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Reduction of Losses and Harmonic Distortion in Distribution Networks with Different Load Models by Placing Shunt Capacitors using Integer Genetic Algorithm

***Abstract**. The non-linear integer problem of capacitor placement in distorted distribution systems is solved with discrete values of capacitor sizes employing integer genetic algorithm with controlled random initialization. The objective function contains yearly system operation costs including cost of energy losses and capacitor prices. Different load models along with harmonic sources modelled as constant current sources are considered in the process of optimization. The results show the importance of load variation, modelling and harmonic sources on the optimal solution.*

***Streszczenie.** W pracy rozwiązany został nieliniowy problem rozmieszczenia pojemności w zakłóconym systemie rozproszonym, wykorzystując genetyczny algorytm ze sterowaną randomiczną inicjalizacją. Funkcja celu zawiera roczny koszt systemu operacyjnego, włączając koszty straty energii i ceny kondensatorów. Modele różnych obciążeń ze źródłami harmonicznymi zamodelowanymi jako źródła prądu o stałej wartości zostały przeanalizowane w procesie optymalizacji. Wyniki pokazują ważność zmienności obciążenia, modelowania i źródeł harmonicznych dla optymalnego rozwiązania. (**Redukcja strat I zakłóceń harmonicznych w systemie rozproszonym z modelowaniem zmiennego obciążenia z pojemnościami bocznikowymi za pomocą algorytmu genetycznego**)*

Keywords: Loss Reduction, Harmonic Distortion, Genetic Algorithm, Capacitors
Słowa kluczowe: redukcja strat, zakłócenia harmoniczne, algorytm genetyczny, algorytm, pojemności.

Introduction

Optimal capacitor placement has been a challenge for power system planners and researchers for many years. The goal is to find a set of locations where optimal values of capacitors can be installed in order to reduce the total power and energy losses while keeping the voltage profile of the network within certain limits. Increased presence of harmonic producing loads and sources in the recent years complicate this problem even more, therefore enforcing additional constraints during the network operation, such as keeping the total harmonic distortion within predefined limits.

The problem of optimal capacitor placement has been treated in various different ways in the past. Approaches ranged from master-slave [1], non-linear programming [2], simulated annealing [3] and heuristic methods [4-9]. In all of the aforementioned methods, loads were modelled as constant power. However, as the system gets more loaded, the voltage dependency of the loads becomes more important in the representation [10], since it makes the algorithm performance and the results, highly dependent on the load model. In addition, here we use constant current and impedance models for load representation and show their importance on voltage level influence and power losses, which are crucial parameters during the decision process of optimal capacitor placement.

Harmonic sources are also considered and modelled as constant current sources [4-9]. Integer genetic algorithm with controlled random initialization using penalty function approach is employed. Iterative load flow calculations are performed using sparse matrix representation of voltage nodal equations.

Problem Formulation

The non-linear problem of capacitor placement in distribution system is solved with discrete values of capacitor sizes and selection of their locations. The objective function's comprised of yearly system operation costs including costs for power and energy losses and capacitor installation (1):

$$F = C_{\mathrm{e}} \cdot \Delta W + C_{\mathrm{p}} \cdot \Delta P + p \cdot \sum_{k \in \boldsymbol{C}} \left(C_{\mathrm{f}} + C_{\mathrm{v}} \cdot Q_{\mathrm{c},k} \right) \quad (1)$$

where C_{e} is electricity price (\$/kWh), ΔW are yearly electricity losses (kWh), C_{p} is price for peak power (\$/kW), ΔP are power losses at maximum load (kW), p is a fraction of total investments that are paid in one year, C_{f} are fixed installation costs (\$), C_{v} are capacitors costs per unit size (\$/kvar), $Q_{\mathrm{c,k}}$ is capacitor size at location k (discrete values of kvar) and $\boldsymbol{C}$ is the set of locations where capacitors are installed (discrete values).

In (1), the most numerically demanding is the calculation of energy losses since it requires non-linear power flow solution for the fundamental harmonic and a series of harmonic voltage calculations with harmonic current injections at locations with harmonic-producing loads. Since we're using genetic algorithms, the procedure for calculation of (1) will be ran in the range of thousands, so the load flow calculations should be made as efficient as possible.

