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doi:10.15199/48.2015.12.63

# The Equivalent Electrical Model for the heat exchanger considering working medium

***Abstract.*** *The Equivalent Electrical Model (EEM), based on the thermo-electric analogy was proposed for the heat exchanger model. The scheme of the EEM in a central part has two arms joined parallel. One of them corresponds to the heat transfer by the medium and the other to the heat transfer omitting the medium. The thermal capacitance of the medium between the active and passive elements and the thermal inductivity - these two elements correspond to the oscillations observed during investigation of the heat exchangers.*

***Streszczenie.*** *Dla modelowego wymiennika ciepła zaproponowano schemat równoważnego modelu elektrycznego (EEM) oparty na analogii termo-elektrycznej. Zastępczy schemat elektryczny w swojej centralnej części ma dwie gałęzie połączone równolegle. Jedna z nich odpowiada wymianie ciepła poprzez medium, a druga z ominięciem medium. Pojemność cieplna medium pomiędzy elementami aktywnym i biernym oraz indukcyjność cieplna - te dwa elementy odpowiadają oscylacjom, obserwowanym w trakcie analizy przebiegu wymiany ciepła w wymiennikach ciepła.**(Model elektryczny wymiennika ciepła uwzględniający rolę medium roboczego)***

**Keywords**: heat transfer, thermo-electric analogy, thermal inductivity, working medium
**Słowa kluczowe**: wymiana ciepła, analogia termo-elektryczna, indukcyjność cieplna, medium robocze

## 1. Introduction

The oscillations of a temperature occurred during some thermal processes are unexpected features of these phenomena. There are some investigations carried out to explain that behaviour of thermal processes. Vernotte [1] has carried out the modifications of classic Fourier equation of the heat conduction. These modifications take into consideration finite velocity of thermal flux propagation. This velocity is very high values in most of the practical cases (especially in the metals). The assumption of their infinity value (as is in the Fourier-Kirchhoff equation) is then acceptable. Some of the experiments show that this assumption is not always correct and may lead to conclusions not according to observed facts. These works take into consideration added by Vernotte part of heat conduction equation:

$$\mathbf{q} = -\lambda \nabla T - \frac{a_t}{w_t^2}\frac{\partial \mathbf{q}}{\partial t} \tag{1}$$

originally using by Vernotte as a thermal relaxation time:

$$\tau = \frac{a_t}{w_t^2} \tag{2}$$

The issues concerning this equation, called "Non-Fourier heat transfer" or "Non-Fourier heat conduction" (*nFhc*) are presented in a lot of articles. Most of them presented analytical solution of an equation (1) by the different boundary conditions and different parameters of the process. The numerical simulations of the results are then presented. The works of Ordonez-Miranda and Alvarado-Gil present these numerical simulations in the different cases [2,3,4,5,6]. In these works, especially in [2] and [6], the thermal relaxation time is taking into consideration. The different kinds of numerical integration schemes and boundary conditions influence on the temperature profile is presented in [7]. The numerical results of using *nFhc* for the heat transfer in the sphere are presented in [8]. The comparison of parabolic (classical Fourier) and hyperbolic (non-Fourier) heat conduction is presented in [9]. The numerical investigation of the crack in the thin layer of solids is presented in [10]. There are oscillations in the transient states of an analyzed phenomenon. The oscillations have greater parameters for small crack length than for large crack length. The smoothed particle hydrodynamics is one of the method using for solve the *nFhc* problems and it is used in [11]. The typical using of the *nFhc* is the short-time laser processing. These problems are presented in [12], but in [13] are also confirmed by an experiment, showed oscillation of the temperature. The works leading to propose in [14] the equivalent electrical model of a natural circulation BWR (Boiling Water Reactor) core were presented in [15] and [16]. The oscillations during resistance heating were observed also by Wesołowski i in. [17].

The experiments analyzing in presented work, investigation of the heat transfer between active element (resistance heating) and passive element, also have showed the thermal oscillations. The thermo-electric analogy was used for their interpretation.

## 2. The research methods

### 2.1. Thermo-electric analogy

Different authors use different terms for the electric model (circuit, network) of a thermal phenomenon, as is presented in the Table 1.

