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Modelling of three-phase transformer's operation using variational methods

Abstract. *This paper presents a mathematical model of power system. The interdisciplinary method, based on a modification of Hamilton's integral variational principle, is used in order to model the system. The analyzed system consists of a nonlinear power transformer that is connected to the unbalanced energy source via asymmetric cable line. The unbalanced RLC circuit is considered as a load of the transformer. The operation of the transformer in transient states is analyzed using the formulated model. The results of computer simulations are presented as graphs.*

Streszczenie*. W pracy przedstawiono model matematyczny układu elektroenergetycznego stosując interdyscyplinarną metodę modelowania, która wykorzystuje modyfikację integralnej zasady wariacyjnej Hamiltona. Analizowany układ składa się z nieliniowego transformatora mocy, który jest podłączony przez asymetryczną linię kablową do niesymetrycznego źródła energii. Transformator pracuje obciążony niesymetrycznym obwodem RLC. Wykorzystując sformułowany model przedstawiono analizę pracy transformatora w stanach przejściowych. Wyniki symulacji komputerowych przedstawiono w postaci graficznej. (****Modelowanie pracy trójfazowego transformatora mocy z wykorzystaniem metod wariacyjnych****).*

Keywords: Hamilton's rule, Euler-Lagrange's system, asymmetrical circuits, power transformer.
Słowa kluczowe: zasada Hamiltona, system Eulera-Lagrange'a, obwody asymetryczne, transformator mocy.

Introduction

A power transformer is one of the main components of power systems. Power conversion processes in this device depend on the design parameters of transformer and the state of power line. It is very important for consumers to ensure not only the sufficient transformer power, but also the adequate power quality, including symmetry of supply voltage. In general case, not only a power transformer should be modelled precisely: also its load and power line. The equations based on distributed parameters should describe physical processes occurring in power line in order to ensure the sufficient adequacy of mathematical model [2]. Such model is very complicated and requires integration of equations with partial derivatives, whereas, in majority calculations the alternative solution is the appropriate connection of RLC elements that represents a cable line accurately enough. In general, two connections of elements are taken into account: serial and serial-parallel. The first type of connection was presented in [1], whereas, the second type is considered in this paper. The ways of derivation of state equations are different for both connections.

Mathematical model

The extended functional of operation by Hamilton was formulated on the basis of non-conservative Lagrangian and used in order to obtain the mathematical model of power system, including power transformer and power line. The variations of the functional were derived and compared to zero, and then the extreme function (extremal) equations together with constraints equations, being the mathematical model of the power system, were obtained as a consequence. The considered power system is depicted in Figs 1 and 2.

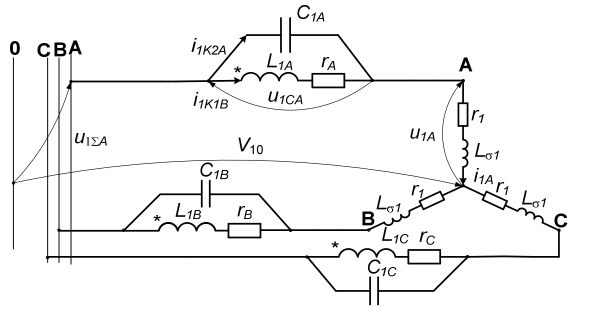

Fig. 1. Electric circuit consisting of primary winding elements

The following additional variables were defined in Figs:

a) line-to-line (phase) voltages of secondary winding $u_{2AB} = u_{2A} - u_{2B}$, $u_{2BC} = u_{2B} - u_{2C}$, $u_{2CA} = u_{2C} - u_{2A}$

b) voltages over cable line inductances for both A-phase windings u_{1LA}, u_{2LA}

c) phase inductances of cable line L_{1A}, L_{1B}, L_{1C}, L_{2A}, L_{2B}, L_{2C}

