Leszek S. CZARNECKI, Prashanna BHATTARAI
School of Electrical Engineering and Computer Sciences, Louisianan State University, Baton Rouge, USA

doi:10.15199/48.2015.06.06

Currents' Physical Components (CPC) in Three-Phase Systems with Asymmetrical Voltage

Abstract. *Energy flow related phenomena in three-phase unbalanced, linear, time-invariant (LTI) loads, supplied with asymmetrical, but sinusoidal voltage, in three-wire systems, are investigated in the paper. It is demonstrated that the load current can be decomposed into Currents' Physical Components (CPC), associated with distinctive physical phenomena in the load. It is also shown how the CPC can be expressed in terms of the supply voltage and equivalent parameters of the load. An equivalent circuit of LTI loads at asymmetrical, but sinusoidal supply voltage is presented as well. This decomposition provides solid fundamentals for defining powers of such loads.*

Streszczenie. *Artykuł przedstawia wyniki badań nad zjawiskami energetycznymi w liniowych, czasowo-niezmienniczych (LTI) odbiornikach niezrównoważonych, zasilanych niesymetrycznym, lecz sinusoidalnym napięciem w obwodach trójprzewodowych. Pokazano, że prąd takich odbiorników może być rozłożony na Składowe Fizyczne, jednoznacznie stowarzyszone z określonymi zjawiskami fizycznymi. Pokazano także, że prądy te mogą być określone poprzez napięcie zasilania i parametry równoważne odbiornika. Przedstawiono także obwód równoważny niezrównoważonych odbiorników LTI, zasilanych niesymmetrycznym, lecz sinusoidalnym napięciem. Rozkład ten tworzy solidne podstawy dla definicji mocy takich odbiorników.* **(*Składowe Fizyczne Prądów w obwodach trójfazowych z niesymetrycznym napięciem zasilania*)**.

Keywords: Current decomposition, unbalanced loads, asymmetrical systems, power definitions, power theory.
Słowa kluczowe: Rozkład prądu, odbiorniki niezrównoważone, systemy niesymetryczne, definicje mocy, teoria mocy.

Introduction

Residential distribution systems, systems in commercial buildings or electrical traction grids can be regarded from a utility perspective as slowly time-varying aggregates of mainly single-phase loads, supplied from a three-phase, three-wire distribution system, usually through a transformer in Δ/Y configuration, as shown in Fig. 1.

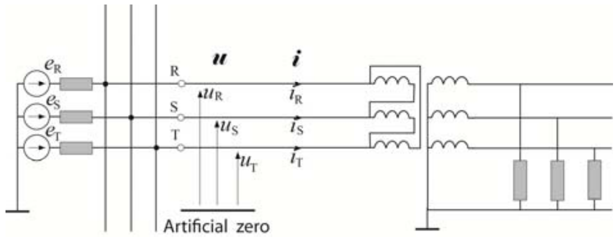

Fig. 1. Three-phase load composed of aggregates of single-phase loads.

Symbols $\boldsymbol{u}$ and $\boldsymbol{i}$ in this figure denote three-phase vectors of line-to-artificial zero voltages and line currents, namely

$$\boldsymbol{u} \stackrel{\text{df}}{=} [u_R,\ u_S,\ u_T]^T, \qquad \boldsymbol{i} \stackrel{\text{df}}{=} [i_R,\ i_S,\ i_T]^T .$$

Even AC arc furnaces can be regarded as three separate single-phase arcs furnaces, in a common cage, i.e., three single-phase loads supplied from a three-wire system.

Due to a potential imbalance, such systems differ as to power properties from system dominated by three-phase loads, usually motors or rectifiers.

It could be a surprising observation that in spite of the fact that considerable amount of energy produced in power systems is distributed just in systems as shown in Fig. 1, the power theory enables now their description in power terms only on the condition that the supply voltage is symmetrical. It could be regarded as a remarkable deficiency of the power theory. Consequently, such loads can be described in power terms only approximately, at the assumption that the supply voltage is symmetrical. Unfortunately, with the lack of power definitions valid at asymmetrical voltage, even the error of such approximation cannot be evaluated.

Studies on powers in asymmetrical three-phase systems have a century long history, but these studies are not concluded even now. They were initiated by Steinmetz [1] and Lyon [3], while the main mathematical tool for these studies, in a form of the concept of symmetrical components, was provided by Fortesque [2].

Difficulties with the development of the power theory of asymmetrical three-phase systems have started with the question on how to select the definition of the apparent power.

The American Institute of Electrical Engineers (AIEE) adopted [4] in 1920 two different definitions of the apparent power, namely: ***arithmetic apparent power:***

$$S = S_A = U_R I_R + U_S I_S + U_T I_T \tag{1}$$

and ***geometric apparent power:***

$$S = S_G = \sqrt{P^2 + Q^2}\ . \tag{2}$$

A debate [6-8] on which one of these two definitions is right was inconclusive. Consequently, both were supported by the IEEE Standard Dictionary of Electrical and Electronics Terms [18]. At the same time, a definition of this power suggested in 1922 by Buchholz in [5], namely

$$S = S_B = \sqrt{U_R^2 + U_S^2 + U_T^2}\ \sqrt{I_R^2 + I_S^2 + I_T^2} \tag{3}$$

was not supported by the IEEE Standard.

Eventually it was proven in [21] that the arithmetic and geometric definitions of the apparent power in systems with unbalanced loads provide an incorrect value of the power factor, while the right value of this factor at sinusoidal voltages and currents is obtained only when the Buchholtz definition (3) is used.

There is considerable amount of literature on various approaches to description power properties of three-phase systems, with some results published even recently [22-26] and studies on this subject are still not completed.

Most of studies [9-11, 13-17, 19] have focused the attention on power definitions at nonsinusoidal supply voltage. Unfortunately, at a wrong definition of the apparent power S, even at sinusoidal voltages and currents, it was not possible to develop neither the right definitions of electric powers of three-phase loads nor the right power equation. This issue for symmetrical supply voltages was eventually solved in [17], however, for asymmetrical supply voltages both the power definitions and the power equation have yet to be developed. Their development is just the subject of this paper. Powers in systems with unbalanced loads have become the object of interest in [12], but still at a symmetrical supply voltage.

The Currents' Physical Components (CPC) provides a conceptual frame for studies in this paper. It is based on three basic ideas:

(i) The supply current of the load is a core quantity in the circuit for the power theory development. This prerequisite is in a contrast to approaches based on a power as such a core quantity.

(ii) The supply current decomposition into mutually orthogonal components. Orthogonality makes the rms value of the load current independent on mutual interactions of the current components.

(iii) The supply current components should be associated with distinctive physical phenomena in the circuit. This last prerequisite gave the name to this theory: Currents' Physical Components (CPC) power theory.

These basic ideas of the CPC power theory were originally applied [13] to single-phase LTI loads with non-sinusoidal supply voltage and next to loads with sequentially increasing complexity with respect to their structure as well as voltages and currents waveforms. This paper can be regarded as a next step in this theory's development, now applied to unbalanced LTI loads with asymmetrical, but sinusoidal supply voltage.

Apart from the CPC approach, the concept of an unbalanced current and unbalanced power are essential for these studies. Originally, the concept of the unbalanced power was introduced in [17] for an unbalanced load with nonsinusoidal, but symmetrical supply voltage. This concept, confined to LTI loads operated at such conditions, is outlined in the following Section.

Original concept of unbalanced power

An equivalent circuit of linear stationary LTI loads as seen from the primary side of a Δ/Y transformer, as shown in Fig. 1, can have the form shown in Fig. 2. As it was proven in [20] there is infinite number of such circuits, equivalent with respect to the load currents.

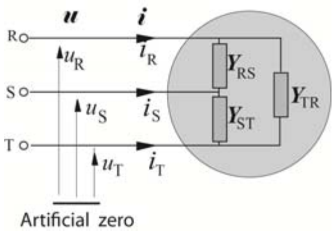

Fig. 2. Equivalent circuit of three-phase loads.

The three-phase vectors of the load voltages and currents, can be expressed in the form

(4) $$\boldsymbol{u}(t) \overset{\text{df}}{=} \begin{bmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{U}_R \\ \boldsymbol{U}_S \\ \boldsymbol{U}_S \end{bmatrix} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{U\, e^{j\omega t}\}$$

(5) $$\boldsymbol{i}(t) \overset{\text{df}}{=} \begin{bmatrix} i_R(t) \\ i_S(t) \\ i_T(t) \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} \boldsymbol{I}_R \\ \boldsymbol{I}_S \\ \boldsymbol{I}_T \end{bmatrix} e^{j\omega t} = \sqrt{2}\,\mathrm{Re}\{I\, e^{j\omega t}\}.$$

In these formulas symbols U and I denote three-phase vectors of complex rms (crms) values $\boldsymbol{U}_R$, $\boldsymbol{U}_S$, and $\boldsymbol{U}_T$ of line voltages, measured with respect to an artificial zero, and line currents $\boldsymbol{I}_R$, $\boldsymbol{I}_S$, and $\boldsymbol{I}_T$.