Two sets of constraints are imposed on bus voltages regarding their RMS value and total harmonic distortion (THD). RMS value of bus voltages is constrained with upper and lower bounds, as in (2):

$$U^{\min} \le \sqrt{\sum_{h} \left[U_i^{(h)} \right]^2} \le U^{\max}, i = 1..N \quad (2)$$

where $U_i^{(\mathrm{h})}$ is the voltage RMS value at bus i for harmonic h and N is the number of system buses. Voltage distortion constraint is considered by specifying an upper limit on THD denoted with $\mathrm{THD}^{\max}$ as in (3):

$$\mathrm{THD}_i = \frac{100}{U_i^{(1)}} \cdot \sqrt{\sum_{h \ne 1} \left[U_i^{(h)} \right]^2} \le \mathrm{THD}^{\max}, i = 1..N \quad (3)$$

where $\mathrm{THD}^{\max}$=5%, $U^{\min}$=0.9 pu and $U^{\max}$=1.1 pu, according to limits specified by IEEE – 519 Std. [11].

Objective function value is a quantity related to a given solution of a problem, which in our case should have a value as low as possible. However, should a solution satisfy certain constraints, its value alone is not enough to quantify the solution as acceptable or not. Therefore, we define an augmented objective function, which is based on the penalty function approach and serves to enable comparison between solutions accounting possible constraint violation. The augmented objective function is defined as in (4):

$$F_{\mathrm{a}} = F + \sum_{l \in VC} W_l \tag{4}$$

where F_{a} is the augmented function, F is the original objective function from (1), VC is a set of violated constraints and W_l is a penalty term corresponding to a given constraint l.

For a variable x_{i} of type l, bounded with upper and lower bounds $x_i^{\max}$ and $x_i^{\min}$, for which the penalty coefficient is w_l, in case of a constraint violation, the corresponding penalty term in (4) will be, as in (5):

$$W_l = \begin{cases} w_l \cdot \left(x_i^{\min} - x_i\right)^2 & \text{if } x_i < x_i^{\min} \\ w_l \cdot \left(x_i - x_i^{\max}\right)^2 & \text{if } x_i > x_i^{\max} \end{cases} \tag{5}$$

The set of violated constraints for this particular case contains two different penalty coefficients related to RMS and THD bounds. One possibility on value determination of these penalty coefficients is discussed in the section with genetic algorithms.

Modelling of Network Elements, Loads and Harmonic Sources

Various types of power system element equivalents exist in the literature. The modeller is usually tasked with opting what models to use based on depth of modelling, scope and purpose of the analysis etc. When performing harmonic analysis in power systems, [12] suggest the most common models employed, as it is done in this paper. For steady state calculations, all system elements are modelled using their π-equivalent circuit (Figure 1).

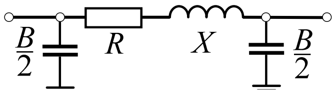

Fig.1 π-equivalent circuit

Line parameters are calculated using analytical expressions based on Carson's theory of the ground fault current earth return path [13]. Skin-effect correction is used for line resistances according to empiric formulas derived in [12], with (6) for overhead lines and (7) for power cables, where h is the harmonic order:

$$R^{(h)} = R^{(1)} \cdot \left(1 + \frac{0.646 \cdot h^2}{192 + 0.518 \cdot h^2}\right) \tag{6}$$

$$R^{(h)} = R^{(1)} \cdot \left(0.187 + 0.532 \cdot \sqrt{h}\right) \tag{7}$$

Transformers are modelled using their π-equivalent circuit also. The magnetising branch is neglected since transformers are considered as non-significant sources of harmonics [12]. Series resistance and reactance are calculated using open and short circuit transformer data. Series resistance is corrected to account for skin-effect correction using (8):

$$R^{(h)} = R^{(1)} \cdot h^{1.15} \tag{8}$$

Different types of load models are recommended in the literature [12]. Due to brevity, only the ones used in this paper will be explained. The generic load model used in this paper is presented on Figure 2. Passive linear load model consists of parallel connection of resistance and reactance, values of which are calculated with (9) and (10) accordingly:

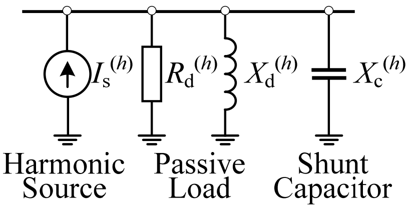

Fig.2 Generic Load Model

$$R_{\mathrm{d}}^{(h)} = \frac{U^2}{k \cdot P_{\mathrm{d}}} \tag{9}$$

$$X_{\mathrm{d}}^{(h)} = \frac{U^2}{k \cdot Q_{\mathrm{d}}} \tag{10}$$

where P_{d} and Q_{d} are load active and reactive power, U is the voltage at the point of common coupling (PCC), while k=0.1·h+0.9 is a correction factor to account for each harmonic influence on the load model parameters.

Shunt capacitors are presented as constant reactance that changes value according to (11) for each harmonic:

$$X_{\mathrm{c}}^{(h)} = h^{-1} \cdot X_{\mathrm{c}}^{(1)} \tag{11}$$

Non-linear and harmonic producing loads are modelled as constant current sources (CCS). CCS model is the most common when dealing with harmonics, since it envelops most of the non-linear load behaviour. The model accounts for all present current injection harmonic amplitudes along with their appropriate phase shifts in relation to PCC voltage. Harmonic current injection values are derived using the 1/h rule of the fundamental component, h being the order of the harmonic. Harmonic phase shifts may or may not be omitted depending on the case [12]. Here, they are excluded from the harmonic source model.

Y-Matrix Based Harmonic Load Flow

Sparse representation of voltage nodal equations for each present harmonic is used in this paper.

$$\underline{\boldsymbol{Y}}^{(h)} \cdot \underline{\boldsymbol{U}}^{(h)} = \underline{\boldsymbol{I}}^{(h)} \tag{12}$$

where $\underline{\boldsymbol{Y}}^{(\mathrm{h})}$ is bus admittance matrix, $\underline{\boldsymbol{U}}^{(\mathrm{h})}$ is vector with bus voltages and $\underline{\boldsymbol{I}}^{(\mathrm{h})}$ is vector with bus current injections for each present harmonic h. Bus current injections for h=1 are calculated with (13):

$$\underline{I}_i^{(1)} = -\left(\underline{S}_i^{(1)} \big/ \underline{U}_i^{(1)}\right)^* + \underline{Y}_{\mathrm{c},i}^{(1)} \cdot \underline{U}_i^{(1)} \tag{13}$$

where $\underline{S}_{\mathrm{i}}^{(1)}$ is load demand at bus i and $\underline{Y}_{\mathrm{c,i}}^{(1)}$ is admittance of the installed capacitor at bus i. Since $\underline{I}_{\mathrm{i}}^{(1)}$ is voltage depended, bus voltages are calculated with (12) using an iterative procedure.

For h>1, bus current injections $\underline{\boldsymbol{I}}^{(\mathrm{h})}$ are determined from the harmonic-producing loads specific spectra. These current injections are assumed voltage independent [12], which enables for direct solution of (12).

Admittance matrix $\underline{\boldsymbol{Y}}^{(h)}$ for each harmonic is calculated using (14) where $\boldsymbol{A}$ is the incidence matrix, which holds information on branch to node connections such that if there's a branch i connecting nodes j and k, then $A(j,i)=1$ and $A(k,i)=-1$, while all other elements of column i in $\boldsymbol{A}$ are zeros. $\underline{\boldsymbol{Y}}_{\mathrm{b}}^{(h)}$ is branch admittance vector comprised of elements from all branch's $\boldsymbol{\pi}$-equivalent circuits.

