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Current Distribution Analysis Along a Conductor Excited by a Pulse Source Using Partial Element Equivalent Circuit Method

***Abstract**. In this paper the time shape of a current pulse along a conductor is investigated by applying the Partial Element Equivalent Circuit (PEEC) method. The pulse is injected by a current source at one end of a perfect conductor placed in a conductive medium. Implementing the thin wire approximation, the current distribution is calculated in the frequency domain, while the time shape of the pulse is reconstructed using Inverse (Fast) Fourier Transform (IFFT). The developed model is verified by comparison with results found in other literature and shows satisfactory agreement.*

***Streszczenie.** W artykule przebadano przebieg czasowy impulsów prądowych wzdłuż przewodu przy zastosowaniu metody obwodu zastępczego z elementami cząstkowymi. Impulsy są aplikowane ze źródła prądowego na końcu idealnego przewodu umieszczonego w przewodzącym środowisku. Implementując aproksymację cienkim przewodem, rozkład prądu obliczany jest w obszarze częstotliwościowym, podczas gdy przebieg czasowy impulsu został zrekonstruowany przy użyciu odwrotnej szybkiej transformaty Fouriera. Opracowany model został zweryfikowany przez porównanie z wynikami znalezionymi w literaturze i wskazano na zadowalającą zgodność. (**Analiza rozkładu prądu wzdłuż przewodnika wzbudzanego przez źródło impulsowe przy użyciu metody zastępczego obwodu częściowych elementów**).*

Keywords: current distribution, PEEC, conductive medium.
Słowa kluczowe: rozkład prądu, PEEC, środowisko przewodzące.

Introduction

The essence of the Partial Element Equivalent Circuit (PEEC) method [1] is to create an equivalent electric circuit from a full-wave solution of the Maxwell's equations for a certain electromagnetic problem. The heterogeneous equivalent circuit is consisted of equivalent elements (impedances, admittances and sources) that take into account the electromagnetic properties of the system, as well as coupling between segments and propagation effects. The circuit based modelling in combination with widely spread SPICElike solvers has the advantage of simple inclusion of additional circuit elements such as RLC components, transmission lines, cables, transformers etc. Another advantage of the PEEC method is the possibility to implement the same circuit model for both time- and frequency- domain analysis. This kind of numerical modelling is useful in several areas of engineering, for example, product research and development of integrated electronic circuits (enabling prevention from Electromagnetic Interference (EMI)), design of complex grounding systems and analysis of transient potentials in them [2], lightning surge analysis [3] etc.

In this paper we develop a Partial Element Equivalent Circuit model for a perfect conductor placed in an unbounded conductive medium, using the thin wire approximation. The conductor is excited at one end by a double – exponential pulse current source. The current and voltage distribution along the conductor is determined in the frequency domain by implementation of the Modified Nodal Approach (MNA) [4]. The time shape of the current pulse along the conductor is then obtained using inverse (fast) Fourier transform (IFFT). The influence of the conductor's length and surrounding medium's characteristics on the current distribution is investigated. The verification process shows that the time shapes of the current obtained with the developed PEEC model are in excellent agreement with the shapes found in other literature [6].

Current distribution calculations

In order to obtain the current distribution in the frequency domain, the Partial Element Equivalent Circuit (PEEC) method is applied on a perfect conductor placed in a conductive medium. The conductor is excited at one end by a pulse current source. A double – exponential current pulse with peak value 1 kA [7] is injected by the current source:

$$i(t) = I(e^{-\alpha t} - e^{-\beta t}) \tag{1}$$

where: $I=1.0167\ kA$, $\alpha=0.0142\ \mu s^{-1}$ and $\beta=5.073\ \mu s^{-1}$. The geometry of the conductor with radius a and length L and the location of the current source are presented in Fig. 1a. The time shape of the source current pulse is presented in Fig. 1b.