For three-phase vectors of sinusoidal quantities, denoted generally by $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$, of the same frequency, a scalar product

(6) $$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\,\boldsymbol{y}(t)\,dt$$

and three-phase rms value

(7) $$\|\boldsymbol{x}\| \overset{\text{df}}{=} \sqrt{(\boldsymbol{x}, \boldsymbol{x})} = \sqrt{\frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\,\boldsymbol{x}(t)\,dt}$$

can be defined [17]. The scalar product, defined by (6) in the time-domain, can be calculated in the frequency-domain, having vectors of crms values of these quantities X and Y, as follows

(8) $$(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{T}\int_0^T \boldsymbol{x}^T(t)\,\boldsymbol{y}(t)\,dt = \mathrm{Re}\{X^T Y^*\}.$$

Two vectors $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ are mutually orthogonal on the condition that

(9) $$(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{Re}\{X^T Y^*\} = 0$$

and consequently, three-phase rms values of such quantities satisfy the relationship

(10) $$\|\boldsymbol{x} + \boldsymbol{y}\|^2 = \|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2.$$

The scalar product of the supply voltage and the load current vectors is equal to the active power P of the load,

(11) $$(\boldsymbol{u}, \boldsymbol{i}) \overset{\text{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{u}^T(t)\,\boldsymbol{i}(t)\,dt = \mathrm{Re}\{U^T I^*\} = P.$$

The power equation developed in [17] for LTI loads of the structure shown in Fig. 1 at sinusoidal and symmetrical and supply voltages, but asymmetrical currents has the form

(12) $$S^2 = P^2 + Q^2 + D_u^2.$$

The apparent power in this equation was defined, according to the Buchholz definition (3), as the product of voltages and currents three-phase rms values, equal to

(13) $$\|\boldsymbol{u}\| \overset{\text{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{u}^T(t)\,\boldsymbol{u}(t)\,dt = \sqrt{U_R^2 + U_S^2 + U_T^2}$$

(14) $$\|\boldsymbol{i}\| \overset{\text{df}}{=} \frac{1}{T}\int_0^T \boldsymbol{i}^T(t)\,\boldsymbol{i}(t)\,dt = \sqrt{I_R^2 + I_S^2 + I_T^2}.$$

Symbols P and Q in the power equation (12) denote common active and reactive powers, which can be directly measured at the load terminals. Symbol D_u denotes the ***unbalanced power***, defined as

(15) $$D_u = A\|\boldsymbol{u}\|^2.$$

The symbol A denotes the magnitude of the ***unbalanced admittance*** of the load, specified in terms of equivalent line-to-line admittances, as follows

(16) $$\boldsymbol{A} = A e^{j\psi} = -(\boldsymbol{Y}_{ST} + \alpha \boldsymbol{Y}_{TR} + \alpha^* \boldsymbol{Y}_{RS}), \quad \alpha = 1 e^{j2\pi/3}.$$

The unbalanced power was also defined in IEEE Std. 1459 [24]. It was defined as

(17) $$S_U = \sqrt{S^2 - (P^p)^2 - (Q^p)^2}$$

where P^{p} and Q^{p} denote the active and reactive powers, but only of the voltage and current symmetrical component of the positive sequence. The power defined by formulae (17) share only the adjective "unbalanced", with that defined by (15). These are two different powers. Formula (17) cannot be rearranged to a power equation, since it neglects energy delivered to the load by the negative sequence component of voltages and currents.

CPC decomposition at asymmetrical voltage

Apparent power S of single-phase loads and in balanced three-phase loads with sinusoidal voltages and currents is equal to the magnitude of the ***complex apparent power*** $\boldsymbol{S}$ which for single-phase systems is defined as

$$\boldsymbol{S} = \boldsymbol{U}\boldsymbol{I}^* = S e^{j\varphi} = P + jQ. \tag{18}$$

When the load is unbalanced and/or voltages are asymmetrical or nonsinusoidal then the apparent power S is no longer the magnitude of the complex apparent power $\boldsymbol{S}$. Unfortunately, similarity of symbols for both powers may cause confusion and even may lead to errors. Since it is a very common custom of denoting the apparent power by S, a clearly different symbol is used in this paper for the power defined as

$$\boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}^* = P + jQ = \boldsymbol{C} = C e^{j\varphi}. \tag{19}$$

Also the adjective "apparent" will not be used. The quantity defined by (19) will be referred to as a ***complex power***.

With respect to active and reactive powers P and Q at the supply voltage $\boldsymbol{u}$, the unbalanced load shown in Fig. 1 is equivalent to a balanced load shown in Fig. 3, on the condition that its phase admittances are equal to

$$Y_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{\|\boldsymbol{u}\|^2} = \frac{\boldsymbol{C}^*}{\|\boldsymbol{u}\|^2}. \tag{20}$$

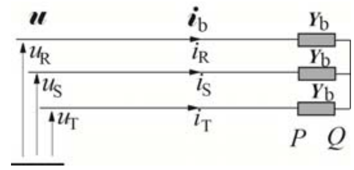

Fig. 3. Balanced load, which is equivalent to the original load with respect to active and reactive powers P and Q.

Indeed, the complex power $\boldsymbol{C}_{\mathrm{b}}$ of such load is

$$\boldsymbol{C}_{\mathrm{b}} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{b}}^* = \boldsymbol{U}^{\mathrm{T}}(Y_{\mathrm{b}}\boldsymbol{U})^* = Y_{\mathrm{b}}^*\|\boldsymbol{u}\|^2 = P + jQ = \boldsymbol{C}. \tag{21}$$

The supply voltage $\boldsymbol{u}$ is sinusoidal, but it can be asymmetrical. Thus it can be decomposed into a sum of symmetrical voltages of the positive $\boldsymbol{u}^{\mathrm{p}}$ and negative $\boldsymbol{u}^{\mathrm{n}}$ sequence, so that

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}})\,\mathrm{e}^{j\omega t}\}. \tag{22}$$

A zero sequence symmetrical component $\boldsymbol{u}^{\mathrm{z}}$ cannot cause any current flow in three-wire systems. Thus it can be neglected or line voltages should be measured with respect to an artificial zero of the system, so the voltage vector $\boldsymbol{u}$ would not contain any zero sequence component.

Let us define unit three-phase vectors of the positive and negative sequence

$$\mathbf{1}^{\mathrm{p}} \stackrel{\mathrm{df}}{=} \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{-j2\pi/3} \\ 1e^{j2\pi/3} \end{bmatrix}, \quad \mathbf{1}^{\mathrm{n}} \stackrel{\mathrm{df}}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix} \tag{23}$$

shown in 4.

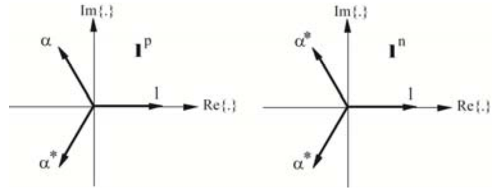

Fig. 4. Unit three-phase vectors $\mathbf{1}^{\mathrm{p}}$ and $\mathbf{1}^{\mathrm{n}}$.

