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Applicability assessment for simplified formulas to compute surface impedance at screened surfaces

Abstract. *It is well known that when determining surface impedance for screened well conducting bodies errors generated due to underestimated curvature radius of the surface occur. This problem is dealt with in the paper, where calculation results for formulas with the screened surface curvature considered and disregarded are compared.*

Streszczenie. *W pracy przeanalizowano zagadnienie błędu popełnianego przy wyznaczaniu impedancji powierzchniowej ekranowanego ciała przewodzącego, spowodowanego założeniem o małej krzywiźnie powierzchni. Porównano rezultaty obliczeń według dwóch wzorów: uwzględniającego i nieuwzględniającego krzywiznę ekranowanej powierzchni.* (***Ocena zakresu stosowalności uproszczonych wzorów do obliczania impedancji powierzchniowej na powierzchniach ekranowanych***).

Słowa kluczowe: quasi-stacjonarne pole elektromagnetyczne, impedancyjne warunki brzegowe, impedancja powierzchniowa, ekrany.
Keywords: quasi-stationary electromagnetic field, impedance boundary conditions, surface impedance, screens.

Introduction

Impedance boundary conditions (IBC) have proven to provide a significant simplification when applied to solving many technical electrodynamics problems at conducting surfaces of the electrodynamic system under analysis [1]. Such conditions express usually approximate relations between electromagnetic field components, or potentials, at boundary surfaces between the conducting and dielectric areas. For low frequency harmonic fields IBC can be noted with two different forms [2, 3], each of them, however, contains a specific factor, such as e.g. surface impedance.

One of the basic assumptions made when developing IBC is the condition that all dimensions of the system conducting bodies are significantly larger than their equivalent penetration depth of the electromagnetic field. Thus, to assume IBC at permeable screen surfaces may seem incorrect as their thickness does not meet these criteria. Nevertheless, the authors of the paper [2] proved that with appropriately modified values of specific factors in IBC this condition can be successfully applied also at screened surfaces. The obtained relations for such factors include such parameters as e.g. the local curvature of the conductor surface. Still, impedance boundary conditions are usually used for surfaces with large curvature radius in comparison to field penetration depth. Such an assumption allows such relations to be significantly simplified.

In this paper an accuracy analysis for simplified formulas to calculate surface impedance for a few typical cases, widely applied in electro techniques was performed. The remaining specific factors appearing in various IBC forms can be uniquely determined from the surface impedance [2].

Exact and simplified formulas for surface impedance of a screened surface

A conducting body coated with a conducting layer, called screen, of different material parameters and a constant thickness d comparable with the field penetration depth $g=\sqrt{\dfrac{2}{\omega\gamma\mu}}$, immersed in a sinusoidal variable electromagnetic field (Fig.1.) is subject to considerations.

It is assumed for electric conductivities γ_s, γ_b and magnetic permeabilities μ_s, μ_b of the screened body and the screen alike to be constant, and for displacement currents within them to be negligibly small; a steady state is assumed for the system.

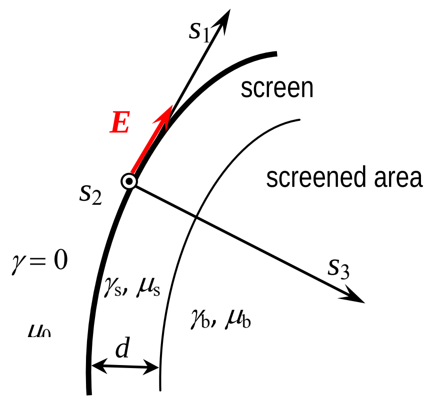

Fig.1. The system under consideration

It can be demonstrated that as long as such assumptions hold the electric field vector in the conducting areas is oriented nearly parallel to the system boundary surfaces. The ratio of the tangential components of the intensities of electric and magnetic fields at the external surface is called surface impedance Z

$$Z \equiv \frac{E_{\parallel}}{H_{\parallel}} \tag{1}$$

In [2] the exact formula for this value was developed, where the surface curvature was taken into account for the spherical model

