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# Implementation of the spherical harmonics in glare sources simulation

***Abstract*** *The UGR index is practically used measure of a discomfort glare for the interior working environment. The basic parameters of the index formula are the angular dependences which define the visibility of light sources. In the paper the new method for light sources description has been presented. The mathematical model using spherical harmonics is discussed. Spherical harmonics allows for convenient and efficient description of the light sources positions as well as their shapes in the field of view.*

***Streszczenie*** *Wskaźnik UGR jest praktycznie stosowaną miarą olśnienia przykrego dla stanowisk pracy we wnętrzach. Podstawowymi parametrami wpływającymi na wartość wskaźnika są zależności kątowe w jakich widoczne są źródła światła. W artykule została przedstawiona nowa metoda opisu źródeł światła z wykorzystaniem harmonicznych sferycznych. Pozwalają one na wygodny i skuteczny opis położenia źródeł światła, jak również ich kształtu w polu widzenia.* ***(Implementacja harmonicznych sferycznych w symulacji źródła olśnienia)***

**Keywords:** discomfort glare, UGR, light source position, spherical harmonics.
**Słowa kluczowe**: olśnienie przykre, UGR, położenie źródła światła, harmoniczne sferyczne.

## Introduction

Discomfort glare is defined as a condition in which a person can feel pain and irritation without any effects of reduction in ability to see. Assessment of such a phenomenon seems difficult in a case of a practical situation of a specific working environment. To measure the effect a Unified Glare Rating (UGR) is applied. Factor defining the rating is determined by the formula (1), that was introduced in CIE documents [1]. The rating is also implemented in the European Standard EN 12464-2 [2].

$$UGR = 8\,log\left(\frac{0.25}{L_u}\sum_i \frac{L_i^2 \omega_i}{P_i^2}\right) \quad (1)$$

where: $L_u$ – background luminance (cd/m2), $L_i$ – luminance of glare source $i$ in the direction of the observer's eye (cd/m2), $\omega$ – solid angle of the glare source $i$ seen from the observer's eye (sr), $P_i$ – position index (Guth's index) for the glare source $i$ according to Guth's analysis.

To calculate the value of UGR, knowledge about a complex set of parameters is required. They need to be associated with proper geometrical conditions. With modern computer technology it is possible to substantially facilitate required computations.

Effective UGR value determination can be achieved with different solutions. Methods using pinhole camera and space division according to set of angles [3] were used. Today two groups of techniques may be discussed. First of them incorporates usage of simulation applications, determining luminance distribution. Getting the value of UGR for certain view position and direction is one of the features of such programs. The second group, that is in use for some time, consider application of an array of photometers to measure luminance distribution. Same as in case of the first group of techniques, the software designed for the mentioned instruments allows for calculation of UGR value.

Main difficulties of measurement of UGR value are due to complexity of geometrical relations of the photometric quantities in formula (1). This work aims to propose a new way of description of positioning lights (sources of glare), to gain simplification of UGR calculations. Method described in the work uses spherical harmonics.

## Spherical Harmonics

Spherical harmonics (SH) [4] is a set of special functions that form a base in spherical coordinates. As the Fourier transform produces a set of harmonic frequencies from the given signal function, the SH allows for approximation and representation of any spherical function in a form of a collection of coefficients. The aforementioned basis is described by formulas (2-4).

$$Y_l^m = \sqrt{2}K_l^m P_l^m(cos\theta)cos(m\varphi), \qquad m > 0 \quad (2)$$

$$Y_l^0 = K_l^0 P_l^0(cos\theta), \qquad m = 0 \quad (3)$$

$$Y_l^m = \sqrt{2}K_l^m P_l^{-m}(cos\theta)sin(-m\varphi), \qquad m < 0 \quad (4)$$

where $P_i^m$ are associated Legendre polynomials, and $K_i^m$ have a form (5).

$$K_l^m = \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} \quad (5)$$

Projecting given spherical function onto the spherical harmonics domain produces a set of coefficients that gives information about how much each of the SH basis functions (given by the index of the coefficient) is present in the original function. Any coefficient is given with a pair of SH band $l$ and octave $m$, and is defined by the weighted integral of the basis function (6).

$$f_l^m = \int_{\Omega 4\pi} Y_l^m(\theta,\varphi) f(\theta,\varphi) d\theta d\varphi \quad (6)$$

where $\theta$ and $\varphi$ describe coordinates on the sphere (in $\Omega 4\pi$ domain) and denote location of samples. A real-life case of integration over a sphere (or a hemisphere) incorporates a usage of summing up $m$ samples of the original function.