The asymmetrical supply voltage $\boldsymbol{u}$ can be expressed with these vectors as

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{p}} + \boldsymbol{u}^{\mathrm{n}} = \sqrt{2}\,\mathrm{Re}\{(\mathbf{1}^{\mathrm{p}}U^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}U^{\mathrm{n}})\,\mathrm{e}^{j\omega t}\} \tag{24}$$

where

$$\begin{bmatrix} U^{\mathrm{p}} \\ U^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\begin{bmatrix} U_{\mathrm{R}} \\ U_{\mathrm{S}} \\ U_{\mathrm{T}} \end{bmatrix}. \tag{25}$$

Since Y_{b} in (20) is admittance of a balanced load, which is equivalent to the original load with respect to the active and reactive powers, it will be referred to as the ***equivalent balanced admittance***. Such an equivalent balanced load draws the current

$$\boldsymbol{i}_{\mathrm{b}} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}_{\mathrm{b}}\,\mathrm{e}^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{Y_{\mathrm{b}}\boldsymbol{U}\,\mathrm{e}^{j\omega t}\} \tag{26}$$

composed of the active current

$$\begin{aligned} \boldsymbol{i}_{\mathrm{a}} &= G_{\mathrm{b}}\boldsymbol{u} = \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{b}}(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}})e^{j\omega t}\} = \\ &= \sqrt{2}\,\mathrm{Re}\{G_{\mathrm{b}}(\mathbf{1}^{\mathrm{p}}U^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}U^{\mathrm{n}})e^{j\omega t}\} \end{aligned} \tag{27}$$

and the reactive current

$$\begin{aligned} \boldsymbol{i}_{\mathrm{r}} &= B_{\mathrm{b}}\boldsymbol{u}(t+T/4) = \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{b}}(\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}})e^{j\omega t}\} = \\ &= \sqrt{2}\,\mathrm{Re}\{jB_{\mathrm{b}}(\mathbf{1}^{\mathrm{p}}U^{\mathrm{p}} + \mathbf{1}^{\mathrm{n}}U^{\mathrm{n}})e^{j\omega t}\}. \end{aligned} \tag{28}$$

The remaining current of the load, after the current of the balanced load is subtracted, is caused by the load imbalance

$$\boldsymbol{i} - \boldsymbol{i}_{\mathrm{b}} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{I} - \boldsymbol{I}_{\mathrm{b}})\mathrm{e}^{j\omega t}\} \stackrel{\mathrm{df}}{=} \boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{\boldsymbol{I}_{\mathrm{u}}\,\mathrm{e}^{j\omega t}\}. \tag{29}$$

Consequently, the load current is decomposed into the active, reactive and unbalanced current components, such that

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}. \tag{30}$$

Mutual orthogonality of the active and reactive currents results from their mutual phase shift by $\pi/2$. Orthogonality of the balanced and unbalanced current has to be proven. Indeed

$$\begin{aligned} (\boldsymbol{i}_{\mathrm{b}}, \boldsymbol{i}_{\mathrm{u}}) &= \mathrm{Re}\{\boldsymbol{I}_{\mathrm{b}}^{\mathrm{T}}(\boldsymbol{I} - \boldsymbol{I}_{\mathrm{b}})^*\} = \mathrm{Re}\{Y_{\mathrm{b}}\boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}^* - Y_{\mathrm{b}}\boldsymbol{U}^{\mathrm{T}}Y_{\mathrm{b}}^*\boldsymbol{U}^*\} = \\ &= \mathrm{Re}\{Y_{\mathrm{b}}(\boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}^* - \boldsymbol{U}^{\mathrm{T}}Y_{\mathrm{b}}^*\boldsymbol{U}^*)\} = \\ &= \mathrm{Re}\{Y_{\mathrm{b}}(\boldsymbol{C} - \boldsymbol{C}_{\mathrm{b}})\} = 0 \end{aligned} \tag{31}$$

thus these three current components are mutually orthogonal and consequently

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_{\mathrm{a}}\|^2 + \|\boldsymbol{i}_{\mathrm{r}}\|^2 + \|\boldsymbol{i}_{\mathrm{u}}\|^2. \tag{32}$$

Each current component in decomposition (30) is distinctively associated with a unique physical phenomenon in the circuit, thus they can be regarded as Currents' Physical Components, (CPC).

This decomposition can be performed, and the three-phase rms values of each particular current can be measured or calculated by measurements of active and reactive powers P and Q at the load terminals as well as crms values $\boldsymbol{I}_{\mathrm{R}}$, $\boldsymbol{I}_{\mathrm{S}}$, and $\boldsymbol{I}_{\mathrm{T}}$ of the load currents.

Multiplying (32) by the square of the three-phase rms value $||\boldsymbol{u}||$ of the supply voltage

$$\{\ ||\boldsymbol{i}\,||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2\ \} \times \ \ ||\boldsymbol{u}\,||^2$$

the power equation

$$S^2 = P^2 + Q^2 + D_{\rm u}^2 \qquad (33)$$

is obtained, with the unbalanced power defined as

$$D_{\rm u} \stackrel{\rm df}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\rm u}||\,. \qquad (34)$$

This power equation is identical with eqn. (12), but developed without the assumption that the supply voltage is symmetrical. Thus, the supply voltage asymmetry does not affect the general form of the power equation of stationary LTI loads with sinusoidal supply voltages.

Power equation (34) and the values of the active, reactive and unbalanced powers provide distinctive information on how permanent flow of energy to the load; the phase-shift between the supply voltage and the load current, as well as the load current asymmetry affect the apparent power S.

The unbalanced current $\boldsymbol{i}_{\rm u}$ in formula (29) is not expressed in terms of the load parameters, however. It only fills a gap between the load current and its active and reactive components. The same applies to the unbalanced power $D_{\rm u}$. Definition (34) has no analogy to definition (15). It is possible to calculate its value, but it cannot be used in a design process of a reactive compensator that would compensate this power. A dependence of the unbalanced power on the circuit parameters is needed for that.

Therefore, let us find how the unbalanced current and power depend on the circuit parameters.

The active and reactive currents in circuits with symmetrical supply voltage are symmetrical currents, such that

$$\boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} = \sqrt{2}\,{\rm Re}\{(G_{\rm e}+jB_{\rm e})\boldsymbol{U}\,{\rm e}^{j\omega t}\} = \sqrt{2}\,{\rm Re}\{\boldsymbol{Y}_{\rm e}\boldsymbol{U}\,{\rm e}^{j\omega t}\} \qquad (35)$$

where, according to [17],

$$\boldsymbol{Y}_{\rm e} \stackrel{\rm df}{=} G_{\rm e} + jB_{\rm e} = \boldsymbol{Y}_{\rm ST} + \boldsymbol{Y}_{\rm TR} + \boldsymbol{Y}_{\rm RS} \qquad (36)$$

is the equivalent admittance of the load. It is a phase admittance of a balanced load, which is equivalent to the original one with respect to the active and reactive powers P and Q.

When the supply voltage is asymmetrical then, according to formulas (27) and (28) the active and reactive currents follow the voltage asymmetry.

The phase admittance $\boldsymbol{Y}_{\rm b}$ of the equivalent balanced load is different than the equivalent admittance $\boldsymbol{Y}_{\rm e}$, because

$$\boldsymbol{Y}_{\rm b} = G_{\rm b} + jB_{\rm b} = \frac{P - jQ}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{Y}_{\rm RS}U_{\rm RS}^2 + \boldsymbol{Y}_{\rm ST}U_{\rm ST}^2 + \boldsymbol{Y}_{\rm TR}U_{\rm TR}^2}{||\boldsymbol{u}||^2}\,. \qquad (37)$$

At symmetrical supply voltage $U_{\rm RS} = U_{\rm ST} = U_{\rm TR} = ||\boldsymbol{u}||$, so that $\boldsymbol{Y}_{\rm b} = \boldsymbol{Y}_{\rm e}$. If the voltage is asymmetrical then admittances $\boldsymbol{Y}_{\rm e}$ and $\boldsymbol{Y}_{\rm b}$ differ mutually by admittance $\boldsymbol{Y}_{\rm d}$,

$$\boldsymbol{Y}_{\rm d} = G_{\rm d} + jB_{\rm d} \stackrel{\rm df}{=} \boldsymbol{Y}_{\rm e} - \boldsymbol{Y}_{\rm b} \qquad (38)$$

dependent on the voltage asymmetry. This asymmetry can be specified quantitatively by a ***complex coefficient of the supply voltage asymmetry***, defined as

$$\frac{\boldsymbol{U}^{\rm n}}{\boldsymbol{U}^{\rm p}} = \frac{U^{\rm n}e^{j\varphi}}{U^{\rm p}e^{j\phi}} = \frac{U^{\rm n}}{U^{\rm p}}e^{j(\varphi-\phi)} \stackrel{\rm df}{=} \boldsymbol{a} = a\,e^{j\psi}\,. \qquad (39)$$

When the voltage asymmetry is specified by this coefficient, then the difference between admittances $\boldsymbol{Y}_{\rm b}$ and $\boldsymbol{Y}_{\rm e}$ is equal to

$$\boldsymbol{Y}_{\rm d} = \frac{2a}{1+a^2}[\boldsymbol{Y}_{\rm ST}\cos\psi + \boldsymbol{Y}_{\rm TR}\cos(\psi - \tfrac{2\pi}{3}) + \boldsymbol{Y}_{\rm RS}\cos(\psi + \tfrac{2\pi}{3})]\,. \qquad (40)$$

Admittance $\boldsymbol{Y}_{\rm d}$ depends not only on the line-to-line admittances of the load, but also on the supply voltage asymmetry. When the load is balanced, i.e., $\boldsymbol{Y}_{\rm RS} = \boldsymbol{Y}_{\rm TR} = \boldsymbol{Y}_{\rm ST} = \boldsymbol{Y}_{\rm e}/3$, then $\boldsymbol{Y}_{\rm d} = 0$, independently on the supply voltage asymmetry. When the supply voltage is symmetrical and consequently, asymmetry coefficient $\boldsymbol{a} = 0$, then $\boldsymbol{Y}_{\rm d} = 0$, independently on the load imbalance. Therefore, admittance $\boldsymbol{Y}_{\rm d}$ is referred to as a ***voltage asymmetry dependent unbalanced admittance*** in this paper. Admittance $\boldsymbol{Y}_{\rm d}$ can have a non-zero value only if the load is unbalanced and the supply voltage is asymmetrical.

