G_{\mathrm{e}})^2 U_{\mathrm{R}n}^2} \tag{43}$$

$$\|\boldsymbol{i}_{\mathrm{r}}\| = \sqrt{3}\sqrt{\sum_{n\in N} B_{\mathrm{e}n}^2 U_{\mathrm{R}n}^2} \tag{44}$$

$$\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{p}})^2 U_{\mathrm{R}n}^2} \tag{45}$$

$$\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{n}})^2 U_{\mathrm{R}n}^2} \tag{46}$$

$$\|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\| = \sqrt{3}\sqrt{\sum_{n\in N} (Y_{\mathrm{u}n}^{\mathrm{z}})^2 U_{\mathrm{R}n}^2} . \tag{47}$$

The three-phase rms value of the load current satisfies the relation

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{s}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}\|^2 \tag{48}$$

on the condition that scalar products of all components in (38) are mutually orthogonal.

The scalar product of three-phase vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ is defined as

$$(\boldsymbol{x}, \boldsymbol{y}) \stackrel{\mathrm{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{x}^{\mathrm{T}}(t)\, \boldsymbol{y}(t)\, dt \tag{49}$$

and can be calculated having vectors X_n and Y_n of crms values of harmonics of $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ as

$$(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} Y_n^{*} . \tag{50}$$

Three phase vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are mutually orthogonal on the condition that their scalar product is zero, i.e.,

$$(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{Re}\sum_{n\in N} X_n^{\mathrm{T}} Y_n^{*} = 0 . \tag{51}$$

Thus, to prove that (48) is valid, it has to be proven that 15 scalar products of six components in (38) are equal to zero.

Let us calculate the scalar product

$$\begin{aligned}(\boldsymbol{i}_{\mathrm{a}}, \boldsymbol{i}_{\mathrm{s}}) &= \mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{a}n}^{\mathrm{T}} \boldsymbol{I}_{\mathrm{s}n}^{*} = \mathrm{Re}\sum_{n\in N} G_{\mathrm{e}}\, \boldsymbol{U}_n^{\mathrm{T}} (G_{\mathrm{e}n} - G_{\mathrm{e}})\, \boldsymbol{U}_n^{*} = \\ &= G_{\mathrm{e}}\Big(\sum_{n\in N} G_{\mathrm{e}n} \|\boldsymbol{u}_n\|^2 - G_{\mathrm{e}} \sum_{n\in N} \|\boldsymbol{u}_n\|^2\Big) = G_{\mathrm{e}}\,(P - P) = 0.\end{aligned} \tag{52}$$

thus the active and the scattered currents are mutually orthogonal.

The active and the reactive currents are mutually orthogonal because they are composed of only components shifted mutually by $\pi/2$. Indeed

$$(\boldsymbol{i}_{\mathrm{a}},\boldsymbol{i}_{\mathrm{r}}) = \mathrm{Re}\sum_{n\in N} \boldsymbol{I}_{\mathrm{a}n}^{\mathrm{T}}\,\boldsymbol{I}_{\mathrm{r}n}^{*} = \mathrm{Re}\sum_{n\in N} G_{\mathrm{e}}\,\boldsymbol{U}_{n}^{\mathrm{T}}(-jB_{\mathrm{e}n})\,\boldsymbol{U}_{n}^{*} = \tag{53}$$
$$= \mathrm{Re}\sum_{n\in N} G_{\mathrm{e}}(-jB_{\mathrm{e}n})\|\boldsymbol{u}_{n}\|^{2} = 0.$$

The same applies to the scattered and the reactive currents, so that

$$(\boldsymbol{i}_{\mathrm{s}},\boldsymbol{i}_{\mathrm{r}}) = 0. \tag{54}$$

The unbalanced currents $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$ are mutually orthogonal because they are of difference sequence, thus

$$(\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}},\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}) = (\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}},\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}) = (\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}},\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}) = 0\ . \tag{55}$$

The active, scattered and the reactive currents can contain harmonics of all sequence thus, their orthogonality to unbalanced currents $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$ remains unclear.

Let us observe that for the supply voltage harmonics of the positive sequence the unbalanced current, according to (26), can contain only components of the negative and zero sequence, thus they are orthogonal to the active, scattered and the reactive currents. For the voltage harmonics of the negative sequence the unbalanced current, according to (27), can contain only components of the zero and positive sequence, thus they are orthogonal to the active, scattered and the reactive currents. Finally, for the voltage harmonics of the zero sequence, the unbalanced current, according to (25), can contain only components of the positive and negative sequence, thus they are also orthogonal to the active, scattered and the reactive currents. It means that the active, scattered and the reactive currents $\boldsymbol{i}_{\mathrm{a}}$, $\boldsymbol{i}_{\mathrm{s}}$ and $\boldsymbol{i}_{\mathrm{r}}$ are orthogonal to unbalanced currents $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$, $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}$. It means that all components of decomposition (38) are mutually orthogonal and the relationship (48) between their three-phase rms value is valid. Thus (38) can be regarded as a decomposition of the load current into the Currents' Physical Components (CPC). Each component is associated with a physical phenomenon observable at the load terminals, and they are mutually orthogonal, meaning they contribute to the load current three-phase rms value independently on each other.