The bigger the coefficient index, acquired from band $l$ and octave $m$ pair, the more precise the original function can be approximated - such a coefficient is an equivalent of a high frequency, as in case of the Fourier transform. The high-frequency functions are usually small, bright spots on the dark background. Given that kind of function describing the light distribution on the sphere, representing it via SH coefficients requires a lot more of them then in case of a function that consists of mainly large, white or gray, smooth patches of light (a low-frequency function). Any spherical function represented by SH coefficients is reconstructed by the formula (7).

$$f(\theta,\varphi) = \sum_{l=0}^{N-1}\sum_{m=-l}^{l} f_l^m Y_l^m(\theta,\varphi) \quad (7)$$

As the band-octave pair notation is a bit complicated, introducing two-dimensional way of addressing coefficients, a simplified version of (7) is actually used. It determines a SH coefficient index to be $i = l(l+1)+m$, where $l$ is a band and $m$ is an octave ordinal. This way the reconstruction equation simplifies to (8).

$$f(\theta,\varphi) = \sum_{i=0}^{N^2-1} f_i Y_i(\theta,\varphi) \quad (8)$$

Application of spherical harmonics is very wide. The computer graphics application ranges from description of area lights [5], to global illumination techniques like precomputed radiance transfer (PRT technique) [6]. Usage of SH is also noticeable in fields of science like quantum mechanics, geophysics or study of magnetism.

The set of SH coefficients representing a spherical function can be treated as a vector. Thus any operations normally done on a vector can be applied to a SH vector as well. A given set of coefficients can be converted to another one, that approximates a different function, by multiplying it with a proper transformation matrix. The simplest changes in SH vector are applied by scaling and translation operations. Such transformations refer respectively to scaling or shifting (translating) the original function itself. Rotation, on the other hand, is more complicated, due to the fact of SH rotation invariance property. Figure 1. shows the visualization of the first SH coefficients basis function.

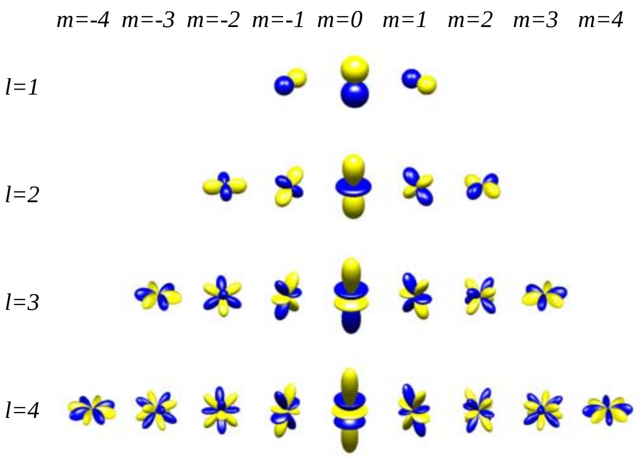


Fig. 1. Visualization of the spherical harmonics for the first bands, for $l = 1,2,3,4$

**Light Source description using SH**

For a use in the analysis of glare, a light source can be described in two ways. First, the simplest one, is a point light source in space with defined luminance and color; sometimes also a direction (for spot light sources). Another approach consider using a shape of an area light projected onto an upper hemisphere. Of course light sources that consist only of one point can be done this way as well, although that would be very inefficient. Either way, this solution forms a spherical function around a certain point $P$ that is in the center of the aforementioned hemisphere. Such function can be represented by an array of spherical harmonics coefficients. To do so, a uniformly distributed collection of normalized directions within the upper hemisphere (i.e. which cosine factor with a normal in point $P$ is above zero) is required to sample the function $L$, representing light distribution on the hemisphere. The samples are used for the process of projecting function $L$ into spherical harmonics space. Note that number of samples used does not directly imply quality of the outcome – in a case of a single point-like light source it would be better to use only one sample, while a large number of them can simply not hit the right spot, resulting in unwanted artifacts. This refers also to very small area lights or larger ones viewed from afar.

For a color, RGB light function $L$, each channel of the color of a sample is considered a separate function. This implies a need for three separate arrays of coefficients to be created. Also this shows that in such a case there will be a three times bigger usage of memory required. In any case, to get the information back from the coefficients a reverse projection is to be applied.

**SH Computations' sampling patterns**

Both the SH projection and the reverse process (reconstruction) of a function from an array of SH coefficients require a set of different samples, or in other words sampling directions, defined in the spherical coordinates. As stated earlier, any set of such samples used for either projection or the reconstruction, have to be uniformly distributed on the hemisphere. A starting point of such a scheme is to prepare a set of samples within a unit square. This way one gets a set of positions in range of [0, 1]. It is not that difficult to make them distributed uniformly in such a case, but projecting them onto the hemisphere requires additional computations [7].