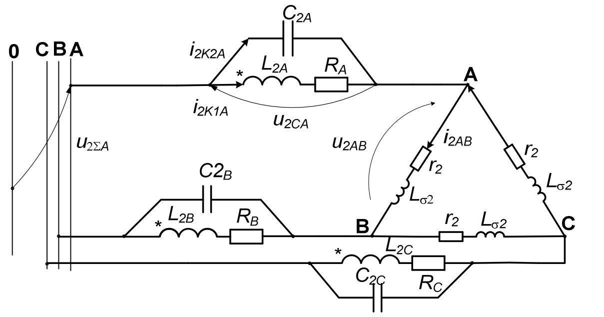

Fig. 2. Electric circuit consisting of secondary winding elements

The equations of non-varying constraints are given below

(1) $u_{\Sigma 1A} + u_{\Sigma 1B} + u_{\Sigma 1C} \neq 0, \quad u_{1LA} + u_{1LB} + u_{1LC} \neq 0,$
$u_{1rA} + u_{1rB} + u_{1rC} \neq 0$,

(2) $u_{\Sigma 2A} + u_{\Sigma 2B} + u_{\Sigma 2C} \neq 0, \quad u_{2LA} + u_{2LB} + u_{2LC} \neq 0,$
$u_{2RA} + u_{2RB} + u_{2RC} \neq 0$,

(3) $i_{2AB} \equiv i_{2A},\, i_{2BC} \equiv i_{2B},\, i_{2CA} \equiv i_{2C},\; i_{2AB} + i_{2BC} + i_{2CA} = 0$,

(4) $r_{1\Sigma A} = r_1 + r_A,\, r_{1\Sigma B} = r_1 + r_B,\, r_{1\Sigma C} = r_1 + r_C,\, r_A \neq r_B \neq r_C$,

(5) $i_{1A} + i_{1B} + i_{1C} = 0, \quad i_{2A} + i_{2B} + i_{2C} = 0$,

(6) $u_{2AB} + u_{2BC} + u_{2CA} = 0,\, R_A \neq R_B \neq R_C$,

(7) $L_{1A} \neq L_{1B} \neq L_{1C},\; L_{2A} \neq L_{2B} \neq L_{2C}$,

(8) $u_{2KA} = u_{2LA} + u_{2RA},\; u_{2KB} = u_{2LB} + u_{2RB}$,
$u_{2KC} = u_{2LC} + u_{2RC}$.

The extended functional of operation by Hamilton using the modified Lagrangian elements was formulated [1, 2]:

(9) $L^* = \tilde{T}^* - P^* + \Phi^* - D^*$,

where L^* is modified Lagrange's function, $\tilde{T}^*$ is kinetic co-energy, P^* is potential energy, Φ^* is dissipation energy, D^* is external forces energy.

The following generalized coordinates were used: electric charges in primary and secondary windings of transformer Q_{nA}, Q_{nB}, Q_{nC}, charges in capacitances $Q_{nK1A}, Q_{nK1B}, Q_{nK1C}$ and charges in cable line wires $Q_{nK2A}, Q_{nK2B}, Q_{nK2C}$. The currents i_{nA}, i_{nB}, i_{nC}, $i_{nK1A}, i_{nK1B}, i_{nK1C}$, $i_{nK2A}, i_{nK2B}, i_{nK2C}$ in windings were assumed as the generalized velocities, where $n = 1, 2$ are indexes of the primary winding and the secondary winding, respectively.

On the basis of Figs 1 and 2, the Lagrangian elements (9) are given as follows [1,2]:

$$\tilde{T}^* = \sum_{j=1}^{3}\left[\int_0^{i_{1j}}\Psi_{1j}di_{1j} + \int_0^{i_{2j}}\Psi_{2j}di_{2j} + \frac{1}{2}\sum_{n=1}^{2}L_{nj}i_{nK1j}^2\right] + \tag{10}$$

$$+\sum_{n=1}^{2}\Big[M_{nAB}i_{nK1A}i_{nK1B} + M_{nAC}i_{nK1A}i_{nK1C} + M_{nBA}i_{nK1B}i_{nK1A} +$$