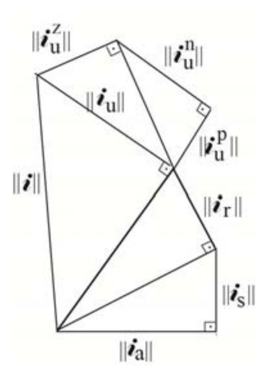

Fig. 6. Diagram of three-phase rms values of CPC.

The relationship (48) can be illustrated, with a polygon shown in Fig. 6, with side length proportional to the three-phase rms values of individual components of the load current.

Orthogonality of six components cannot be illustrated, of course, on a plane. Only two sides can be drawn on a plane as orthogonal and these are only two first terms on the right side of (48). Its sequence can be changed, without affecting the length of diagonal, meaning $\|\boldsymbol{i}\|$, however.

Illustration. Let us assume that the load shown in Fig. 7 is supplied with a symmetrical voltage of the fundamental harmonic rms value U_1 = 240 V, distorted with the 3rd, 5th and 7th order harmonics of relative rms value U_3 = 2% U_1, U_5 = 3% U_1 and U_7 = 1.5% U_1, thus N = {1, 3, 5, 7}.

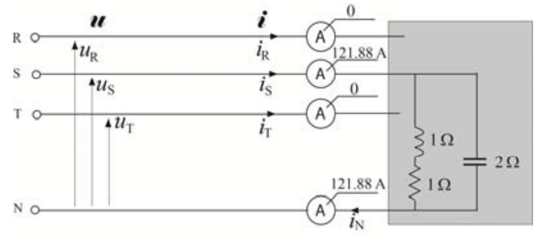

Fig. 7. Example of unbalanced load.

The rms values of the line-to-neutral voltage harmonics and the load admittances are compiled in Table1.

Table 1. Rms values of voltage harmonics and load admittance

n	U_n [V]	$\boldsymbol{Y}_{\mathrm{S}n} = G_{\mathrm{S}n}+jB_{\mathrm{S}n}$ [S]
1	240	0.5
3	4.8	0.1+j1.2
5	7.2	0.0385+j2.308
7	3.6	0.02+j3.36

The active power of the load is

$$P = \sum_{n\in N} G_{\mathrm{S}n}U_{\mathrm{S}n}^{2} = 28.804\ \mathrm{kW}\ .$$

The supply voltage three rms value

$$\|\boldsymbol{u}\| = \sqrt{\sum_{n\in N}\|\boldsymbol{u}_n\|^2} = \sqrt{3}\sqrt{\sum_{n\in N}U_n^2} = 416.01\ \mathrm{V}$$

thus the equivalent conductance of the load is

$$G_{\mathrm{e}} = \frac{P}{\|\boldsymbol{u}\|^2} = 0.165\ \mathrm{S}$$

and the three-phase rms value of the active current is

$$\|\boldsymbol{i}_{\mathrm{a}}\| = G_{\mathrm{e}}\|\boldsymbol{u}\| = \frac{P}{\|\boldsymbol{u}\|} = 69.22\ \mathrm{A}\ .$$

The values of equivalent conductance $G_{\mathrm{e}n}$, susceptance $B_{\mathrm{e}n}$, and magnitude of unbalanced admittances $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{p}}$, $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{n}}$ and $\boldsymbol{Y}_{\mathrm{u}n}^{\mathrm{z}}$ for harmonic frequencies, calculated according to (12), (14) and (25-27), are compiled in Table 2.

Table 2. Equivalent parameters of the load for harmonics

n	$G_{\mathrm{e}n}$[S]	$B_{\mathrm{e}n}$[S]	$Y_{\mathrm{u}n}^{\mathrm{p}}$ [S]	$Y_{\mathrm{u}n}^{\mathrm{n}}$ [S]	$Y_{\mathrm{u}n}^{\mathrm{z}}$ [S]
1	0.167	0	0	0.167	0.167
3	0.033	0.400	0.401	0.401	0
5	0.013	0.769	0.769	0	0.769
7	0.007	1.120	0	1.12	1.12