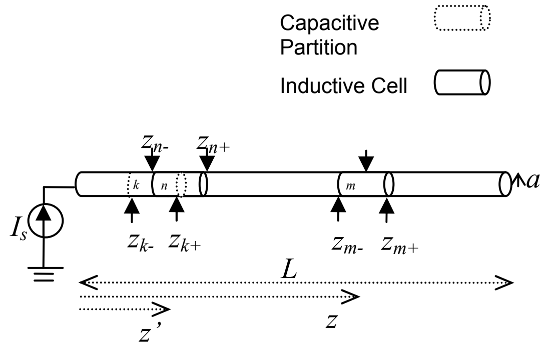

Fig.1a. The geometry of the analyzed conductor

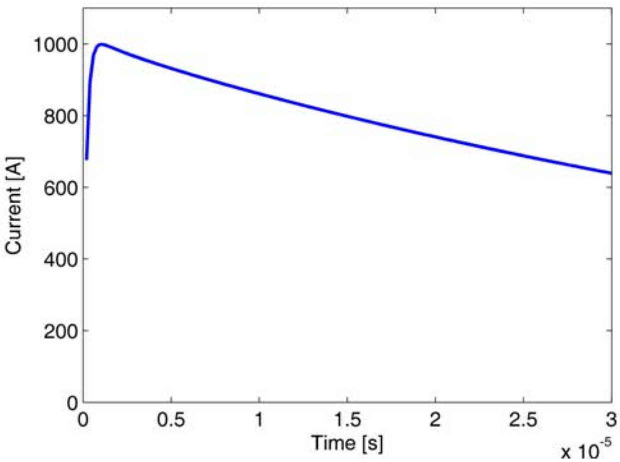

Fig.1b. The shape of the source current pulse

Equivalent Circuit Parameters Extraction

As shown in Fig. 1, the geometry of the conductor is discretized in two ways. The length of the segments is $l=L/N$, where N is the number of segments. For successful thin wire modeling, the following conditions should be satisfied [8]: $L/a>30$, $a/\lambda<<1$, $l/a\geq 5$ and $l/\lambda>1/20$, where λ is the wavelength of the source current. The segments notated as m and n are called inductive cells and are used for calculating the partial inductances of the system. The segment notated as k is called a capacitive partition and is used for determining the potential coefficients. The capacitive partition is shifted by half the segment length and as a result the total number of partitions is $N+1$ (equals the number of nodes) [1]. The potential difference across the m-th segment is expressed via partial inductances L_{mn}, as follows:

$$\varphi_{m+} - \varphi_{m-} = \sum_{n=1}^{N} j\omega I_n \cdot L_{mn} \tag{2}$$

In general, Eqn. (2) shows that the potential difference across the m-th segment is related to the current of the n-th segment via a partial inductance:

$$L_{mn} = -\frac{\mu}{4\pi}\int_{z_{m-}}^{z_{m+}}\int_{z_{n-}}^{z_{n+}} g(z,z')dz'dz \tag{3}$$

The potential difference across the m-th segment can also be expressed via potential coefficients, P_{k+} and P_k :

$$\varphi_{m+} - \varphi_{m-} = \sum_{k=1}^{N+1} Q_k\left(P_{k+} - P_{k-}\right) \tag{4}$$

In general, Eqn. (4) implicates that the potential of node j is related to the charge of the k-th capacitive partition via a potential coefficient

$$P_{jk} = \frac{1}{4\pi\underline{\varepsilon} l_k}\int_{z'_{k-}}^{z'_{k+}} g(z_j,z')dz' \tag{5}$$

In Eqns. (3) and (5), $g(z,z')$ is the Green's function for the unbounded conductive medium

$$g(z,z') = \frac{e^{-\gamma R}}{R} \tag{6}$$

where R is the distance between segment m and segment n

$$R = \sqrt{a^2 + (z-z')^2} \tag{7}$$

and the propagation constant is

$$\gamma = \sqrt{-\omega^2 \underline{\varepsilon}\mu}\ . \tag{8}$$

Extensive details of the equivalent circuit extraction can be found in [5].

The potentials of the nodes and the currents of the segments are determined implementing the Modified Nodal Approach theory [2], [3]. The resulting matrix equation is

$$\begin{bmatrix} -\mathbf{A} & -j\omega\mathbf{L} \\ j\omega\mathbf{P}^{-1} & \mathbf{A}^{\mathrm{T}} \end{bmatrix}\begin{bmatrix} \boldsymbol{\varphi} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ \mathbf{I}_s \end{bmatrix}. \tag{9}$$

where $\mathbf{A}$ is the incident matrix, $\mathbf{L}$ is the matrix of partial inductances, $\mathbf{P}$ is the matrix of potential coefficients, φ is a vector of node potentials, $\mathbf{I}$ is a vector of segment currents, $\mathbf{V}_s$ is a vector of external voltage sources and $\mathbf{I}_s$ is a vector of external current sources.

Reconstruction of the current time shapes

To illustrate the developed model, the conductor with radius a=7mm and length L=10m was placed in a medium with conductivity σ=0.01 S/m and permittivity ε_r=10. The source current pulse duration was limited to 900 μs and sampled at a 0.05 μs rate. The values for the current for every frequency of interest were obtained by solving the equivalent circuit matrix equation of the conductor (9) in frequency domain. The time shape of the current pulse along the conductor was then obtained using inverse (fast) Fourier transform (IFFT). The rise of the current pulse at the beginning of time is presented in Fig. 2, in three points located at 0 m, 5 m and 7 m distance from the source.

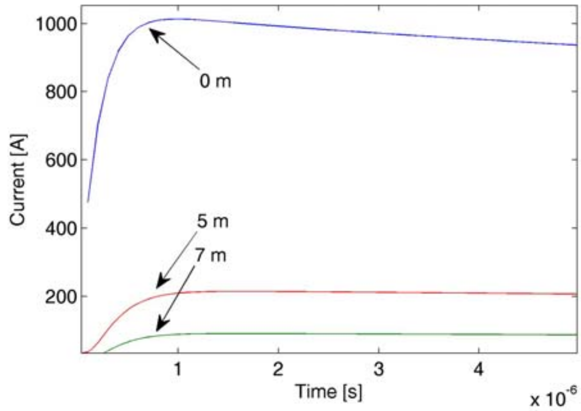

Fig.2. The rise of the current pulse at different locations

It may be observed from the above figure that the peak of the current at 5 m and 7 m distance along the conductor is drastically reduced, due to leakage currents. The figure also shows a delayed start of the current pulse at 5 m and 7 m distance along the conductor.

