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Control of a Finite Element Based Dynamic System

***Abstract**. This paper presents the formulation of the circuit-coupled finite element method embedded in closed loop control system. The controller checks the output of the dynamic system after each time step and controls the input (current or voltage) to reach the steady state faster. The analysed dynamic systems are a voltage fed solenoid with iron core, and a three phase switched reluctance motor. The results of the voltage driven solenoid are compared with the results from the analytical model. The control parameters for the proportional-integral-derivative controller were estimated using the step response of the solenoid. The controller of switched reluctance motor is a speed and position based control logic.*

***Streszczenie.** W artykule zaprezentowano sformułowanie sprzężone obwodowo-elementowoskończeniowe wykorzystanoe w systemie sterowania z zamkniętą pętlą. Kontroler sprawdza wyjście z systemu dynamicznego po każdym kroku czasowym i steruje wejście (prąd lub napięcie) w celu szybszego dojścia do stanu ustalonego. Analizowane systemy dynamiczne są napięciowo zasilanymi solenoidami z rdzeniem żelaznym w połączeniu z trójfazowym przełączalnym silnikiem reluktancyjnym. Wyniki otrzymane dla układu napędowego porównane zostały z wynikami otrzymanymi w modelu analitycznym. Parametry sterowania sterownika różniczkowo-całkowego zostały estymowane za pomocą analizy skokowej odpowiedzi solenoidu. Sterownik silnika reluktancyjnego przełączalnego bazuje na logice sterowania szybkości i położenia. (**Sterowanie systemu dynamicznego bazujące na metodzie elementów skończonych**).*

Keywords: Field-circuit coupling finite element formulation, Closed loop control, PID controller, Switched reluctance motor.
Słowa kluczowe: polowo-obwodowe sformułowanie elementów skończonych, sterowanie w zamkniętej pętli, sterownik PID, przełączalny silnik reluktancyjny.

Introduction

To set up the state space representation [1, 2] of a physical system in some academic cases are easy, however it is impossible Thus, the design of the controller is not so accurate because of the simplifications. The behaviour of closed loop control [1, 2] hardly depends on the controller. Ergo, the appropriate model of the system is a very important task of the controller design.

To the appropriate model, the wide range of devices in the electrical engineering fields, the finite element method (FEM) [3, 4] is a very useful technique, when the problems with complex geometry cannot be solved by analytical methods. But the finite element model of the physical system is not enough to replace the state space model of the system. Modelling of physical behaviour of the dynamic devices is a very complex task, because electromagnetic field has strong interaction with mechanical motion and external circuits. These interactions need to be given a coupled-field treatment for realistic and accurate analysis. Further, most of the electric equipments are voltage fed, so to take into account the circuit equation is necessary for the accurate analysis, and integration into a control loop.

One of the possibilities is the integration of the finite element based model and the control system into one simulation tool is the Matlab-Simulink system simulator [5, 6, 7]. The controller can be realized in the Matlab/Simulink environment easily. The finite element tool, which is implemented under the Matlab computing environment in C language and in own scripting language of the Matlab can also be embedded in the Simulink environment [5, 7].

Closed Loop System Simulation

In a feedback control system there are different ways of controlling the output of the system such as proportional-integral-derivative (PID) control, adaptive control, neural network prediction control and fuzzy logic control [1]. In this case the PID controller [1], [2] has been used.

There are many PID control configurations, but the most common implementation of this controller, as shown in Fig. 1, is the feedback-loop with a single input and single output. The reference is the desired current of the coil or velocity and system output is the calculated current of the coil or velocity from the FEM model. The signal error enters the PID control block and the resulting excitation signal is the sum of the error signal affected by the proportional, integral and derivative actions. The output excitation signal of the PID controller corresponds to the applied voltage, which is the input of the dynamic system [1], [2].

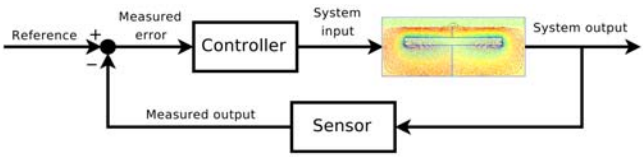

Fig.1. Block diagram of the closed loop control.

The fractional order PID tuning algorithm [8] has been used for the estimation of controller parameters, which is a built-in function in Matlab-Simulink.

The feedback control loop is implemented in Matlab-Simulink (as it is shown in Fig. 1), where the controller is a built-in PID controller function block, which can be used for the tuned parameters with filtered derivatives [1].

$$C_{PID}(s) = K_P + K_I \frac{1}{s} + K_D \frac{Ns}{s+N} \quad (1)$$

where K_P, K_I, K_D are the controller parameters, the proportional gain, the integral gain and the derivative gain, respectively, and N is the filter coefficient [1]. The controller determined the applied voltage, which is the input of the finite element model.