The vector of crms values of unbalanced current $\boldsymbol{I}_{\rm u}$ in the load supply lines can be decomposed, as shown in Appendix A, as follows

$$\boldsymbol{I}_{\rm u} = \boldsymbol{Y}_{\rm d}\boldsymbol{U} + \boldsymbol{1}^{\rm n}\boldsymbol{A}^{\rm p}\boldsymbol{U}^{\rm p} + \boldsymbol{1}^{\rm p}\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm n} \qquad (41)$$

where

$$\boldsymbol{A}^{\rm p} \stackrel{\rm df}{=} -(\boldsymbol{Y}_{\rm ST} + \alpha\boldsymbol{Y}_{\rm TR} + \alpha^*\boldsymbol{Y}_{\rm RS}) \qquad (42)$$

$$\boldsymbol{A}^{\rm n} \stackrel{\rm df}{=} -(\boldsymbol{Y}_{\rm ST} + \alpha^*\boldsymbol{Y}_{\rm TR} + \alpha\boldsymbol{Y}_{\rm RS}) \qquad (43)$$

are unbalanced admittances for the positive and the negative sequence voltages. The term

$$\boldsymbol{1}^{\rm n}\boldsymbol{A}^{\rm p}\boldsymbol{U}^{\rm p} \stackrel{\rm df}{=} \boldsymbol{J}^{\rm n} \qquad (44)$$

in formula (41) stands for a vector of crms values of symmetrical currents of negative sequence proportional the positive sequence voltage, while

$$\boldsymbol{1}^{\rm p}\boldsymbol{A}^{\rm n}\boldsymbol{U}^{\rm n} \stackrel{\rm df}{=} \boldsymbol{J}^{\rm p} \qquad (45)$$

stands for a vector of crms values of symmetrical currents of positive sequence proportional the negative sequence voltage.

Thus, the vectors of the active, reactive and unbalanced currents, $\boldsymbol{i}_{\rm a}$, $\boldsymbol{i}_{\rm r}$ and $\boldsymbol{i}_{\rm u}$ can be specified in terms of four admittances, $\boldsymbol{Y}_{\rm b}$, $\boldsymbol{Y}_{\rm d}$, $\boldsymbol{A}^{\rm p}$ and $\boldsymbol{A}^{\rm n}$, which can be expressed in terms of line-to-line admittances $\boldsymbol{Y}_{\rm RS}$, $\boldsymbol{Y}_{\rm ST}$, and $\boldsymbol{Y}_{\rm TR}$, line-to-line supply voltage rms values and the coefficient of its asymmetry $\boldsymbol{a}$.

Equivalent circuit

The original unbalanced LTI load supplied with asymmetrical voltage can be regarded as a parallel connection of two balanced loads with phase admittance $\boldsymbol{Y}_{\rm b}$ and $\boldsymbol{Y}_{\rm d}$, respectively, and two symmetrical current sources, connected as shown in Fig. 5, which inject two three-phase currents $\boldsymbol{j}^{\rm n}$ and $\boldsymbol{j}^{\rm p}$. The balanced loads draw asymmetrical currents of the crms value proportional to admittance $\boldsymbol{Y}_{\rm b}$ and $\boldsymbol{Y}_{\rm d}$. Three-phase currents $\boldsymbol{j}^{\rm n}$ and $\boldsymbol{j}^{\rm p}$ are symmetrical currents proportional to the positive and negative sequence components of the supply voltage $\boldsymbol{u}^{\rm p}$ and $\boldsymbol{u}^{\rm n}$, but of opposite sequence to those voltages, according to formulae (44) and (45). All parameters of such equivalent circuit are expressed in terms of line-to-line admittances $\boldsymbol{Y}_{\rm RS}$, $\boldsymbol{Y}_{\rm ST}$, and $\boldsymbol{Y}_{\rm TR}$, of the Δ configured equivalent circuit shown in Fig. 2.

Such a circuit can be regarded as an ***equivalent circuit of unbalanced loads*** supplied with sinusoidal, but asymmetrical voltage. It visualizes the complex nature of the unbalanced current $\boldsymbol{i}_{\rm u}$. This nature could be irrelevant when only its rms value or/and the unbalanced power have to be known. Knowledge of this nature could yet be crucial for a process of design of a reactive compensator that would be capable to compensate the unbalanced current. A study on a possibility of reactive compensation of such loads is beyond of the scope of this paper, however.

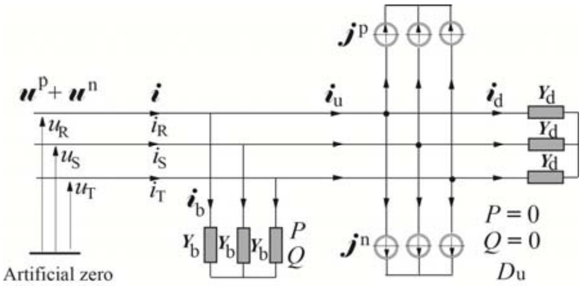

Fig. 5. Equivalent circuit of unbalanced load.

The balanced branch with admittance $\boldsymbol{Y}_b$ has the active and reactive powers equal to P and Q, respectively, because this admittance was calculated, according to formula (20), just to satisfy such a condition.

The branch with the unbalanced current $\boldsymbol{i}_u$, has to have zero active and reactive powers, P and Q, because these two powers of the original load are equal, according to formula (20), to the powers of the branch with current $\boldsymbol{i}_b$, while the whole equivalent circuit has to satisfy the balance principle with respect to the active and reactive powers. The only non-zero power of this branch could be the unbalanced power D_u.

Illustration. Let us calculate physical components of the load current in the circuit shown in Fig. 6, with strongly asymmetrical supply voltage and strongly unbalanced load. The active and reactive powers of the load are equal, respectively, to P = 10.0 kW and Q = 10.0 kvar.

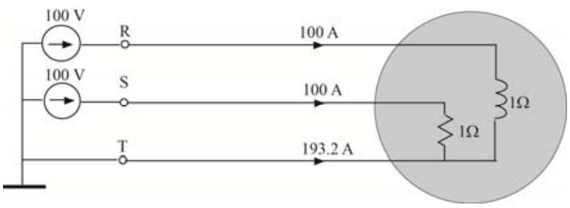

Fig. 6. Example of unbalanced load with asymmetrical supply voltage.

Complex rms values of the positive and negative sequence symmetrical components, $\boldsymbol{U}^{p}$ and $\boldsymbol{U}^{n}$, of the supply voltage, calculated according to (25), are equal to

$$\begin{bmatrix} \boldsymbol{U}^{p} \\ \boldsymbol{U}^{n} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\begin{bmatrix} 100 \\ 100\, e^{-j120^{\circ}} \\ 0 \end{bmatrix} = \begin{bmatrix} 66.66 \\ 33.33\, e^{\,j60^{\circ}} \end{bmatrix} \mathrm{V}.$$

The three-phase rms values of the supply voltage symmetrical components are

$$\|\boldsymbol{u}^{p}\| = \sqrt{3}\, U^{p} = \sqrt{3}\times 66.66 = 115.47\ \mathrm{V}$$

$$\|\boldsymbol{u}^{n}\| = \sqrt{3}\, U^{n} = \sqrt{3}\times 33.33 = 57.73\ \mathrm{V}$$

and consequently, three-phase rms value of the supply voltage is

$$\|\boldsymbol{u}\| = \sqrt{\|\boldsymbol{u}^{p}\|^2 + \|\boldsymbol{u}^{n}\|^2} = \sqrt{115.47^2 + 57.73^2} = 129.1\ \mathrm{V}$$

while the vector of the supply voltage with respect to the artificial zero is

$$U = \mathbf{1}^{p}\boldsymbol{U}^{p} + \mathbf{1}^{n}\boldsymbol{U}^{n} = \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} 66.7 + \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} 33.3\, e^{j60^{\circ}} = \begin{bmatrix} 88.2\, e^{j19.1^{\circ}} \\ 88.2\, e^{-j139.1^{\circ}} \\ 33.3\, e^{j120^{\circ}} \end{bmatrix} \mathrm{V}.$$