$$+M_{nBC}i_{nK1B}i_{nK1C} + M_{nCA}i_{nK1C}i_{nK1A} + M_{nCB}i_{nK1C}i_{nK1B}\Big],$$

$$P^* = \frac{1}{2}\left[\frac{(Q_{1A}-Q_{1K1A})^2}{C_{1A}} + \frac{(Q_{1B}-Q_{1K1B})^2}{C_{1B}} + \frac{(Q_{1C}-Q_{1K1C})^2}{C_{1C}}\right] + \tag{11}$$

$$+\frac{1}{2}\left[\frac{(Q_{2A}-Q_{2C}-Q_{2K1A})^2}{C_{2A}} + \frac{(Q_{2B}-Q_{2A}-Q_{2K1B})^2}{C_{2B}} +\right.$$

$$\left.+\frac{(Q_{2C}-Q_{2B}-Q_{2K1C})^2}{C_{2C}}\right] = \frac{1}{2}\sum_{n=1}^{2}\sum_{j=1}^{3}C_{nj}u_{nCj}^2,$$

$$= \frac{1}{2}\sum_{n=1}^{2}\sum_{j=1}^{3}C_{nj}u_{nCj}^2, \quad \Phi^* = \frac{1}{2}\sum_{n=1}^{2}\sum_{j=1}^{3}r_n i_{nj}^2 + \sum_{j=1}^{3}r_j i_{1K1j}^2 + \sum_{j=1}^{3}R_j i_{2K1j}^2$$

$$D^* = \sum_{j=1}^{3}\int_0^t u_{\Sigma 1j}i_{1j}d\tau + \int_0^t\Big((u_{\Sigma 2A}-u_{\Sigma 2B})i_{2A} + (u_{\Sigma 2B}-u_{\Sigma 2C})i_{2B} + \tag{12}$$

$$+(u_{\Sigma 2C}-u_{\Sigma 2A})i_{2C}\Big)d\tau - \int_0^t V_{01}(i_{1A}+i_{1B}+i_{1C})d\tau - \int_0^t\Big((u_{2CA}-u_{2CB})i_{2A} +$$

$$+(u_{2CB}-u_{2CC})i_{2B} + (u_{2CC}-u_{2CA})i_{2C}\Big)d\tau, \quad n = 1, 2, \quad j = A, B, C,$$

where $\mathbf{\Psi}_1, \mathbf{\Psi}_2$ are column vectors of flux linkages of transformer primary and secondary windings, $\mathbf{r}_1, \mathbf{r}_2$ are resistance matrixes, $\mathbf{i}_1, \mathbf{i}_2$ are column vectors of currents, $\mathbf{u}_1, \mathbf{u}_2$ are column vectors of voltages.

The modified Lagrangian was formulated on the basis of the equations (10) – (12). Substituting it into the functional of operation by Hamilton [1] and determining the functional variations, the Euler-Lagrange's equations were derived [1]. The following dependencies were obtained by solving them:

$$\frac{d\mathbf{\Psi}_1}{dt} + \mathbf{r}_1\mathbf{i}_1 - \mathbf{u}_1 - \mathbf{u}_{1\Sigma} + \mathbf{u}_{1C} + \mathbf{V}_{10} = 0, \tag{13}$$

$$\frac{d\Psi_{2A}}{dt} + r_2 i_{2A} - (u_{2\Sigma A} - u_{2\Sigma B}) + (u_{2CA} - u_{2\Sigma B}) = 0, \tag{14}$$

$$\frac{d\Psi_{2B}}{dt} + r_2 i_{2B} - (u_{2\Sigma B} - u_{2\Sigma C}) + (u_{2CB} - u_{2\Sigma C}) = 0, \tag{15}$$

$$\frac{d\Psi_{2C}}{dt} + r_2 i_{2C} - (u_{2\Sigma C} - u_{2\Sigma A}) + (u_{2CC} - u_{2\Sigma A}) = 0, \tag{16}$$