$$Z_k = j\omega\mu_0 R_1 \frac{1-F}{2+F} \tag{2}$$

where $F = 1-3\dfrac{a_{12}c_{22}-a_{13}c_{21}}{c_{11}c_{22}-c_{12}c_{21}}$, $c_{11}=a_{12}-a_{22}$, $c_{12}=a_{13}-a_{23}$, $c_{21}=a_{32}a_{44}-a_{42}a_{34}$, $c_{22}=a_{33}a_{44}-a_{43}a_{34}$, $a_{12}=\dfrac{\mu_2}{\mu_1}(1-\alpha_s R_1)\exp(\alpha_s R_1)$,

$$a_{13}=\frac{\mu_2}{\mu_1}(1+\alpha_s R_1)\exp(-\alpha_s R_1),$$
$$a_{22}=(1-\alpha_s R_1+\alpha_s^2 R_1^2)\exp(\alpha_s R_1),$$
$$a_{23}=(1+\alpha_s R_1+\alpha_s^2 R_1^2)\exp(-\alpha_s R_1),$$
$$a_{32}=\left(\frac{R_1}{R_2}\right)^3(1-\alpha_s R_2)\exp(\alpha_s R_2),$$
$$a_{33}=\left(\frac{R_1}{R_2}\right)^3(1+\alpha_s R_2)\exp(-\alpha_s R_2),$$
$$a_{34}=-\frac{\mu_3}{\mu_2}[(1-\alpha_b R_2)\exp(\alpha_b R_2)-(1+\alpha_b R_2)\exp(-\alpha_b R_2)]$$
$$a_{42}=\left(\frac{R_1}{R_2}\right)^3(1-\alpha_s R_2+\alpha_s^2 R_2^2)\exp(\alpha_s R_2),$$
$$a_{43}=\left(\frac{R_1}{R_2}\right)^3(1+\alpha_s R_2+\alpha_s^2 R_2^2)\exp(-\alpha_s R_2),$$
$$a_{44}=-(1-\alpha_b R_2+\alpha_b^2 R_2^2)\exp(\alpha_b R_2)+ \\ +(1+\alpha_b R_2+\alpha_b^2 R_2^2)\exp(-\alpha_b R_2),$$

R_1 – curvature radius of the external boundary surface

$R_2 = R_1 - d$ – curvature radius of the internal boundary surface

The approximate solution is obtained with additional assumptions made for all body dimensions and for the surface curvature radius at each point to be significantly larger than the equivalent field penetration depth. By assuming the local coordination system s_1, s_2, s_3 in the vicinity of any selected point of the screen surface as in Fig. 1, the general solution of the complex Maxwell equations can be presented as

(3) $$\boldsymbol{E}=[E_1,0,0] \quad \boldsymbol{H}=[0,H_2,H_3]$$

(4) $$E_1^s=E^+(s_1,s_2)e^{-\alpha_s s_3}+E^-(s_1,s_2)e^{\alpha_s s_3}$$
$$H_2^s=\frac{1}{Z_s}\left(E^+(s_1,s_2)e^{-\alpha_s s_3}+E^-(s_1,s_2)e^{\alpha_s s_3}\right)$$

in the screen, and

(5) $$E_1^b=E^0(s_1,s_2)e^{-\alpha_b s_3}, \quad H_2^s=\frac{1}{Z_b}E^0(s_1,s_2)e^{-\alpha_b s_3}$$

in the screened area,

where $\boldsymbol{E}$, $\boldsymbol{H}$ are the complex amplitudes of electric and magnetic field intensities, respectively,

$$\alpha_s=\sqrt{\mathrm{j}\omega\gamma_s\mu_s}, Z_s=\sqrt{\frac{\mathrm{j}\omega\mu_s}{\gamma_s}}, \alpha_b=\sqrt{\mathrm{j}\omega\gamma_b\mu_b},$$
$$Z_b=\sqrt{\frac{\mathrm{j}\omega\mu_b}{\gamma_b}}$$

By introducing a classical electrodynamics boundary conditions the relations between the functions E^+, E^-, E^0 from the formulas (4), (5) are obtained. Based on this, modified surface impedance at the screen surface can be written as

(6) $$Z\equiv\frac{E_1}{H_2}=Z_s\frac{Z_s\sinh(\alpha_s d)+Z_b\cosh(\alpha_s d)}{Z_s\cosh(\alpha_s d)+Z_b\sinh(\alpha_s d)}$$

where d stands for the screen thickness.