The equivalent balanced admittance $\boldsymbol{Y}_b$ of the load in the circuit shown in Fig. 3 is equal to

$$\boldsymbol{Y}_b = G_b + jB_b = \frac{P - jQ}{\|\boldsymbol{u}\|^2} = 0.600 - j\,0.600\ \mathrm{S}$$

hence, the active current vector has the waveform

$$\boldsymbol{i}_a = \sqrt{2}\,\mathrm{Re}\{\mathbf{I}_a\, e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{G_b\,(\mathbf{1}^{p}\boldsymbol{U}^{p} + \mathbf{1}^{n}\boldsymbol{U}^{n})\, e^{j\omega t}\} =$$

$$= \sqrt{2}\,\mathrm{Re}\{0.60(\mathbf{1}^{p}\times 66.7 + \mathbf{1}^{n}\times 3.33 e^{j60^{\circ}})\, e^{j\omega t}\} =$$

$$= \sqrt{2}\,\mathrm{Re}\{\begin{bmatrix} 52.9\, e^{j19.1^{\circ}} \\ 52.9\, e^{-j139.1^{\circ}} \\ 20.0\, e^{j120^{\circ}} \end{bmatrix} e^{j\omega t}\}\ \mathrm{A}.$$

The vector of the reactive current is

$$\boldsymbol{i}_r = \sqrt{2}\,\mathrm{Re}\{I_r\, e^{j\omega t}\} = \sqrt{2}\,\mathrm{Re}\{jB_b\,(\mathbf{1}^{p}\boldsymbol{U}^{p} + \mathbf{1}^{n}\boldsymbol{U}^{n})\, e^{j\omega t}\} =$$

$$= \sqrt{2}\,\mathrm{Re}\{\begin{bmatrix} 52.9\, e^{-j70.9^{\circ}} \\ 52.9\, e^{j130.9^{\circ}} \\ 20.0\, e^{j30^{\circ}} \end{bmatrix} e^{j\omega t}\}\ \mathrm{A}.$$

The unbalanced current vector can be presented in the form

$$\boldsymbol{i}_u = \sqrt{2}\mathrm{Re}\{(I - I_a - I_r)\mathrm{e}^{j\omega t}\} = \sqrt{2}\mathrm{Re}\{\begin{bmatrix} 95.2\, e^{-j135^{\circ}} \\ 95.2\, e^{-j75^{\circ}} \\ 164.9\, e^{j75^{\circ}} \end{bmatrix} e^{j\omega t}\}\ \mathrm{A}.$$

Three-phase rms values of the current components are equal to

$$\|\boldsymbol{i}_a\| = G_b\,\|\boldsymbol{u}\| = 0.60\times 129.1 = 77.46\ \mathrm{A}$$

$$\|\boldsymbol{i}_r\| = |B_b|\,\|\boldsymbol{u}\| = 0.60\times 129.1 = 77.46\ \mathrm{A}$$

$$\|\boldsymbol{i}_u\| = \sqrt{I_{uR}^2 + I_{uS}^2 + I_{uT}^2} = \sqrt{95.2^2 + 95.2^2 + 164.9^2} = 212.9\mathrm{A}.$$

The supply current has the three-phase rms value

$$\|\boldsymbol{i}\| = \sqrt{I_R^2 + I_S^2 + I_T^2} = \sqrt{100^2 + 100^2 + 193.2^2} = 239.4\ \mathrm{A}$$

and indeed

$$\sqrt{\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_r\|^2 + \|\boldsymbol{i}_u\|^2} = \sqrt{77.46^2 + 77.46^2 + 212.9^2} = 239.4\mathrm{A}$$

which confirms numerical correctness of the current decomposition into physical components.

The load power factor $\lambda = P/S = \|\boldsymbol{i}_a\|/\|\boldsymbol{i}\| = 0.32$. The unbalanced power is equal to

$$D_u = \|\boldsymbol{u}\|\times\|\boldsymbol{i}_u\| = 129.1\times 212.9 = 27.5\ \mathrm{kVA}.$$

To find parameters of the equivalent circuit of the load let us calculate unbalanced admittances $\boldsymbol{A}^{p}$ and $\boldsymbol{A}^{n}$, namely

$$\boldsymbol{A}^{\mathrm{p}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}}) = -[1+\alpha(-j1)] = 1.932\,e^{-j165^{\mathrm{o}}}\ \mathrm{S}$$

$$\boldsymbol{A}^{\mathrm{n}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{TR}} + \alpha\boldsymbol{Y}_{\mathrm{RS}}) = -[1+\alpha^{*}(-j1)] = 0.518\,e^{-j105^{\mathrm{o}}}\mathrm{S}\ .$$

The complex coefficient of the supply voltage asymmetry is equal to

$$\boldsymbol{a} = a\,e^{j\psi} = \frac{\boldsymbol{U}^{\mathrm{n}}}{\boldsymbol{U}^{\mathrm{p}}} = \frac{33.33\,e^{j60^{\mathrm{o}}}}{66.66} = 0.50\,e^{j60^{\mathrm{o}}}$$

therefore, the asymmetry dependent unbalanced admittance is equal to

$$\boldsymbol{Y}_{\mathrm{d}} = \frac{2a}{1+a^{2}}[\boldsymbol{Y}_{\mathrm{ST}}\cos\psi + \boldsymbol{Y}_{\mathrm{TR}}\cos(\psi - \frac{2\pi}{3}) + \boldsymbol{Y}_{\mathrm{RS}}\cos(\psi + \frac{2\pi}{3})] =$$

$$= \frac{2\times 0.5}{1+0.5^{2}}[\cos(60^{\mathrm{o}}) - j\cos(60^{\mathrm{o}} - 120^{\mathrm{o}})] = 0.566\,e^{-j45^{\mathrm{o}}}\ \mathrm{S}.$$

With these parameters of the equivalent circuit, the vector of the unbalanced current crms values is equal to

$$\boldsymbol{I}_{\mathrm{u}} = \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \mathbf{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}} + \mathbf{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} = 0.566\,e^{-j45^{\mathrm{o}}}\begin{bmatrix}\boldsymbol{U}_{\mathrm{R}}\\ \boldsymbol{U}_{\mathrm{S}}\\ \boldsymbol{U}_{\mathrm{T}}\end{bmatrix} +$$

$$+1.93\,e^{-j165^{\mathrm{o}}}\begin{bmatrix}1\\ \alpha\\ \alpha^{*}\end{bmatrix}\boldsymbol{U}^{\mathrm{p}} + 0.518\,e^{-j105^{\mathrm{o}}}\begin{bmatrix}1\\ \alpha^{*}\\ \alpha\end{bmatrix}\boldsymbol{U}^{\mathrm{n}} = \begin{bmatrix}95.2\,e^{-j135^{\mathrm{o}}}\\ 95.2\,e^{-j75^{\mathrm{o}}}\\ 164.9\,e^{j75^{\mathrm{o}}}\end{bmatrix}\mathrm{A}.$$

It could be checked that the complex power of the branch of the equivalent circuit with the unbalanced current $\boldsymbol{i}_{\mathrm{u}}$ is

$$\boldsymbol{C}_{\mathrm{u}} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{I}_{\mathrm{u}}^{*} = P_{\mathrm{u}} + jQ_{\mathrm{u}} = 0.$$

This confirms numerical correctness of calculation of the unbalanced current, thus correctness of calculation of the parameters of the equivalent circuit.

****

Formula (41) shows that the unbalanced current is an intricate quantity. Unbalanced admittances $\boldsymbol{A}^{\mathrm{p}}$ and $\boldsymbol{A}^{\mathrm{n}}$ depend only on the load parameters, while the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\mathrm{d}}$ depends moreover on the supply voltage asymmetry. Admittances $\boldsymbol{A}^{\mathrm{p}}$ and $\boldsymbol{A}^{\mathrm{n}}$ can have different values and consequently, dependence of the unbalanced current on the supply voltage positive and negative sequence components can be different.