The finite element model is implemented in a Matlab function, which contains the finite element procedure [5, 6]. The finite element procedure means load the finite element mesh from GMSH, and data of materials, assembling the coupled equation system and solve it. The data load and the assembly of permanent part of equation system have been done only in the first time step to improve the running performance of the loop.

Coupled Finite Element Formulation

The magnetic problems are modelled in two-dimensional space, using the FEM to discretize the domain, which is based on the weak formulation of the partial differential equations, which can be obtained by Maxwell's equations and the weighted residual method [3]. The magnetic vector potential formulation has been applied, and the temporal derivatives are discretized by the backward Euler's scheme. The field and circuit equations of stranded conductors are combined together using the direct coupling method [4, 9]. Eq. (2) shows the matrix system of the field equations [4, 9]:

(2)
$$\mathbf{S}\cdot\mathbf{A}(t)+\mathbf{N}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}(t)-\mathbf{P}\cdot\mathbf{I}(t)=\mathbf{0}$$
$$\mathbf{Q}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}(t)+\mathbf{R}\cdot\mathbf{I}(t)-\mathbf{L}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{I}(t)=\mathbf{U}(t)$$

where $\mathbf{A}$ is the vector of magnetic vector potential; $\mathbf{I}$ is the vector of currents in the windings; $\mathbf{U}$ is the vector of voltages at the terminal of the winding; $\mathbf{S}$ is the matrix related to permeability; $\mathbf{N}$ is the matrix related to electric conductivity; $\mathbf{P}$ is the matrix associated with constant coil current; $\mathbf{Q}$ is the matrix associated with flux linkage; $\mathbf{R}$ is the matrix of d.c. resistance of windings, $\mathbf{L}$ is the matrix of the end-windings inductances.

In order to simulate the rotation of the rotor in the two-dimensional case, we used one of the most common methods, the so called moving band technique [9]. The new angular speed and rotor displacement are evaluated by the mechanical oscillation equation [4]:

(3)
$$\omega_r(t)=\omega_r(t-\Delta t)+\frac{\left[T_e-T_L-D_r\cdot\omega_r(t-\Delta t)\right]}{J_r}\cdot\Delta t,$$
$$\alpha_r(t)=\alpha_r(t-\Delta t)+\omega_r(t)\cdot\Delta t,$$

where J_r is the rotor inertia moment, D_r is the friction damping coefficient, T_e is the electromagnetic torque, T_L is the load torque acting on the mechanical axis, ω_r is the rotor speed, and α_r is the rotor angular position. At each time step, the electromagnetic torque is calculated via the Maxwell's stress tensor [4].

The first problem is solved in static magnetic and eddy current case by the coupled finite element formulation. The above described formulation is the eddy current field formulation. The formulation of static magnetic field is same, the only difference is the induced current term (second term of first equation) of equation (2).

The geometry has been meshed with the help of first order triangular element by GMSH [10].

Application examples

To demonstrate the importance of the presented method, an axisymmetric magnetic problem, and a planar problem are tested. The former is an iron core solenoid, and the latter is a switched reluctance motor, as it can be seen in Fig. 2.

The first studied magnetic system is a solenoid with an iron core. This device is axisymmetric and consists of two iron pot core. The number of turns of the winding is 50, and the coil resistance is 2 Ω. The B-H relationship of the iron is considered to be linear, the relative permeability is 3000, and the conductivity of iron is $1\cdot10^5$ S/m. The geometrical parameters of the problem can be found in [4]. The excitation is a step pulse of 10V, which is switched on at *t*=0 s.

The analytical solution of the solenoid is based on the theory of magnetic circuits with two simplifications. The effect of the eddy current, and the reluctance of the iron core are neglected. The equation for the current of the coil for this RL circuit is [4],

(4)
$$i(t)=\frac{u(t)}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

where *u*(*t*) is the excitation voltage; *R* is the resistance of the coil; *L* = 0.9869 mH is the calculated inductance, and *t* is the time.

The second magnetic system is a three-phase (6/4) switched reluctance motor (SRM). The current excited static magnetic version of this problem is an Agros2D [11] example. The geometry and parameters are in [11].

The number of turns of the windings is 50, and the coil resistance is 0.4 Ω. The B-H relationship of the iron is considered to be linear, the relative permeability is 1000, and the conductivity of iron is $1.39\cdot10^6$ S/m. The amplitude of the square wave voltage excitation is 1.57 V.