(A slightly different form of the formula (6) and detailed calculation instruction can be found in [3]).

Computation results

The applicability of formula (6) to determining modified surface impedance at the screened surface depends mainly on the degree to which the assumption about the small curvature is true. To the error due to this simplification, however, contribute many parameters such as field frequency, material parameters of the screen and the screened body, screen thickness, or last but not least, the surface curvature radius. In the present paper a few typical screening cases applied in low frequency electrical devices, e.g. transformers tanks were analysed. It was assumed for the parameters of the screened body are of typical construction steel, while the screen is made of aluminium, copper (electromagnetic screens), or transformer steel (as for magnetic screens). The following values for parameters were assumed

- field frequency f = 50 Hz,
- screened area μ_r = 500, γ = 8 MS/m,
- screen: Al, μ_r = 1, γ = 35 MS/m,
- steel screen μ_r = 2000, γ = 8 MS/m.

The plots shown in Figs. 2 and 3 present the comparison between the module of the surface impedance $|Z|$ and its independent variable φ calculated from the formulas (2) and (6) against the radius of the surface curvature for the cases under consideration. They represent screens of Al and Cu of the thickness d = 10 mm, as well as the steel screen of a thickness d = 1 mm.

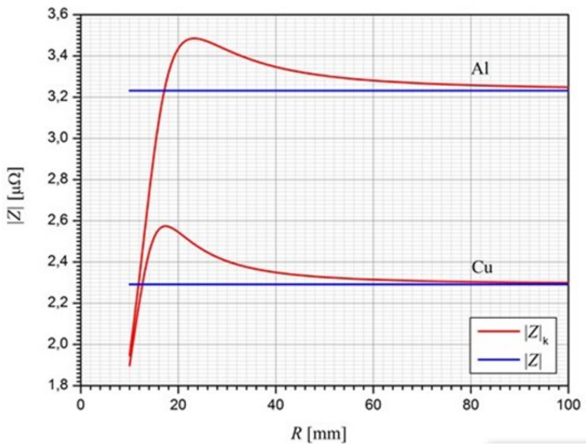

Fig.2a. Comparison of surface impedances calculated according to exact (2) and simplified formula (6) for Al and Cu screens; modules

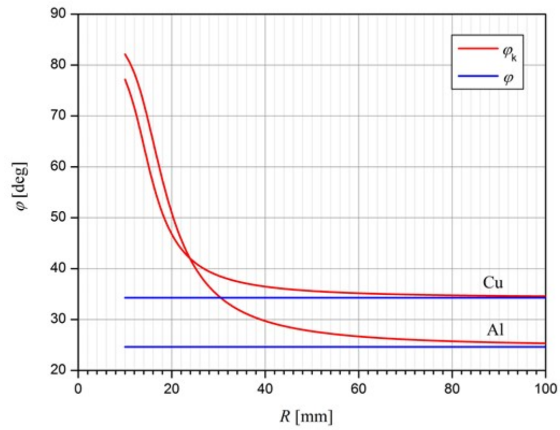

Fig.2b. Comparison of surface impedances calculated according to exact (2) and simplified formula (6) for Al and Cu screens; independent variables

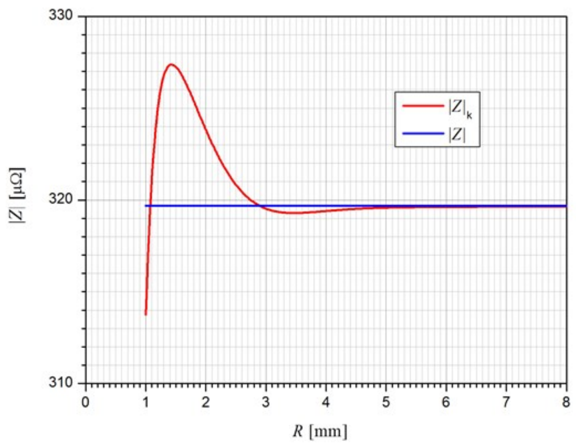

Fig.3a. Comparison of surface impedances calculated according to exact (2) and simplified formula (6) for a steel screen; modules