Since the equivalent balanced admittance $\boldsymbol{Y}_{\mathrm{b}} = \boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{d}}$ and consequently, $G_{\mathrm{b}} = G_{\mathrm{e}} - G_{\mathrm{d}}$ and $B_{\mathrm{b}} = B_{\mathrm{e}} - B_{\mathrm{d}}$, the active and reactive powers can be decomposed into components independent of the voltage asymmetry and dependent on it, namely

$$P = G_{\mathrm{b}}||\boldsymbol{u}||^{2} = (G_{\mathrm{e}} - G_{\mathrm{d}})||\boldsymbol{u}||^{2} = P_{\mathrm{s}} - P_{\mathrm{d}} \tag{46}$$

where P_{S} denotes the load active power at a symmetrical supply voltage, but with the same rms value as the asymmetrical one. The power P_{d} occurs because of the supply voltage asymmetry, but it disappears, independently of this asymmetry, when the load is balanced.

Similarly, the reactive power

$$Q = -B_{\mathrm{b}}||\boldsymbol{u}||^{2} = -(B_{\mathrm{e}} - B_{\mathrm{d}})||\boldsymbol{u}||^{2} = Q_{\mathrm{s}} - Q_{\mathrm{d}} \tag{47}$$

where Q_{s} denotes the reactive power at symmetrical supply voltage, while the power Q_{d} occurs because of the supply voltage asymmetry in presence of the load imbalance.

Observe that the unbalanced current contains both positive and negative sequence components, since the vector

$$\mathbf{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}} + \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U}^{\mathrm{n}} \overset{\mathrm{df}}{=} \boldsymbol{I}_{\mathrm{u}}^{\mathrm{n}} \tag{48}$$

is a vector of crms values of the supply currents of the negative sequence, while the vector

$$\mathbf{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}} + \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U}^{\mathrm{p}} \overset{\mathrm{df}}{=} \boldsymbol{I}_{\mathrm{u}}^{\mathrm{p}} \tag{49}$$

is a vector of crms values of the positive sequence currents. Thus, the unbalanced current can be expressed in the form

$$\boldsymbol{i}_{\mathrm{u}} = \sqrt{2}\,\mathrm{Re}\{(\boldsymbol{I}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{I}_{\mathrm{u}}^{\mathrm{n}})e^{j\omega t}\} = \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}} \tag{50}$$

so that, the load current can be decomposed into four components

$$\boldsymbol{i} = \boldsymbol{i}_{\mathrm{a}} + \boldsymbol{i}_{\mathrm{r}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}} + \boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}\ . \tag{51}$$

These components are mutually orthogonal, so that their three-phase rms values satisfy the relationship

$$||\boldsymbol{i}||^{2} = ||\boldsymbol{i}_{\mathrm{a}}||^{2} + ||\boldsymbol{i}_{\mathrm{r}}||^{2} + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||^{2} + ||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||^{2}\ . \tag{52}$$

The active current $\boldsymbol{i}_{\mathrm{a}}$ is associated exclusively with permanent energy transfer from the supply source to the load, meaning with the load active power P. The reactive current $\boldsymbol{i}_{\mathrm{r}}$ is associated exclusively with the phase-shift between the supply voltage and the load current, meaning with the load reactive power Q. These two currents are asymmetrical currents and their asymmetry reproduces the asymmetry of the supply voltage. Currents $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}$ and $\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}$ are symmetrical currents, which occur exclusively due to the load imbalance. They do not contribute to the active and reactive powers P and Q of the load, but only to an increase of its three-phase rms value. Therefore, these four components of the load current can be regarded as the Currents' Physical Components (CPC).

Multiplying eqn. (52) by the square of the three-phase rms value of the supply voltage, the power equation is obtained in the form

$$S^{2} = P^{2} + Q^{2} + D_{\mathrm{u}}^{\mathrm{p}2} + D_{\mathrm{u}}^{\mathrm{n}2} \tag{53}$$

with

$$D_{\mathrm{u}}^{\mathrm{p}} \overset{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{p}}||, \qquad D_{\mathrm{u}}^{\mathrm{n}} \overset{\mathrm{df}}{=} ||\boldsymbol{u}||\,||\boldsymbol{i}_{\mathrm{u}}^{\mathrm{n}}||. \tag{54}$$

Conclusions

The paper shows that the basic ideas of the Currents' Physical Components power theory can be applied to unbalanced three-phase LTI loads supplied with sinusoidal, but asymmetrical voltage. It enables decomposition of the load current into orthogonal components associated with distinctive physical phenomena in the circuit and to describe the load in power terms. Results presented in this paper enable to remove one of deficiencies of the power theory of electrical circuits, namely, the lack of a power equation in the situation when an LTI unbalanced load is supplied with sinusoidal, but asymmetrical voltage.

Appendix

Equivalent admittances

Let an LTI unbalanced load has an equivalent circuit as shown in Fig. 2.

The complex power $\boldsymbol{C}$ of a three-phase load is defined as

$$\boldsymbol{C} = \boldsymbol{U}^{\mathrm{T}}\boldsymbol{i}^{*} = \boldsymbol{U}_{\mathrm{R}}\boldsymbol{I}_{\mathrm{R}}^{*} + \boldsymbol{U}_{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}}^{*} + \boldsymbol{U}_{\mathrm{T}}\boldsymbol{I}_{\mathrm{T}}^{*}$$

hence

$$\begin{aligned}\boldsymbol{C} &= \boldsymbol{U}_{\mathrm{R}}\boldsymbol{I}_{\mathrm{R}}^{*} + \boldsymbol{U}_{\mathrm{S}}\boldsymbol{I}_{\mathrm{S}}^{*} + \boldsymbol{U}_{\mathrm{T}}\boldsymbol{I}_{\mathrm{T}}^{*} = \\ &= \boldsymbol{U}_{\mathrm{R}}(\boldsymbol{I}_{\mathrm{RS}}^{*} - \boldsymbol{I}_{\mathrm{TR}}^{*}) + \boldsymbol{U}_{\mathrm{S}}(\boldsymbol{I}_{\mathrm{ST}}^{*} - \boldsymbol{I}_{\mathrm{RS}}^{*}) + \boldsymbol{U}_{\mathrm{T}}(\boldsymbol{I}_{\mathrm{TR}}^{*} - \boldsymbol{I}_{\mathrm{ST}}^{*}) = \\ &= \boldsymbol{U}_{\mathrm{RS}}\boldsymbol{I}_{\mathrm{RS}}^{*} + \boldsymbol{U}_{\mathrm{ST}}\boldsymbol{I}_{\mathrm{ST}}^{*} + \boldsymbol{U}_{\mathrm{TR}}\boldsymbol{I}_{\mathrm{TR}}^{*} = \\ &= \boldsymbol{U}_{\mathrm{RS}}\boldsymbol{Y}_{\mathrm{RS}}^{*}\boldsymbol{U}_{\mathrm{RS}}^{*} + \boldsymbol{U}_{\mathrm{ST}}\boldsymbol{Y}_{\mathrm{ST}}^{*}\boldsymbol{U}_{\mathrm{ST}}^{*} + \boldsymbol{U}_{\mathrm{TR}}\boldsymbol{Y}_{\mathrm{TR}}^{*}\boldsymbol{U}_{\mathrm{TR}}^{*} = \\ &= \boldsymbol{Y}_{\mathrm{RS}}^{*}U_{\mathrm{RS}}^{2} + \boldsymbol{Y}_{\mathrm{ST}}^{*}U_{\mathrm{ST}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}^{*}U_{\mathrm{TR}}^{2} \\ &= (P_{\mathrm{RS}} + jQ_{\mathrm{RS}}) + (P_{\mathrm{ST}} + jQ_{\mathrm{ST}}) + (P_{\mathrm{TR}} + jQ_{\mathrm{TR}}) = P + jQ.\end{aligned}$$

It can be also expressed directly as the sum of complex powers of three single-phase loads configured in Δ as shown in Fig. 2, namely

$$\begin{aligned}\boldsymbol{C} &= \boldsymbol{C}_{\mathrm{RS}} + \boldsymbol{C}_{\mathrm{ST}} + \boldsymbol{C}_{\mathrm{TR}} = \boldsymbol{U}_{\mathrm{RS}}\boldsymbol{I}_{\mathrm{RS}}^{*} + \boldsymbol{U}_{\mathrm{ST}}\boldsymbol{I}_{\mathrm{ST}}^{*} + \boldsymbol{U}_{\mathrm{TR}}\boldsymbol{I}_{\mathrm{TR}}^{*} = \\ &= \boldsymbol{Y}_{\mathrm{RS}}^{*}U_{\mathrm{RS}}^{2} + \boldsymbol{Y}_{\mathrm{ST}}^{*}U_{\mathrm{ST}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}^{*}U_{\mathrm{TR}}^{2}.\end{aligned}$$