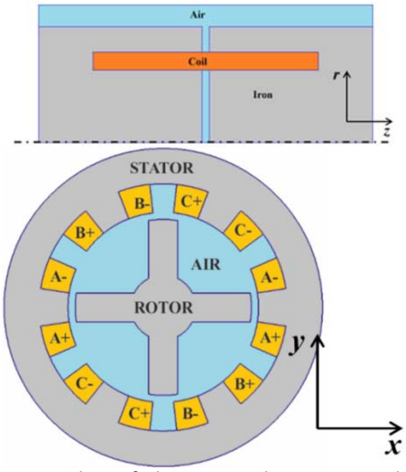

Fig.2. The geometries of the magnetic systems, the iron core solenoid and the 6/4 switched reluctance motor.

Results and Discussion

Before implementing the closed loop control of solenoid, the behavior of it is analyzed by the step response. The input variable of the system is the excitation voltage and the output variable is the current of the coil. To determine the step response of the system used a constant voltage of 10V for 0.01s. The transfer function of this system in complex frequency domain [2]:

(5)
$$W(s)=\frac{1}{sL+R},$$

where L and R are the inductance and the resistance of the system, and s is a complex variable [1], [2].

Fig. 3 shows the step responses of the magnetic system. The transient behavior of the solutions is different, because as the model of the system is more and more adequate, the calculation of the inductance of the solenoid is also more accurate. The inductance of the magnetic system is 0.9869 mH, 1.5464 mH and 1.5641 mH for the analytic solution, static magnetic field solution and eddy current field solution, respectively. The settling time is also depend from the inductance, because the time constant of the system is $\tau = L / R$ [2]. So, if the inductance is larger, the settling time (approximately 5τ) will be longer. This conclusion is also supported by the enlarged part of the Fig. 3.

The transfer function of the magnetic system (equation (5)) has been used for the tuning in the case of both FEM models. The transfer function of the static FEM models can be prepared easily by the calculated inductances and by the given coil resistance. In the case of dynamic FEM model, the least square method [12] used for the system identification, because the resistance of the system is also varies in time because the effect of eddy current. This statement is supported very well Fig. 4, where the solution of the static magnetic field can be seen. This figure shows the magnetic vector potential distribution in the steady state. The result of eddy current field in the steady state is same as the static magnetic case. The eddy current is induced only in the transient state.

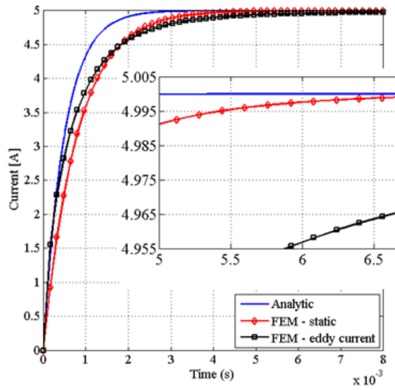

Fig.3. Step response of the solenoid.

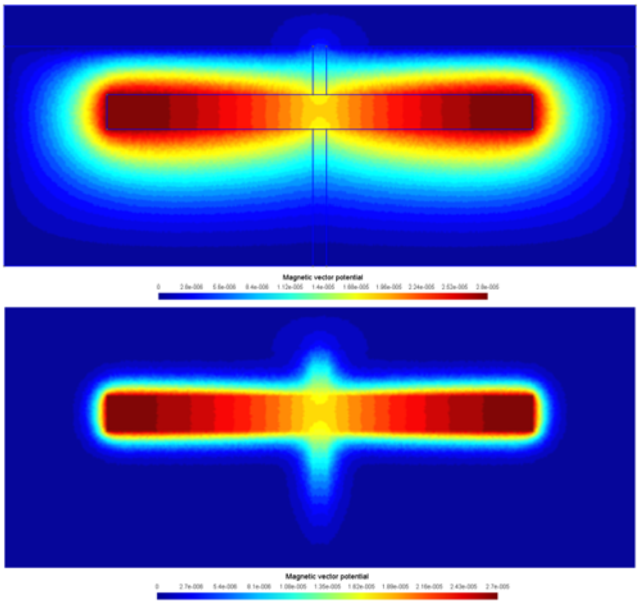

Fig.4. The magnetic vector potential distribution in static magnetic (upper) and eddy current (lower) case in the steady state.

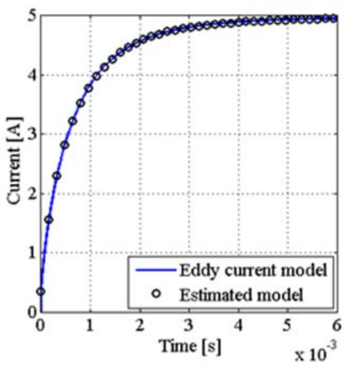

Fig.5. Step responses of the dynamic finite element model and of the estimated eddy current model, $W_{EC}(s)$.