$$\mathbf{u}_{nL} + \mathbf{r}\mathbf{i}_{nK1} - \mathbf{u}_{nC} = 0, \tag{17}$$

$$-\mathbf{u}_{nC} + \mathbf{C}_n^{-1}\mathbf{Q}_{nK2} = 0, \tag{18}$$

where:

$$u_{nLA} = L_{nA}\frac{di_{nK1A}}{dt} + M_{nAB}\frac{di_{nK1B}}{dt} + M_{nAC}\frac{di_{nK1C}}{dt}, \tag{19}$$

$$u_{nLB} = L_{nB}\frac{di_{nK1B}}{dt} + M_{nBA}\frac{di_{nK1A}}{dt} + M_{nBC}\frac{di_{nK1C}}{dt}, \tag{20}$$

$$u_{nLC} = L_{nC}\frac{di_{nK1C}}{dt} + M_{nCA}\frac{di_{nK1A}}{dt} + M_{nCB}\frac{di_{nK1B}}{dt}. \tag{21}$$

The sum of terms in matrix equation (13) is given below:

$$V_{10} = -\frac{1}{3}\Big(-((u_{1\Sigma A} - u_{1\Sigma B}) + (u_{1\Sigma B} - u_{1\Sigma C}) + \tag{22}$$

$$+(u_{1\Sigma C} - u_{1\Sigma A})) + u_{1CA} + u_{1CB} + u_{1CC}\Big).$$

On the basis of the non-varying constraints:

$$\Psi_{nA} + \Psi_{nB} + \Psi_{nC} = 0, \quad i_{nA} + i_{nB} + i_{nC} = 0 \tag{23}$$

it was obtained:

$$\frac{d}{dt}\begin{bmatrix}\Psi_{1A}\\ \Psi_{1B}\end{bmatrix} = \frac{1}{3}\begin{bmatrix}2u_{1\Sigma A} - u_{1\Sigma B} - u_{1\Sigma C}\\ 2u_{1\Sigma B} - u_{1\Sigma A} - u_{1\Sigma C}\end{bmatrix} - \tag{24}$$

$$-\frac{1}{3}\begin{bmatrix}2u_{1CA} - u_{1CB} - u_{1CC}\\ 2u_{1CB} - u_{1CA} - u_{1CC}\end{bmatrix} - \begin{bmatrix}r_2 & \\ & r_2\end{bmatrix}\begin{bmatrix}i_{1A}\\ i_{1B}\end{bmatrix},$$

$$\frac{d}{dt}\begin{bmatrix}\Psi_{2A}\\ \Psi_{2B}\end{bmatrix} = \begin{bmatrix}(u_{2\Sigma A} - u_{2\Sigma B})\\ (u_{2\Sigma B} - u_{2\Sigma C})\end{bmatrix} - \begin{bmatrix}(u_{2CA} - u_{2CB})\\ (u_{2CB} - u_{2CC})\end{bmatrix} - \tag{25}$$

$$-\begin{bmatrix}r_2 & \\ & r_2\end{bmatrix}\begin{bmatrix}i_{2A}\\ i_{2B}\end{bmatrix}$$

The column vectors of voltages over inductances and voltages over capacitances of cable line at primary winding and secondary winding are given as follows [1]:

$$\mathbf{u}_{nL}^{(3)} = \begin{bmatrix}L_{nA} & M_{nAB} & M_{nAC}\\ M_{nBA} & L_{nB} & M_{nBC}\\ M_{nCA} & M_{nCB} & L_{nC}\end{bmatrix}\frac{d}{dt}\begin{bmatrix}i_{nK1A}\\ i_{nK1B}\\ i_{nK1C}\end{bmatrix} \equiv \mathbf{L}_{nS}^{(3)}\frac{d}{dt}\mathbf{i}_{nK1}^{(3)}, \tag{26}$$