(14)
$$\underline{\boldsymbol{Y}}^{(h)} = \boldsymbol{A} \cdot \mathrm{diag}\left(\underline{\boldsymbol{Y}}_{\mathrm{b}}^{(h)}\right) \cdot \boldsymbol{A}^{\mathrm{T}}$$

Knowing all elements of (12), load flow calculation procedure for $h=1$ is performed in the following steps:

1. *Set all voltages to 1 pu (flat start);*
2. *Calculate current injections with (13) and solve for voltages in (12);*
3. *Compare voltages with corresponding ones from previous iteration and terminate the process if the maximum difference is less than 10^{-4} pu, otherwise go to Step 2.*

For $h>1$, current injections (13) are considered voltage independent, which enables for (12) to be directly solved. Once all voltages are known, one can easily calculate branch currents and corresponding power and energy losses as a simple sum of losses in all branches.

Integer Genetic Algorithm

Genetic algorithm (GA) is a search heuristic that mimics the processes of natural selection. Each variable referred to as *gene* is integrated into a vector that represents a solution to the given problem and is called *chromosome*. GA always deals with a set of solutions called *population*. The process of chromosome transformation in a given population obtains a new population called *generation*. Chromosome transformation is performed using three genetic operators called: *selection*, *crossover* and *mutation*. For the purposes of this paper, we use the GA optimisation toolbox from Matlab$^{\mathrm{TM}}$.

Capacitor locations along with their appropriate sizes which are needed for the minimization of (1) are determined using an integer GA. For each bus (excluding the slack bus denoted with 1) we define an integer gene g_i with minimum value of zero and maximum value of $(N_{\max}+1)\cdot N_{\mathrm{types}}-1$, where $N_{\max}$ is the maximum number of capacitors per bus and N_{types} is the number of standard capacitor types available for the given case (15):

(15)
$$0 \le g_i \le \left(N_{\max}+1\right)\cdot N_{\mathrm{types}}-1,\ i=2..N$$

Once we know the gene's value, we can obtain the capacitor types (16) and capacitors size (17):

(16)
$$t_i = \mathrm{int}\left(\frac{g_i}{N_{\max}+1}\right)+1$$

(17)
$$n_i = g_i - \left(N_{\max}+1\right)\cdot\left(t_i-1\right)$$

During the initialization stage, to prevent a possible overcompensation, we keep certain gene values at zero. Gene's zero values are determined randomly. For this purpose, we define a real parameter $d=(0,1)$ that describes the density of capacitors in the system. The product of this parameter times the number of buses $d\cdot N$ gives the expected number of buses where capacitors can be installed. Location of these buses is not known in advance and is determined by the GA. During the initialization stage, for each gene we generate a uniformly distributed random number $r=[0,1)$ and in cases of $r<d$ the gene's assigned with a random value according to (15), otherwise the gene's value is set to zero.

When penalty terms are used, the effectiveness of GA's greatly influenced by their value. If the penalty terms are low, the GA will have problem eliminating chromosomes with violated constraints, since it will have problems detecting them. On the other hand, if the penalty terms are high, one may expect a large number of chromosomes with violated constraints, hence making the choice of selecting fit individuals for solution improvement rather difficult. The task of choosing appropriate penalty terms is based on trial-error basis and there is no general rule. Here, we've adopted the following strategy:

1. *Select a penalty coefficient for (2) such that if there's an RMS voltage violation of 0.1pu at all buses, the penalty term equals the objective function (1);*
2. *Select a penalty coefficient for (3) such that if there's a THD violation of 1% at all buses, the penalty term equals the objective function (1);*

Case Study

The proposed procedure is applied to 12.5 kV, IEEE18 distorted distribution system [8, 14]. The system contains a 3 MW six-pulse converter at bus 5 (Figure 3). This converter causes a maximum THD=8.49% which is well above the limit given with [11]. Current harmonic injections are calculated as fractions of the fundamental component, applying the rule of $1/h$ where $h \in \{5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37, 41, 43, 47, 49\}$. Phase shifts of harmonic currents are omitted since there's a single source of harmonics in the system [12].