Table1. The names using for the electric model corresponding to the thermal system

| | Name | Publication |
|---|---|---|
| 1 | electrical model | [18], [19] |
| 2 | equivalent electrical model | [14] |
| 3 | equivalent electrical circuit | [20] |
| 4 | circuit representation | [21] |
| 5 | circuital model | [22] |
| 6 | thermal scheme | [23] |
| 7 | equivalent thermal network | [24], [25], [26], [27] |

The thermo-electric analogy presented, for example, in [18], interpreted the thermal phenomena as analogical to the electric phenomena, with complete conformity of the equations describing the phenomena thermal to the electric. This analogy is used for modelling thermal systems by the electric networks. Such a model of a thermal system needs a few words concerning the nomenclature of this model. The terms presented in the Tab.1., numbered from 1 to 5 seem to be more suitable than these numbered from 6 to 7. The final scheme of the phenomenon is the electric circuit, not a „thermal" network. This electric circuit is only analogical to the thermal system. The final representation of thermal object is named „equivalent electrical model" in the presented work.

### 2.1. Methods of investigation and analyze

The detailed method of different kinds of heat exchanger investigation and methods of analysis, using signal identification were presented in [26]. The shell-and-tube and plate heat exchangers were investigated. The model of heat exchanger was also investigated. The resistance heating element (bar or spiral made from copper or kanthal) was the primary side of this exchanger, the secondary side was the element (usually copper bar) situated in close neighbourhood of the primary side, without contact between them. The air was the medium of the heat flow. This heat exchanger model was investigated more detailed because of fully controlled set of heat transfer parameters influencing its course. The temperatures of active and passive elements were measured by the set of thermocouples every second.

The results of the experiments were analyzed using the methods of signal analysis and their implementation in MATLAB packet and *System Identification Toolbox*. The temperature of the primary side was assumed as the input in analyzed system and the output was the temperature of the secondary side of the model of heat exchanger. The step response of system was the main characteristic which was investigated. The discrete models were created (the discrete transmittance $H(z)$ was calculated for them) and next they were transformed into the continuous models (for which the continuous transmittance $H(s)$ was calculated).

### 3. The equivalent electrical model for the heat exchanger with considering the character of the radiation

The analysis carried out using the experimental results led to the conclusion that observed in many times phenomenon of the oscillation may be the base of the preparing equivalent electrical model (EEM) taking into consideration thermal resistance, thermal capacity, as well as thermal inductivity. The EEM was prepared for the heat exchanger working in the transient states and it was the scheme of the heat transfer from the active element through the medium to the passive element [27,28]. The radiation resistance and convection resistance were joined in the one thermal resistance in the scheme proposed in [28]. It was not correct, because the convection uses a medium, and the radiation does not. The EEM of the heat exchanger has been corrected and is presented in the Figure 1. This circuit represents the heat transfer from active element (represented as a node A) to passive element (node P). The node A represents the primary side and the node P secondary side of the heat exchanger. The voltage $U_2$ represents the difference between the temperature of heating spiral and the reference temperature (which can be assumed as equal to 0 degrees Celsius). The temperature of the heated bar, which is represented by the voltage $U_2$ in the EEM is calculated the same way. The resistance heating of the spiral (node A) is represented by the left side of the diagram, heating element passive from active is represented by the right side of the diagram (separated by the dashed line). The current $I$ represents the heat flux caused by the resistance heating the spiral.

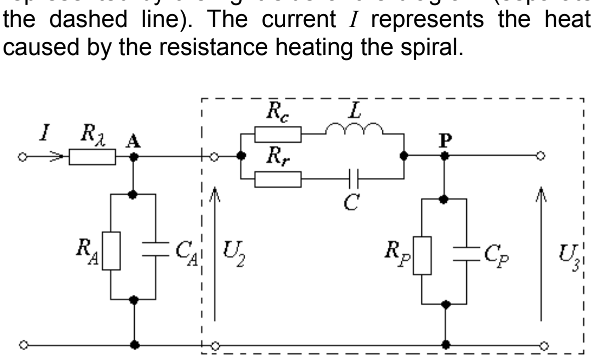


Fig. 1. The equivalent electrical circuit with considering the radiation and convection character

The meaning of circuit elements is the following:
1. The $C_A$ and $C_P$ represent the thermal capacity of the active and passive elements respectively
2. The thermal resistance $R_A$ and $R_P$ represent the resistance of waste to the surroundings
3. The $R_\lambda$ represents the conductance through the active element
4. The central part of the scheme represents the heat transfer between the two elements - using thermal resistance of convection $R_c$ and thermal resistance of radiation $R_r$. The first of them uses medium and the second omits the medium
5. The thermal capacity $C$ of medium between the elements and the thermal inductivity $L$ are the two elements giving the possibility of the oscillation in presented circuit.