$$\frac{d}{dt}\mathbf{u}_{1C}^{(3)} \equiv \frac{d}{dt}\begin{bmatrix}u_{1CA}\\ u_{1CB}\\ u_{1CC}\end{bmatrix} = \left[\mathbf{C}_1^{(3)}\right]^{-1}\mathbf{i}_{1K2}^{(3)} = \left[\mathbf{C}_1^{(3)}\right]^{-1}(\mathbf{A}_1\mathbf{i}_1 - \mathbf{i}_{1K1}^{(3)}), \tag{27}$$

$$\frac{d}{dt}\mathbf{u}_{2C}^{(3)} \equiv \frac{d}{dt}\begin{bmatrix}u_{2CA}\\ u_{2CB}\\ u_{2CC}\end{bmatrix} = \left[\mathbf{C}_2^{(3)}\right]^{-1}\mathbf{i}_{1K2}^{(3)} = \left[\mathbf{C}_2^{(3)}\right]^{-1}(\mathbf{A}_2\mathbf{i}_2 - \mathbf{i}_{2K1}^{(3)}), \tag{28}$$

where $^{(3)}$ indicates the rank of matrix, as well as

$$\mathbf{A}_1 = \begin{bmatrix}1 & 0\\ 0 & 1\\ -1 & -1\end{bmatrix}, \ \mathbf{A}_2 = \begin{bmatrix}2 & 1\\ -1 & 1\\ -2 & -1\end{bmatrix}. \tag{29}$$

While for two-phase system:

$$\mathbf{u}_{1C} = \mathbf{B}_1\mathbf{u}_{1C}^{(3)} \equiv \begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\end{bmatrix}\begin{bmatrix}u_{1CA}\\ u_{1CB}\\ u_{1CB}\end{bmatrix}, \tag{30}$$

$$\mathbf{u}_{1\Sigma} = \mathbf{B}_1\mathbf{u}_{1\Sigma}^{(3)} \equiv \begin{bmatrix}2 & -1 & -1\\ -1 & 2 & -1\end{bmatrix}\begin{bmatrix}u_{1\Sigma A}\\ u_{1\Sigma B}\\ u_{1\Sigma B}\end{bmatrix}, \tag{31}$$

$$\mathbf{u}_{2C} = \mathbf{B}_2\mathbf{u}_{2C}^{(3)} \equiv \begin{bmatrix}1 & -1 & 0\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}u_{2CA}\\ u_{2CB}\\ u_{2CB}\end{bmatrix}, \tag{32}$$

(33) $$\mathbf{u}_{2\Sigma} = \mathbf{B}_2 \mathbf{u}_{2\Sigma}^{(3)} \equiv \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{2\Sigma A} \\ u_{2\Sigma B} \\ u_{2\Sigma B} \end{bmatrix},$$

where: $\mathbf{A}_n, \mathbf{B}_n$ are topological matrixes.

According to Figs 1 and 2 it can be written:

(34) $$\frac{d\mathbf{i}_{nK1}}{dt} = \left[\mathbf{L}_n^{(3)}\right]^{-1} \left(\mathbf{u}_{nC}^{(3)} - \mathbf{r}\mathbf{i}_{nK1}\right), \text{ r = R}$$

On the basis of the equation of constraints [1], the dependencies between transformer phase currents and fluxes can be determined as follows:

(35) $$\mathbf{\Psi}_n = \mathbf{L}_{n\sigma}\mathbf{i}_n + \mathbf{\psi}, \quad \mathbf{\alpha}_{n\sigma} = \mathbf{L}_{n\sigma}^{-1}$$

Time derivative of current vector:

(36) $$\frac{d\mathbf{i}_n}{dt} = \mathbf{\alpha}_{\sigma n}\left(\frac{d\mathbf{\Psi}_n}{dt} - \frac{d\mathbf{\psi}}{dt}\right) = \mathbf{\alpha}_{\sigma n}\left(\mathbf{u}_{n\Sigma} - \mathbf{u}_{nC} - \mathbf{r}_n\mathbf{i}_n - \frac{d\mathbf{\psi}}{dt}\right).$$

The operating flux vector is expressed as follows [1]:

(37) $$\mathbf{\psi}_1 = \mathbf{\tau}^{-1}(\mathbf{i}_1 + \mathbf{i}_2)$$

whereas time derivative thereof:

(38) $$\frac{d\mathbf{\psi}}{dt} = \mathbf{G}_1\left(\mathbf{u}_{1\Sigma} - \mathbf{u}_{1C} - \mathbf{r}_1\mathbf{i}_1\right) + \mathbf{G}_2\left(\mathbf{u}_{2\Sigma} - \mathbf{u}_{2C} - \mathbf{r}_2\mathbf{i}_2\right),$$

where:

(39) $$\mathbf{G} = \begin{array}{|c|c|} \hline (\alpha_{\sigma 1A} + \alpha_{\sigma 2A} + \rho_A)^{-1} & \\ \hline & (\alpha_{\sigma 1B} + \alpha_{\sigma 2B} + \rho_B)^{-1} \\ \hline \end{array},$$

$\rho = di/d\psi$ is inverse differential inductance of the transformer, ψ is operating flux.

Substituting in the equation (36) for time derivative of flux (38), mathematical model of power transformer was obtained:

(40) $$\frac{d\mathbf{i}_1}{dt} = \mathbf{A}_1(\mathbf{u}_{1\Sigma} - \mathbf{u}_{1C} - \mathbf{r}_1\mathbf{i}_1) + \mathbf{A}_{12}(\mathbf{u}_{2\Sigma} - \mathbf{u}_{2C} - \mathbf{r}_2\mathbf{i}_2),$$

(41) $$\frac{d\mathbf{i}_2}{dt} = \mathbf{A}_{21}(\mathbf{u}_{1\Sigma} - \mathbf{u}_{1C} - \mathbf{r}_1\mathbf{i}_1) + \mathbf{A}_2(\mathbf{u}_{2\Sigma} - \mathbf{u}_{2C} - \mathbf{r}_2\mathbf{i}_2),$$

(42) $$\mathbf{A}_{11} = \mathbf{\alpha}_{\sigma 1}(\mathbf{1} - \mathbf{\alpha}_{\sigma 1}\mathbf{G}), \quad \mathbf{A}_{12} = \mathbf{A}_{21} = -\mathbf{\alpha}_{\sigma 1}\mathbf{\alpha}_{\sigma 2}\mathbf{G},$$
$$\mathbf{A}_{22} = \mathbf{\alpha}_{\sigma 2}(\mathbf{1} - \mathbf{\alpha}_{\sigma 2}\mathbf{G}).$$

The differential equations (27), (28), (34), (40), (41) have to be integrated together, taking into account the dependencies (19) – (22), (29) – (33), (39), (42).

Results of computer simulation

Simulations of three-phase power transformer were made and they are presented below. The unbalanced electric power source, that fed the transformer via cable line, was considered. The unbalanced load was connected to the transformer secondary winding also via cable line. The power transformer with parameters U_{H1} = 6 kV, U_{H2} = 0,4 kV, S_H = 160 kVA, P_S = 410 W, i_O = 2%, P_{ZW} = 3650 W is considered in the experimental investigations.

Three computational examples are shown in the paper. The first one deals with symmetrical system (power supply and transformer load). The second example deals with asymmetry of resistance, inductance and capacitance in circuits of both transformer windings. The third example deals with asymmetry of resistance, inductance and capacitance in circuits of both transformer windings, as well as asymmetry of feeding voltage of the transformer primary winding.

Fig. 3 shows time changes of transformer primary winding current for symmetrical system (the first computational example). Analyzing this Figure it can be seen that: 1) transient processes after connecting the transformer to the power source are invisible due to small values of electromagnetic time constants, 2) in steady state the magnitudes of all phase currents are identical. The currents reach the rated values.