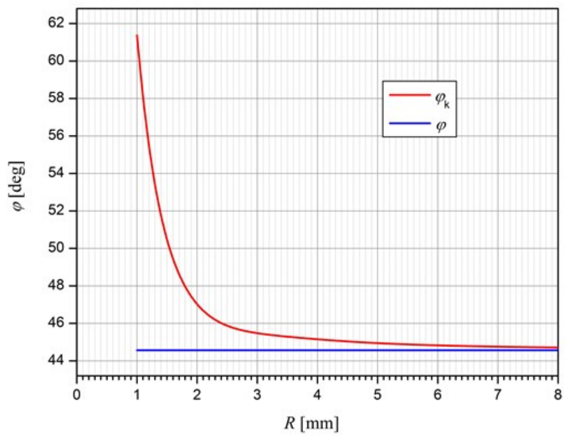

Fig.3b. Comparison of surface impedances calculated according to exact (2) and simplified formula (6) for a steel screen; independent variables

The plots show a striking lack of monotonicity for $Z_k(R)$ function.

Diagrams presented in Figs. 4, 5, and 6 show the dependence of the percentage error made by ignoring the surface curvature with relation to the surface curvature radius R and the screen thickness d. To estimate the error it was assumed

$$\delta = \frac{|Z_k - Z|}{|Z_k|} \cdot 100\% \tag{7}$$

where Z_k stands for the value of the modified surface impedance with surface curvature taken into account and calculated as provided in [2]), and Z impedance calculated form formula (6).

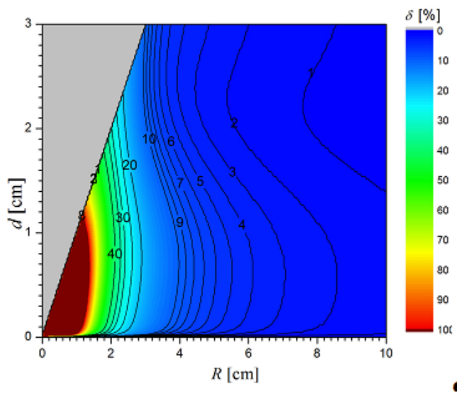

Fig.4. Dependence δ as a function of screen curvature radius R and the screen thickness d for Al screen

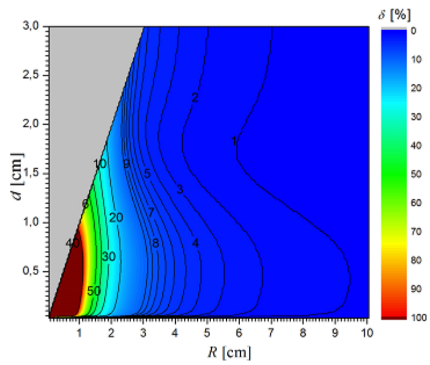

Fig.5. Dependence δ as a function of screen curvature radius R and the screen thickness d for Cu screen

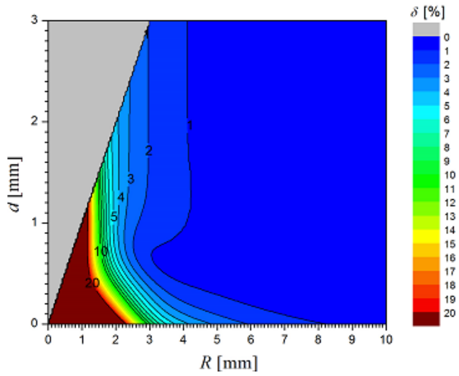

Fig.6. Dependence δ as a function of screen curvature radius R and the screen thickness d for steel screen

Conclusions

The problem of errors that occur when determining a surface impedance of a screened conducting body, due to small surface curvature assumption was investigated in the presented paper.

A comparison of computations according to two formulas, namely with and without the screened surface curvature assumed (see Fig. 2 – 6) was carried out.

Analysis of errors resulting from neglecting the screened surface curvature allows drawing the following conclusions

- the error is very sensitive to screened surface curvature radius value and varies within the range up to the value circa 5 times bigger than the electromagnetic field penetration for electromagnetic screens (Al, Cu) (see Fig. 4, and 5), as well as to 3 times bigger than the electromagnetic field penetration for magnetic screens (see Fig. 6);
- within the range of the screen thicknesses applicable in engineering practice, the error dependence on the screen thickness parameter is relatively small.

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