The continuous transmittance of the right side of this circuit (with the $U_2$ input and the $U_3$ output) has a form:

(3) $$G(s)=\frac{U_3(s)}{U_2(s)}=\frac{L(s)}{M(s)}=\frac{A_1 s^2+B_1 s+C_1}{s^3+A_2 s^2+B_2 s+C_2}$$

when $L(s)$ - numerator, $M(s)$ - denominator, $A_1$, $B_1$, $C_1$, $A_2$, $B_2$, $C_2$ - polynomials coefficients:

(4) $$A_1=\frac{1}{R_c C_P},\; B_1=\frac{R_c+R_r}{R_c C_p L},\; C_1=\frac{1}{R_c C_p C L},$$

(5) $$A_2=\frac{LC(R_p+R_c)+R_p C_p(L+R_r R_c C)}{R_p R_c C_p C L},$$

(6) $$B_2=\frac{R_p(R_c+R_r)C+L+R_r R_c C+R_p R_r C_p}{R_p R_c C_p C L},$$

(7) $$C_2=\frac{R_p+R_r}{R_p R_c C_p C L}.$$

The proposed diagram of the EEM and their transmittance were verified by the comparison with the experiments. Two experiments were taken into consideration, their parameters are as follows:

Experiment 1 (heat flux 1200 [W], medium temperature of the stabilization of the active element 535 [°C], medium temperature of the stabilization of the passive element 90 [°C])

Experiment 2 (1600 W, 650°C, 110°C).

The experiments were approximated using Parameters Models with the structure Output Error (OE). The comparison between the models with the structure near to proposed EEM (OE 331 for both of the experiments) and near to EEM proposed in [28, 29] (OE 221) have been carried out. This comparison showed better fitting to the experiment the model OE 331 than OE 221. The structures of these models are as follows:

OE 331

(8) $$G(s)=\frac{a_1 s^2+b_1 s+c_1}{s^3+a_2 s^2+b_2 s+c_2}$$

OE 221

(9) $$G(s)=\frac{a_3 s+b_3}{s^2+b_4 s+c_4}$$

Fitting is represented by the two parameters. One of them is FIT (which is maximized to 100) and the other is FPE (which is minimized). The values of these parameters for both of the experiment are presented in the Table. 2.

Table 2. Values of the fitting parameters for models OE

| | Model | | | |
|---|---|---|---|---|
| | OE 331 | | OE 221 | |
| | FIT | FPE | FIT | FPE |
| Experiment 1 | 91,25 | 5,51 | 79,86 | 46,14 |
| Experiment 2 | 90,64 | 9,27 | 80,82 | 43,03 |

The structure of the presented EEM is in accordance with the structure of models OE 331 received from the experiments. The course of the STEP response for this model for the Experiment 2 is presented in the Figure 2. The character of the STEP course confirmed the oscillatory character of the observed phenomenon.

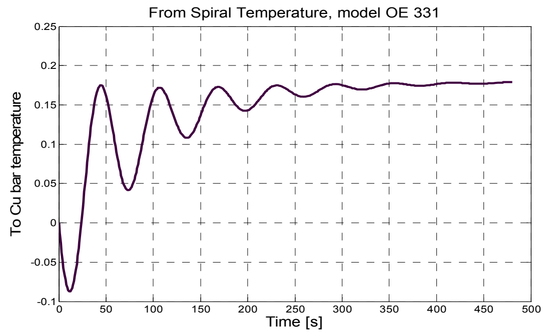


Fig. 2. The course of STEP response for model OE 331

## 4. Summary

The proposed diagram of the equivalent electrical model for the heat exchanger seems to be the correct analog of the real heat transfer, considering both their oscillatory character and the structure of the heat transfer. The comparison between proposed EEM scheme with the experimental results showed the satisfying conformity the transmittance structures with the structure of the models obtained from the experiment.

**Nomenclature:**

$a_i$, $b_i$, $c_i$, $d_i$ - polynomials coefficients in (8) and (9)
$a_t$ - thermal diffusivity
$A_i$, $B_i$, $C_i$ - transfer function (3) coefficients
*FIT*, *FPE* - valuation of the model
$G(s)$ - continuous transfer function
$H(s)$, $H(z)$ - transmittance continuous, discrete
OE - *Parametric Model* with the structure (8) and (9)
$\mathbf{q}$ - heat flux
$s$ - generalized variable, correspond to ($j\omega$)
$t$ - time
$T$ - temperature
$w_t$ - heat flux propagation velocity
$\lambda$- thermal conductivity
$\tau$- thermal relaxation time

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