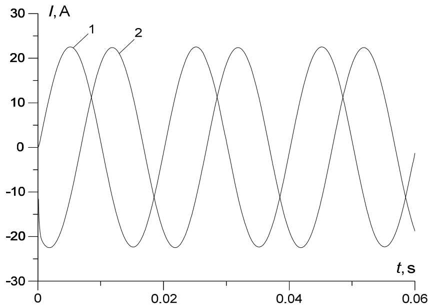

Fig. 3. Time changes of current in the transformer primary winding for the first computational example: 1 is A-phase current; 2 is B-phase current

Fig. 4 shows time changes of short-circuit current in the transformer secondary winding for 3-phase symmetrical system (the first computational example). The current increased over seventeen times in comparison with the previous experiment.

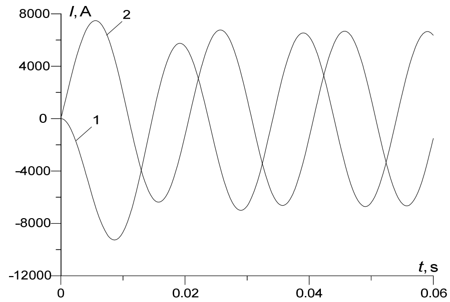

Fig. 4. Time changes of short-circuit current in the transformer secondary winding for the first computational example: 1 is A-phase current; 2 is B-phase current

Figs 5 and 6 show time changes of transformer primary and secondary winding currents for asymmetrical system (the second computational example). The rated data of power devices and power transformer load are given as follows: primary winding resistances: $r_A = 22{,}5\ \Omega$, $r_B = 112{,}5\ \Omega$, $r_C = 225\ \Omega$, primary winding inductances: $L_{1A} = 0{,}0675$ H, $L_{1B} = 0{,}0045$ H, $L_{1C} = 0{,}0225$ H, primary winding capacitances $C_{1A} = 4{,}5\ \mu$ F, $C_{1B} = 2{,}2\ \mu$ F, $C_{1C} = 0{,}4\ \mu$ F, secondary winding resistances: $R_A = 0{,}95\,\Omega, R_B = 2\,\Omega, R_C = 0{,}4\,\Omega$, secondary winding inductances: $L_{2A} = 0{,}003$ H, $L_{2B} = 0{,}0001$ H, $L_{2C} = 0{,}001$ H, secondary winding capacitances $C_{2A} = 100\ \mu$ F, $C_{2B} = 1000\ \mu$ F, $C_{2C} = 500\ \mu$ F. Oscillations of currents can be seen for short time after feeding the transformer due to the oscillation of instantaneous power between inductances and capacitances of cable and transformer windings.

Figs 7 and 8 show time changes of transformer primary and secondary winding currents for asymmetrical system (the second computational example): primary winding voltages $u_A = 300\sin(314t)$, $u_B = 250\sin(314t - 96°)$, $u_C = 330\sin(314t + 72°)$, primary winding resistances: $r_A = 22{,}5\,\Omega$, $r_B = 112{,}5\,\Omega$, $r_C = 225\,\Omega$, primary winding inductances: $L_{1A} = 0{,}0675$ H, $L_{1B} = 0{,}0045$ H, $L_{1C} = 0{,}0225$ H, primary winding capacitances $C_{1A} = 4{,}5\ \mu$ F,

$C_{1B} = 2,2\ \mu$ F, $C_{1C} = 0,4\ \mu$ F, secondary winding resistances: $R_A = 0{,}95\,\Omega$, $R_B = 2\,\Omega$, $R_C = 0{,}4\,\Omega$, secondary winding inductances: $L_{2A} = 0,003$ H, $L_{2B} = 0,0001$ H, $L_{2C} = 0,001$ H, secondary winding capacitances $C_{2A} = 100\,\mu$F, $C_{2B} = 1000\,\mu$F, $C_{2C} = 500\,\mu$F.