Given set of normalized samples $(x,y)$: $0 \le x \le 1$, $0 \le y \le 1$ is first projected into a spherical system space, treating their positions as points on the surface of the unit sphere. Thus the actual latitude $\theta_a$ and longitude $\varphi_a$ of any sample is given with formulas (9) and (10).

$$\theta_a = 2a\cos(\sqrt{(1-x)}), \qquad 0 \le x \le 1 \quad (9)$$

$$\varphi_a = 2\pi y, \qquad 0 \le y \le 1 \quad (10)$$

In case of a hemisphere one can omit the multiplier 2 before arc cosine function, remembering also to rule out any samples that will be out of such a domain (samples that has negative dot product between their position vector and the hemisphere's “north pole” vector). The two spherical coordinates are used in both SH projection and reconstruction, to compute a value of the spherical harmonics base function for the sample. They are also fit for creation of an actual 3D point on the sphere/hemisphere using spherical to Cartesian system space transformation.

Eventually to sample the value of a spherical function, one need to bring back the sample positions to normalized space. It is required because such functions are in the case of our solution represented in a form of a rectangular texture. In most situations textures are sampled by normalized coordinates, in which case the actual size of the texture doesn't matter. In our approach as mentioned above basic samples are produced as uniformly distributed points within a unit square. Given the latitude $\theta_a$ and longitude $\varphi_a$ angles one need to convert them with simple division between their value and the maximum extent for the angle, i.e. for $\theta_a$ one need to divide it by $\pi$ for a whole sphere, and by $\pi/2$ for a hemisphere, while $\varphi_a$ requires division by $2\pi$ in any case. The actual implementation requires only the first operation to be performed, for the normalized position from which the longitude angle was acquired is already given from the base uniform distribution.

Regarding the samples placement, our implementation defines two forms of solving the issue. First of them is straight forward – given the number of wanted samples $N$, the application computes every next sample to be equally distant from the last one by the factor of $1/\sqrt{N}$ horizontally as well as vertically, starting from the “lower left corner” of the data set (first element in the data array). The positions are recalculated into actual array of indices further in the sampling process. It is done including the linear interpolation of their values. Second approach introduces the application of jittered sampling [8]. A small, pseudo-random value is added to every sampling position. Such application has an impact on the quality of calculated coefficients and eventually the reconstructed image.

Finally, it is important to add that none of the methods like importance sampling [9] or stratified sampling [9] can be applied in our solution. Function $L$, representing the light dome, is given as a set of raw data, and thus there are no indications that would imply usage of any “intelligent sampling” algorithm. Only the naive, uniform sampling applies. Additionally the samples shouldn’t be generated completely with a use of random generator of any kind, for their positions heavily influence the outcome of SH projection, especially in case of a close-to-point-like light sources. Using randomly generated sampling positions can lead to some unwanted artifacts to appear. Only randomness used is the one applied with the aforementioned jittered sampling pattern.

**Implementation**

Implementation of our approach includes two main stages of computations, done in separate applications. The separation was due to the fact of easier method of storing output images using Matlab/Octave scripts.

First of the stages incorporates spherical harmonics creation from a light function, given by a texture data (TGA image format). The program, written in C++, loads selected texture, samples it with a provided number of samples and produces a set of SH coefficients, in a form fit for the second stage of the whole process. The generated array of coefficients is loaded into a Matlab/Octave script, the second stage, that performs the reconstruction operations. It stores the outcome also into a TGA image.

In case of the projection stage, our solution uses double precision in representation of any variable used in the computations. After loading raw texture data and preparations regarding sampling positions for them, a given number of sample objects is created. Every sample object contains data about the sample direction/position in both Cartesian and spherical system of coordinates. Before passing the raw texture data to projection routine, they are put into a simple array.

Every element of such an array is actually an interpolated value of the source function, given in the mentioned data from texture. For each sample, the normalized positions are scaled to image size and clamped to nearest integer value above and below (floor and ceil operations). With thus acquired addresses a linearly interpolated value is stored into the array of data passed to projection.

The projection itself computes SH coefficients on-line, according to the formula (6), i.e. the routine does not use any form of precomputed data regarding the spherical harmonics base functions. All values are calculated on the fly. Even though a double precision for variables is applied here, for higher indices of SH coefficients the base values can become NaN (not-a-number), “spoiling” such a sample. This situation occur when an outcome of set of numerical computations, eventually being an input for certain types of functions (like square root), gets a negative zero value. Such a case is caused simply by the fact how floating point numbers are represented. Avoiding NaNs incorporates checking mentioned outcomes’ absolute value against a very small number E - should them be less than E, replace them with zero. Implementation of the SH coefficients generation has been secured with the simple algorithm.