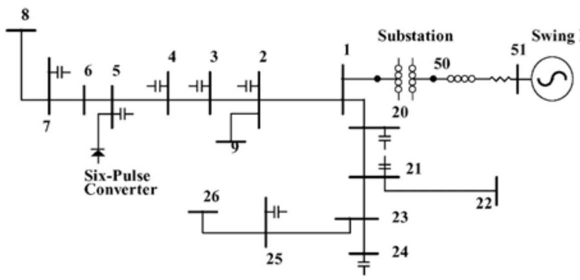

Fig.3 IEEE18 Distorted Distribution System

Capacitors can be installed at 10 potential locations: 2, 3, 4, 5, 7, 20, 21, 24, 25 and 50. Installation costs C_{f} are neglected, while $C_{\mathrm{e}}=0.05$ \$/kWh, $C_{\mathrm{p}}=120$ \$/kW and $p=0.1$. Capacitors are available at 150, 300, 450 and 600 kvar with corresponding unit prices of 5, 3.25, 2.53 and 2.2 \$/kvar. Capacitor density is set to 0.5 ($d=0.5$) so that on average we expect at 5 out of 10 candidate locations capacitor placement.

The GA operates a population of 50 chromosomes for 300 generations with maximum number of generations reached as a termination criteria and crossover probability of 0.6. All other settings for GA are left by default as it is in Matlab$^{\mathrm{TM}}$.

In most applications regarding capacitor placement problem, loads are modelled as constant power. The approach is taken from transmission system analysis where that particular load model fits well since the voltages are practically constant. However, distribution system loads might be at locations without voltage regulation where the effect of centralized regulation is rather weak, hence voltage profile is far from constant. Constant power model for these cases is considered questionable; in fact actual measurements have shown that a simplified constant impedance model is far more accurate for this particular case. Therefore it is meaningful to determine new solutions for capacitor placement based on voltage dependent load models.

Table 1. Simulation Results for the IEEE 18 Distorted Distribution System

	Location	Before optimization [14]	Results from [6]	Results from [8]	Case 1	Case 2	Case 3
Capacitor size (kvar) and location	2	1,050	300	300	7×150	0	0
	3	600	0	0	0	3×150	2×150
	4	600	1,650	1,950	4×300	5×150	0
	5	1,800	3,300	3,000	7×600	7×600	9×600
	7	600	1,050	1,050	2×300	2×300	0
	20	600	600	900	2×600	7×150	3×300
	21	1,200	900	750	1×150	1×150	3×150
	24	1,500	150	150	2×150	2×150	0
	25	900	150	150	2×150	2×150	3×150
	50	1,200	300	0	0	1×150	0
Total capacitors (kvar)		10,050	8,400	8,250	9,000	7,950	7,500
Minimum voltage (pu)		1.029	1.003	1.005	1.013	1.018	0.999
Maximum voltage (pu)		1.055	1.050	1.050	1.057	1.058	1.089
Maximum THD (%)		8.5	4.9	5.0	5.0	5.0	5.0
Losses [kW]		282.93	249.31	248.18	247.66	237.70	253.71
Total cost ($/year)		159,853.14	140,903.73	140,302.80	140,864.31	135,331.29	104,496.93
Benefits ($/year)			18,949.41	19,550.34	18,988.83	24,521.85	55,356.21

Three particular cases are observed for this test system:

1. *Case 1* where loads are modelled as constant powers;
2. *Case 2* where loads are modelled as constant impedances;
3. *Case 3* where loads are modelled as constant power and variable in time.

Load factors for *Case 3* are 0.6, 0.8 and 1.0, with appropriate durations of 2628, 4818 and 1314 hours. Table 1 shows the simulation results in comparison to others from the literature [6, 8]. One can see that the differences in Total cost for *Case 1* are rather negligible and certainly lower then the uncertainty of the input data. The differences in Total cost for *Case 2* and *3* are rather significant, pointing out that due attention should be given to load modeling and load variation in time. The latter two cases provide for higher benefits while having far less capacitors installed.

Conclusion

Integer genetic algorithm is proposed for the discrete optimisation problem of capacitor placement and sizing under distorted conditions. The objective function accounts for power and energy losses and capacitor prices. Constraints accound for THD and RMS limits according to IEEE-519 Std. [11]. The proposed approach is applied to IEEE 18 distorted distribution system under various load models and load variation. Results and performance of the proposed algorithm are better and they show that due attention should be given to load modelling and time variation, since it greatly influences the outcome of the solution. This means that the problem of optimal capacitor placement should not only be treated as a field where new improved optimization tools will be benchmarked, without taking into account more accurate models for all network components.

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