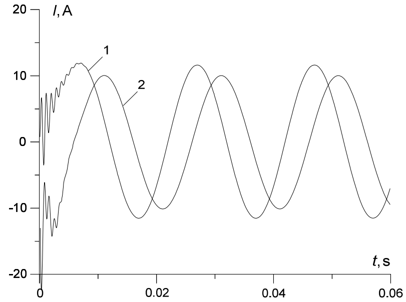

Fig. 5. Time changes of current in the transformer primary winding for the second computational example: 1 is A-phase current; 2 is B-phase current

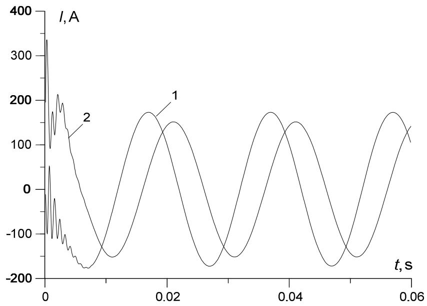

Fig. 6. Time changes of current in the transformer secondary winding for the second computational example: 1 is A-phase current; 2 is B-phase current

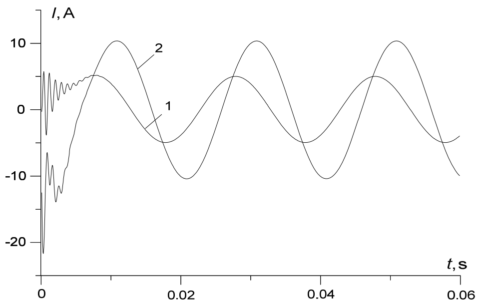

Fig. 7. Time changes of current in the transformer primary winding for the third computational example: 1 is A-phase current; 2 is B-phase current

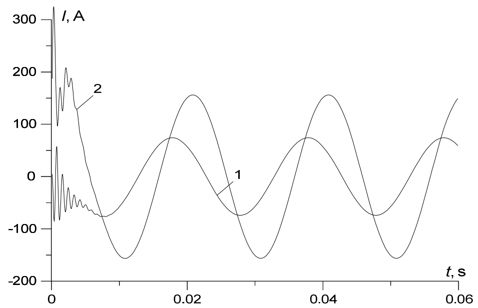

Fig. 8. Time changes of current in the transformer secondary winding for the third computational example: 1 is A-phase current; 2 is B-phase current

Figs 9 and 10 show time changes of transformer primary and secondary winding currents for asymmetrical system and single-phase short-circuit of A-phase secondary winding from $t = 0,06$ (the third computational example).

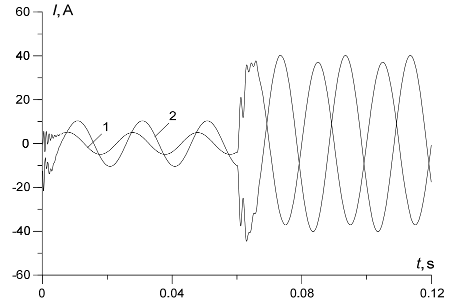

Fig. 9. Time changes of currents in the transformer primary winding for asymmetrical system and single-phase short-circuit of A-phase secondary winding (the third computational example): 1 is A-phase current; 2 is B-phase current

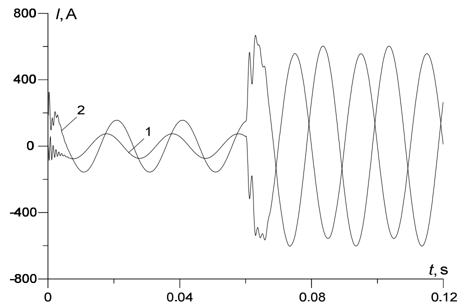

Fig. 10. Time changes of currents in the transformer secondary winding for asymmetrical system and single-phase short-circuit of A-phase secondary winding (the third computational example): 1 is A-phase current; 2 is B-phase current

Conclusions

1. On the basis of variational method [1] the mathematical model of power system, incl. power transformer loaded and fed asymmetrically, by forming the extended functional of operation by Hamilton [2] was formulated.

2. Variational approaches allow for the adequate modelling of the complicated and asymmetrical states of transformer operation for various connections of cable elements: serial [3], parallel and serial-parallel.

3. Analysis of transformer short-circuit states shows that the most dangerous for the system are 2-phase and 3-phase short-circuits.

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