The three-phase rms value of the scattered current, calculated with (43) is

$$\|\boldsymbol{i}_{\mathrm{s}}\| = \sqrt{3}\sqrt{\sum_{n\in N}[(G_{\mathrm{e}n}-G_{\mathrm{e}})U_n]^2} = 2.43\ \mathrm{A}$$

and the reactive current, calculated with (44)

$$\|\boldsymbol{i}_{\mathrm{r}}\| = \sqrt{3}\sqrt{\sum_{n\in N}(B_{\mathrm{e}n}U_n)^2} = 12.32\ \mathrm{A}.$$

The three-phase rms values of the unbalanced current components calculated with (45-47) are equal to

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}|| = \sqrt{3}\sqrt{\sum_{n\in N}(Y_{\mathrm{u}n}^{\mathrm{p}}U_n)^2} = 10.16\ \mathrm{A}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}|| = \sqrt{3}\sqrt{\sum_{n\in N}(Y_{\mathrm{u}n}^{\mathrm{n}}U_n)^2} = 69.71\ \mathrm{A}$$

$$||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}|| = \sqrt{3}\sqrt{\sum_{n\in N}(Y_{\mathrm{u}n}^{\mathrm{z}}U_n)^2} = 70.29\ \mathrm{A}$$

and consequently, the three-phase rms value of the unbalanced current is

$$||\boldsymbol{i}_{\mathrm{u}}|| = \sqrt{||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||^2} = 99.52\ \mathrm{A}\ .$$

The three-phase rms value of the load current, calculated as the root of sum of squares of rms values of the line currents, is

$$||\boldsymbol{i}|| = \sqrt{||i_{\mathrm{R}}||^2 + ||i_{\mathrm{S}}||^2 + ||i_{\mathrm{T}}||^2} = \sqrt{\sum_{n\in N}(Y_{\mathrm{S}n}U_n)^2} = 121.88\ \mathrm{A}\ .$$

This value can be used for verification of the decomposition of the load current into the active, scattered, reactive and the unbalanced currents, since the root of the sum of squares their three-phase rms values should result in the same value of $||\boldsymbol{i}||$. Indeed

$$||\boldsymbol{i}|| = \sqrt{||\boldsymbol{i}_{\mathrm{a}}||^2 + ||\boldsymbol{i}_{\mathrm{s}}||^2 + ||\boldsymbol{i}_{\mathrm{r}}||^2 + ||\boldsymbol{i}_{\mathrm{u}}||^2} = 121.88\ \mathrm{A}$$

which confirms numerical correctness of the load current decomposition (38).

Powers

Developed above the load current decomposition into the Currents' Physical Components in four-wire systems with nonsinusoidal supply voltage provides straightforward explanation for the Steinmetz observation that the apparent power S at nonsinusoidal conditions can be higher than the active power P. That observation, raised originally in single-phase circuits with an electric arc, applies of course to systems of any complexity.

For systems analyzed in this paper, after multiplying (38) by the square of the three-phase rms value of the supply voltage $||\boldsymbol{u}||$, the difference in square between the apparent power S and the active power P is equal to

$$S^2 - P^2 = D_{\mathrm{s}}^2 + Q^2 + D_{\mathrm{u}}^{\mathrm{p}2} + D_{\mathrm{u}}^{\mathrm{n}2} + D_{\mathrm{u}}^{\mathrm{z}2}\ . \tag{56}$$

All powers on the right side of (56), namely, the ***scattered power***:

$$D_{\mathrm{s}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{s}}||, \tag{57}$$

the ***reactive power***:

$$Q \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{r}}||, \tag{58}$$

the ***unbalanced power of the positive sequence***:

$$D_{\mathrm{u}}^{\mathrm{p}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||, \tag{59}$$

the ***unbalanced power of the negative sequence***:

$$D_{\mathrm{u}}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||, \tag{60}$$

and the ***unbalanced power of the zero sequence***:

$$D_{\mathrm{u}}^{\mathrm{z}} \stackrel{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{z}}||, \tag{61}$$

are only formal products, like the apparent power S, of the supply voltage and CPC three-phase rms values.

Nonetheless, since these powers are associated with distinctive phenomena in the load and the load properties, eqn. (56) provides quantitative explanation for the question of why the apparent power S in four-wire systems with LTI loads and nonsinusoidal supply voltage can be higher than the active power P.

Conclusions

The paper provides explanation of power properties of linear time-invariant loads supplied with nonsinusoidal voltage in three-phase systems with a neutral conductor. It demonstrates that the load current can be decomposed into mutually orthogonal components associated with distinctive physical phenomena. This decomposition is based on the voltage and current measurement on the load terminals. The current components can be expressed in terms of measurable parameters of the load. This decomposition enables development of the power equation of three-phase LTI loads in four-wire systems with nonsinusoidal supply voltage.

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