The complex power for individual branches can be expressed as follows

$$\boldsymbol{C}_{\mathrm{RS}} = \boldsymbol{U}_{\mathrm{RS}}\boldsymbol{I}_{\mathrm{RS}}^{*} = \boldsymbol{U}_{\mathrm{RS}}\boldsymbol{Y}_{\mathrm{RS}}^{*}\boldsymbol{U}_{\mathrm{RS}}^{*} = \boldsymbol{Y}_{\mathrm{RS}}^{*}(U_{\mathrm{R}}^{2} + U_{\mathrm{S}}^{2} - 2\,\mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}}\boldsymbol{U}_{\mathrm{S}}^{*}\}).$$

Since

$$U_{\mathrm{T}}^{2} = (-\boldsymbol{U}_{\mathrm{R}} - \boldsymbol{U}_{\mathrm{S}})(-\boldsymbol{U}_{\mathrm{R}} - \boldsymbol{U}_{\mathrm{S}})^{*} = U_{\mathrm{R}}^{2} + U_{\mathrm{S}}^{2} + 2\mathrm{Re}\{\boldsymbol{U}_{\mathrm{R}}\boldsymbol{U}_{\mathrm{S}}^{*}\}$$

thus,

(A1) $$\boldsymbol{C}_{\mathrm{RS}} = \boldsymbol{Y}_{\mathrm{RS}}^{*}(2U_{\mathrm{R}}^{2} + 2U_{\mathrm{S}}^{2} - U_{\mathrm{T}}^{2}) = \boldsymbol{Y}_{\mathrm{RS}}^{*}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{T}}^{2}).$$

Similarly

(A2) $$\boldsymbol{C}_{\mathrm{ST}} = \boldsymbol{U}_{\mathrm{ST}}\boldsymbol{Y}_{\mathrm{ST}}^{*}\boldsymbol{U}_{\mathrm{ST}}^{*} = \boldsymbol{Y}_{\mathrm{ST}}^{*}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{R}}^{2})$$

(A3) $$\boldsymbol{C}_{\mathrm{TR}} = \boldsymbol{U}_{\mathrm{TR}}\boldsymbol{Y}_{\mathrm{TR}}^{*}\boldsymbol{U}_{\mathrm{TR}}^{*} = \boldsymbol{Y}_{\mathrm{TR}}^{*}(2\,||\boldsymbol{u}||^{2} - 3U_{\mathrm{S}}^{2}).$$

The equivalent balanced admittance of the load, defined by eqn. (20), can be expressed with eqs. (A1) – (A3) in the form

(A4) $$\begin{aligned}\boldsymbol{Y}_{\mathrm{b}} &= \frac{\boldsymbol{C}^{*}}{||\boldsymbol{u}||^{2}} = \frac{\boldsymbol{C}_{\mathrm{RS}}^{*} + \boldsymbol{C}_{\mathrm{ST}}^{*} + \boldsymbol{C}_{\mathrm{TR}}^{*}}{||\boldsymbol{u}||^{2}} = \\ &= 2\boldsymbol{Y}_{\mathrm{e}} - \frac{3}{||\boldsymbol{u}||^{2}}(\boldsymbol{Y}_{\mathrm{ST}}U_{\mathrm{R}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}U_{\mathrm{S}}^{2} + \boldsymbol{Y}_{\mathrm{RS}}U_{\mathrm{T}}^{2}) \overset{\mathrm{df}}{=} \boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{d}}\end{aligned}$$

where

$$\boldsymbol{Y}_{\mathrm{e}} = G_{\mathrm{e}} + jB_{\mathrm{e}} = \boldsymbol{Y}_{\mathrm{ST}} + \boldsymbol{Y}_{\mathrm{TR}} + \boldsymbol{Y}_{\mathrm{RS}}.$$

is the equivalent admittance of the load when it is supplied with a symmetrical voltage, and

(A5) $$\boldsymbol{Y}_{\mathrm{d}} \overset{\mathrm{df}}{=} \frac{3}{||\boldsymbol{u}||^{2}}(\boldsymbol{Y}_{\mathrm{ST}}U_{\mathrm{R}}^{2} + \boldsymbol{Y}_{\mathrm{TR}}U_{\mathrm{S}}^{2} + \boldsymbol{Y}_{\mathrm{RS}}U_{\mathrm{T}}^{2}) - \boldsymbol{Y}_{\mathrm{e}}.$$

Let us express this admittance in terms of crms values of symmetrical components of the positive sequence $\boldsymbol{U}^{\mathrm{p}}$ and the negative sequence $\boldsymbol{U}^{\mathrm{n}}$**.** Since

(A6) $$\boldsymbol{U}_{\mathrm{R}} = \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}},\; \boldsymbol{U}_{\mathrm{S}} = \alpha^{*}\boldsymbol{U}^{\mathrm{p}} + \alpha\boldsymbol{U}^{\mathrm{n}},\; \boldsymbol{U}_{\mathrm{T}} = \alpha\boldsymbol{U}^{\mathrm{p}} + \alpha^{*}\boldsymbol{U}^{\mathrm{n}}$$

then

$$\begin{aligned}U_{\mathrm{R}}^{2} &= U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2\mathrm{Re}\{\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\} \\ U_{\mathrm{S}}^{2} &= U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2\mathrm{Re}\{\alpha^{*}\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\} \\ U_{\mathrm{T}}^{2} &= U^{\mathrm{p}2} + U^{\mathrm{n}2} + 2\mathrm{Re}\{\alpha\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\}.\end{aligned}$$

The crms values $\boldsymbol{U}^{\mathrm{p}}$ and $\boldsymbol{U}^{\mathrm{n}}$ have the form

$$\boldsymbol{U}^{\mathrm{p}} = U^{\mathrm{p}}e^{j\phi}, \qquad \boldsymbol{U}^{\mathrm{n}} = U^{\mathrm{n}}e^{j\varphi}$$

therefore, if we denote

$$\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}} = U^{\mathrm{p}}U^{\mathrm{n}}e^{j(\varphi-\phi)} \overset{\mathrm{df}}{=} \boldsymbol{W} = We^{j\psi}$$

admittance $\boldsymbol{Y}_{\mathrm{d}}$, given by (A5), can be expressed as

(A7) $$\boldsymbol{Y}_{\mathrm{d}} = 2\,\frac{\boldsymbol{Y}_{\mathrm{ST}}\mathrm{Re}\{\boldsymbol{W}\} + \boldsymbol{Y}_{\mathrm{TR}}\mathrm{Re}\{\alpha^{*}\boldsymbol{W}\} + \boldsymbol{Y}_{\mathrm{RS}}\mathrm{Re}\{\alpha\boldsymbol{W}\}}{U^{\mathrm{p}2} + U^{\mathrm{n}2}}.$$

When the supply voltage asymmetry is specified by complex asymmetry coefficient $\boldsymbol{a}$, then

$$\frac{\mathrm{Re}\{\boldsymbol{U}^{\mathrm{p}*}\boldsymbol{U}^{\mathrm{n}}\}}{U^{\mathrm{p}2} + U^{\mathrm{n}2}} = \frac{U^{\mathrm{p}}U^{\mathrm{n}}}{U^{\mathrm{p}2} + U^{\mathrm{n}2}}\mathrm{Re}\{e^{j(\varphi-\phi)}\} = \frac{a}{1+a^{2}}\cos\psi$$

and consequently, the asymmetry dependent unbalanced admittance $\boldsymbol{Y}_{\mathrm{d}}$ can be rearranged to the form

(A8) $$\boldsymbol{Y}_{\mathrm{d}} = \frac{2a}{1+a^{2}}[\boldsymbol{Y}_{\mathrm{ST}}\cos\psi + \boldsymbol{Y}_{\mathrm{TR}}\cos(\psi - \frac{2\pi}{3}) + \boldsymbol{Y}_{\mathrm{RS}}\cos(\psi + \frac{2\pi}{3})].$$

The crms value in line R current is equal to

$$\boldsymbol{I}_{\mathrm{R}} = \boldsymbol{Y}_{\mathrm{RS}}(\boldsymbol{U}_{\mathrm{R}} - \boldsymbol{U}_{\mathrm{S}}) - \boldsymbol{Y}_{\mathrm{TR}}(\boldsymbol{U}_{\mathrm{T}} - \boldsymbol{U}_{\mathrm{R}})$$

and can be rearranged to the form

(A9) $$\boldsymbol{I}_{\mathrm{R}} = \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{R}} - (\boldsymbol{Y}_{\mathrm{ST}}\boldsymbol{U}_{\mathrm{R}} + \boldsymbol{Y}_{\mathrm{TR}}\boldsymbol{U}_{\mathrm{T}} + \boldsymbol{Y}_{\mathrm{RS}}\boldsymbol{U}_{\mathrm{S}}).$$