Finally the coefficients are used by a simple Matlab/Octave script, fulfilling the formulation (8), to reconstruct the image of the light source. Precision of the variables used here is a standard one used in Matlab or Octave (no specific flags for the computations applied). Same as in case of projection the reconstruction process does not use precomputed data.

**The light source on the hemisphere**

The application developed for the paper allows for description of light sources via approximation of luminance distribution on a hemisphere. The program relies on the data stored as a textures, containing values of a certain spherical function. The purpose of the application is to calculate a vector of SH coefficients, of a predetermined length. In the performed tests the length was set to 100 coefficients, which is 10 harmonic bands. A number of different light sources were analyzed, varying in size of the patch of light placed on the hemisphere. Any tested source was defined as luminance distribution given by the function of N-th power of the scalar product between provided vector (determining the light spot’s position on the hemisphere, defined as a pair of angles, spherical coordinates) and a one given as a vector linking the center of the sphere with a point on it’s surface. This simple mathematical form provides a useful way of simulating light sources of different sizes, situated on the hemisphere. Figure 2. illustrates a set of examples of used light source functions

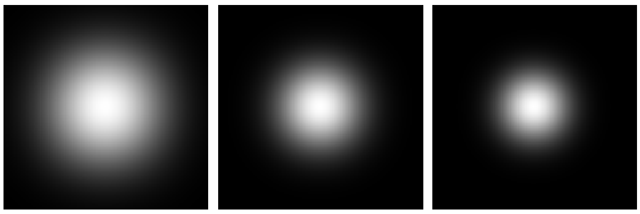

Fig. 2. Simulated light source defined as the N-th power of the scalar product between the vector of "linking" the center hemisphere with a point on its surface, and a given vector, defined by coordinates on the sphere. From the left: N=10, N=20, N=30.

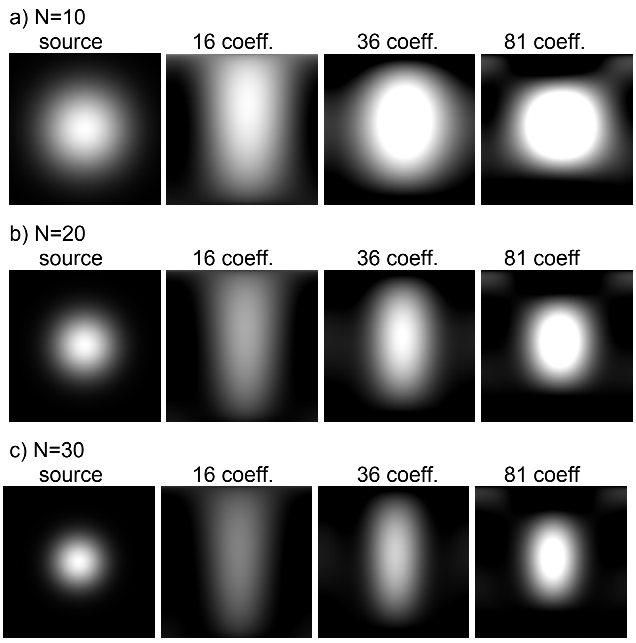


Fig 3. Examples of simulated light source represented in the form of spherical harmonics. Original sources are modeled for: a) N=10, b) N=20, c) N=30. In the columns SH representations using different numbers of coefficients have been shown.

For each test, a separate set of SH coefficients was created. With such vectors, the second stage of our application created ten images (for every vector), showing the reconstruction of the original image for a certain number of coefficients. The images were created for continuous SH bands, meaning for 1, 4, 9, 16 etc. SH coefficients. Figure 3. contains an example of three test images and three reconstructions (each for different band) for every each one of them.

Results clearly show that the most significant and visible changes in the reconstruction image are present in case of the test images that contain largest patches of light. The reconstruction for original image created for N = 10 shows the situation. On the other hand, when the original image contained much smaller streak of light, e.g. N = 100, much more coefficients is required to achieve similar effect. Basically the number of used SH coefficients highly influences the quality of the reconstruction. It also depends on the size of the light spot upon the original image. The smaller it gets, the more of the generated SH coefficients is to be used for getting a suitable outcome. In some cases even the maximum assumed in the test may not be enough.

**Summary**

The paper describes application of spherical harmonics in simulation of light sources located upon a hemisphere. The method used allows for description and approximation of light distribution functions with a set of coefficients. From the point of view of the computational complexity the method is an attractive one. Additionally, a mechanism of defining lights sources of different sizes has been proposed. It relays on a scalar product of certain vectors.

The conducted experiments showed that in discussed case usage of spherical harmonics is a good idea, but the method itself seems to be a slowly convergent process. It requires a large number of coefficients to produce proper outcome. This leads to conclusion, an additional algorithm is required for determination of a proper number of coefficients to be used in a case of a particular light distribution function

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