The results for the current time shape along the analyzed conductor obtained by the PEEC model presented in this paper are compared with those calculated by applying the Electromagnetic Model (EM) based on the Method of Moments and Mixed Potential Integral Equation, found in [6]. Figs. 3 and 4 present the current at 5 m and 7 m distance along the conductor of 10 m and 100 m length respectively. Results for the current are compared for a medium with specific conductivity a) 0.01 S/m and b) 0.1 S/m. The conductor has radius a=7mm and was placed in a medium with permittivity ε_r=10. The source current pulse duration is 900 μs and is sampled at a 0.2 μs rate

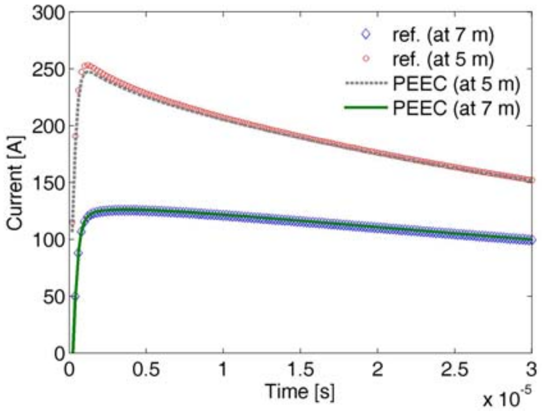

Fig.3a. Comparison of the time shapes of the current along a 10 m long conductor for medium conductivity of 0.01 S/m

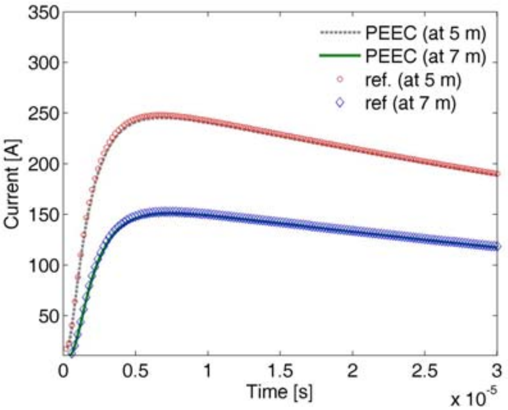

Fig.3b. Comparison of the time shapes of the current along a 10 m long conductor for medium conductivity of 0.1 S/m

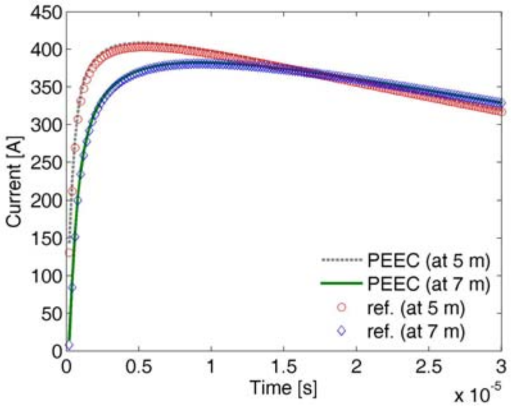

Fig.4a. Comparison of the time shapes of the current along a 100 m long conductor for medium conductivity of 0.01 S/m

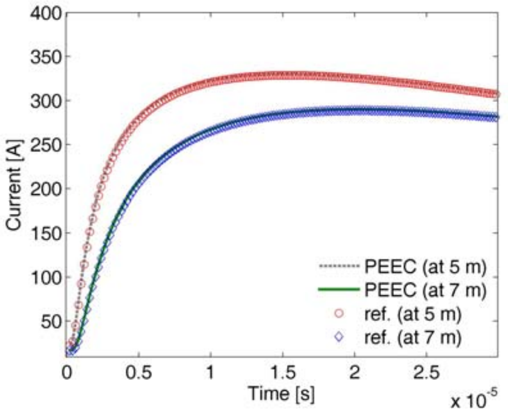

Fig.4b. Comparison of the time shapes of the current along a 100 m long conductor for medium conductivity of 0.1 S/m

It may be observed from Figs. 3 and 4 that the results for the current curves obtained with the PEEC model, developed here, are in excellent agreement with the curves obtained by the method found in ref. [6].

Conclusions

The Partial Element Equivalent Method has been applied to current distribution calculation along a perfect conductor placed in a conductive medium excited by a pulse current source. Coupling between segments and propagation effects were taken into account by the equivalent circuit elements. The obtained equivalent circuit was used in combination with the Modified Nodal Approach in order to determine the current distribution along the conductor. Parametric analysis of the time shape of the current pulse at different locations was performed using inverse Fourier transform (IFFT). The verification process showed that the time shapes of the current obtained with the developed PEEC model are in excellent agreement with the shapes from other literature.

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