If crms values of line voltages $\boldsymbol{U}_{\mathrm{R}}$**,** $\boldsymbol{U}_{\mathrm{S}}$ and $\boldsymbol{U}_{\mathrm{T}}$ are expressed in terms of symmetrical components, i.e., with formula (A7), then formula (A9) can be rearranged to

$$\boldsymbol{I}_{\mathrm{R}} = \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{R}} + \boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{R}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{R}}^{\mathrm{n}}$$

where

(A10) $$\boldsymbol{A}^{\mathrm{p}} \overset{\mathrm{df}}{=} -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{RS}})$$

(A11) $$\boldsymbol{A}^{\mathrm{n}} \overset{\mathrm{df}}{=} -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha^{*}\boldsymbol{Y}_{\mathrm{TR}} + \alpha\boldsymbol{Y}_{\mathrm{RS}}).$$

Similarly, the crms value of lines S and T currents can be presented in the form

$$\begin{aligned}\boldsymbol{I}_{\mathrm{S}} &= \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{S}} + \boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{T}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{T}}^{\mathrm{n}} \\ \boldsymbol{I}_{\mathrm{T}} &= \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U}_{\mathrm{T}} + \boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}_{\mathrm{S}}^{\mathrm{p}} + \boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}_{\mathrm{S}}^{\mathrm{n}}.\end{aligned}$$

These three crms values of supply line currents can be expressed in the vector form

(A12) $$\boldsymbol{i} = \begin{bmatrix}\boldsymbol{I}_{\mathrm{R}} \\ \boldsymbol{I}_{\mathrm{S}} \\ \boldsymbol{I}_{\mathrm{T}}\end{bmatrix} = \boldsymbol{Y}_{\mathrm{e}}\boldsymbol{U} + \mathbf{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}} + \mathbf{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}$$

so that, the vector of unbalanced currents is equal to

$$\boldsymbol{i}_{\mathrm{u}} = \boldsymbol{i} - \boldsymbol{i}_{\mathrm{b}} = (\boldsymbol{Y}_{\mathrm{e}} - \boldsymbol{Y}_{\mathrm{b}})\boldsymbol{U} + \mathbf{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}} + \mathbf{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}$$

or it can be rearranged as follows

(A13) $$\boldsymbol{i}_{\mathrm{u}} = \boldsymbol{Y}_{\mathrm{d}}\boldsymbol{U} + \boldsymbol{J}^{\mathrm{n}} + \boldsymbol{J}^{\mathrm{p}}$$

where

(A14) $$\boldsymbol{J}^{\mathrm{n}} = \mathbf{1}^{\mathrm{n}}\boldsymbol{A}^{\mathrm{p}}\boldsymbol{U}^{\mathrm{p}}, \qquad \boldsymbol{J}^{\mathrm{p}} = \mathbf{1}^{\mathrm{p}}\boldsymbol{A}^{\mathrm{n}}\boldsymbol{U}^{\mathrm{n}}.$$

REFERENCES

[1] Steinmetz Ch.P., Theory and calculation of electrical apparatur, *McGraw-Hill Book Comp*., New York, 1917.

[2] Fortescue C.L., Method of symmetrical components as applied to the solution of poly-phase networks, *AIEE* 37, (1918), pp. 1027-1140.

[3] Lyon W.V., Reactive power and unbalanced circuits, *Electrical World*, (1920), pp. 1417-1420.

[4] AIEE Committee, Apparent power in three-phase systems, *Trans. of AIEE*, (1920), Vol. 39, pp. 1450-1455.

[5] Buchholz F., Drehstrom-Scheinleistung bei unlaichmaβiger Belastung der drei Zweige, *Licht und Kraft*, (1922), pp. 9-11.

[6] Smith V.G., Reactive and fictitious power, Trans. of AIEE, (1933), Vol. 52, pp. 748-751.

[7] Fortesque C.L., Powers, reactive volt-ampers, power factor; Trans. of AIEE, (1933), Vol. 52, pp. 758-762.

[8] Curtis H.L., Silsbee F.B., Definitions of power and related quantities, *Trans. of AIEE*, (1935), Vol. 54, pp. 394-404.

[9] Quade, W., Zusammensetzung der Wirk-, Blind-, und Scheinleistung bei Wechselstromen beliebiger Kurvenform und neue Leistungsdefinition fur unsymetrische Mehrphasensysteme belibieger Kurvenform, *ETZ*, (1937), pp. 1312-1320.

[10] Rosenzweig I., Symbolic multidimensional vector analysis as a method of analysis of multi-phase systems, (in Polish), *Czasopismo Techniczne,* (1939), Tom L VI, pp. 1-29.

[11] Nedelcu V.N., Die enheitliche Leistungstheorie der unsymmetrischen mehrwelligen Mehrphasensysteme, *ETZ-A*, (1963), No. 5, pp. 153-157.

[12] Nowomiejski Z., Cichowska Z., Unbalanced three-phase systems, (in Polish). *Zaszyty Naukowe Polit. Śl., ELEKTRYKA,* (1964), *V*. 17 No. 98, pp. 25-76.

[13] Czarnecki L.S., Orthogonal current components of linear loads current, (in Polish) *Zaszyty Naukowe Polit. Śl., ELEKTRYKA,* (1983), No. 86, pp. 5-17, Gliwice, Poland.

[14] Akagi H., Kanazawa Y., Nabae A., Instantaneous reactive power compensator comprising switching devices without energy storage components, *IEEE Trans. IA*, (1984), IA-20, No. 3, pp. 625-630.

[15] Fryze S., Theorethical and physical fundamentals for the active, reactive and apparemt power definitions in multi-phase systems with distorted voltages and currents, (in Polish), *Zeszyty Naukowe Pol. Śl. ELEKTRYKA,* (1985), No. 100, pp. 29-46, posthumos publication.

[16] Koch, K., Bestimmung von Groβen in Mehr-Leitr Systemen, *etz Archiv*, (1986), Vol. 8, No. 10, pp. 313-318.

[17] Czarnecki L.S., Orthogonal decomposition of the current in a three-phase non-linear asymmetrical circuit with nonsinusoidal voltage, *IEEE Trans. on Instr. and Meas.,* (1988), Vol. IM-37, No. 1, pp. 30-34.

[18] Institute of Electrical and Electronic Engineers, *The New Standard Dictionary of Electrical and Electronics Terms,* 1992.

[19] Depenbrock M., The FDB-method, a generalized applicable tool for analyzing power relations, *IEEE Transations on Power Deliv.*, (1993), Vol. 8, No. 2, pp. 381-387.

[20] Czarnecki L.S., Equivalent circuits of unbalanced loads supplied with symmetrical and asymmetrical voltage and their identification, *Archiv fur Elektrot.*, (1995), 78 pp. 165-168.

[21] Czarnecki L.S., Energy flow and power phenomena in electrical circuits: illusions and reality, *Archiv fur Elektrotechnik*, (82), (1999), No. 4, pp. 10-15.

[22] de Leon F., Cohen J., A practical approach to power factor definitions: transmission losses, reactive power compensation, and machine utilization, *Proc. of the Power Eng. Soc. 2006 Meeting*, IEEE Dig. Obj. Ident.: 10.1109/PES.2006.1709175.

[23] Morsi W.G., El-Hawary M.E., Defining power components in nonsinusoidal unbalanced polyphase systems: the issues, *IEEE Transactions on Power Delivery,* (2007), Vol. 22, No. 4, pp. 2428-2438.

[24] IEEE, Standard definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced and unbalanced conditions. *IEEE 1459-2010*.

[25] Czarnecki L.S., Powers in three-wire systems with neutral conductor, *Przegląd Elektrotechniczny,* (2013), R. 89, No. 9, pp. 267-273.

[26] Czarnecki L.S., Haley P.M., Unbalanced power in four-wire systems and its reactive compensation, *IEEE Trans. on Power Delivery,* (2015), Vol. 30, No. 1, pp. 53-63.

Authors: *Prof. dr hab. inż. Leszek S. Czarnecki, IEEE Life Fellow, Alfredo M. Lopez Distinguished Professor, School of Electrical Eng. and Computer Science, Louisiana State Univ., Baton Rouge, USA, LA 70803, lsczar@cox.net, www.lsczar.info*

Prashanna Bhattarai, M.S., School of Electrical Engineering and Computer Science, Louisiana State University, Baton Rouge, USA, LA 70803, pbhatt3@lsu.edu