The results of the identification of the eddy current finite element model can be shows Fig. 5, and the transfer function of estimated model is the following:

(6)

$$W_{EC}(s) = \frac{0.03368s^3 + 1309s^2 + 5.244\cdot 10^6 s + 2.389\cdot 10^9}{s^3 + 8858s^2 + 1.403\cdot 10^7 s + 4.786\cdot 10^9}.$$

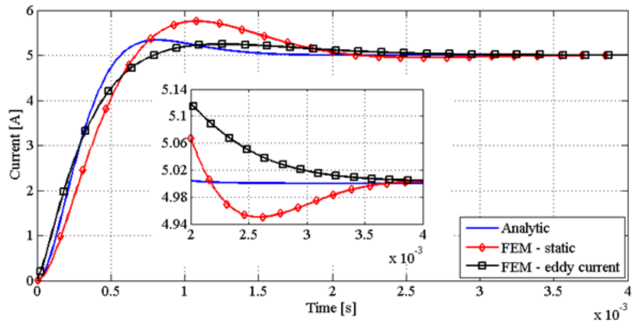

Fig.6. Responses obtained with controllers tuned with analytic model.

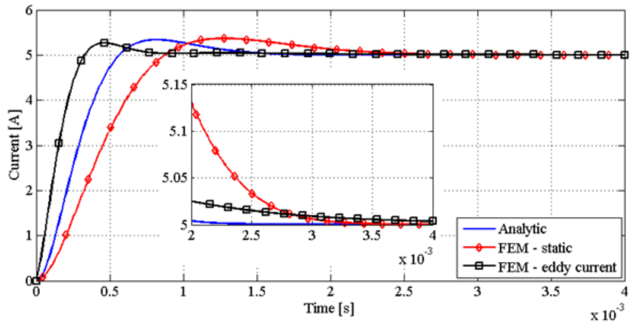

Fig.7. Responses obtained with controllers tuned with each model (analytic with analytic model, static with static model, eddy current with estimated eddy current model).

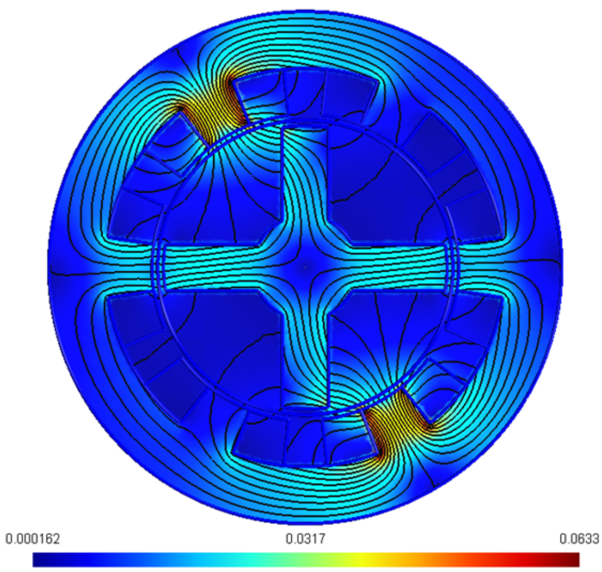

Fig.8. The magnetic flux distribution and the equipotential lines into the switched reluctance motor.

The implemented feedback control loop is analyzed in two cases. The first case, when the PID controller is tuned for the analytic solution. This case shows, why the model accuracy is important in the controller design. The second one shows, when the controller of static and dynamic system are tuned for the static and dynamic model, respectively. So, the controller tuned for each model to show the maximum performance of the closed loop control.

Fig. 6 and Fig. 7 show the temporal behavior of the current of the coil of the closed loop system.

These figures show the first 4 ms of the simulations, because after this time to reach steady state in each case. There is big difference between the simulations of control system of FEM models, because the effect of eddy current is not negligible for the studied problem. Fig. 6 shows the feedback control behavior, if the analytical model used for the control design. The finite element model is more in line with reality than the analytic model. In this case, the tuned PID controller does not work so well, because the overshoot is increased at the static model, and the rising time is decreased of both numerical models. Further, the steady state is reached with some oscillation, as you can see in the enlarged area. However, if the more accurate model (FEM model) used for the estimation of the controller parameters, the transient behavior of the magnetic system is much better. The overshoot and settling time are much smaller, and the steady state is reached without any oscillation, as it can be shows Fig. 7.

A simple control strategy used with the direct field-circuit coupling FEM model (see in Fig. 8) of the 6/4 SRM. The speed and position of the rotor were used for controlling the switches of the excitation after each time step.

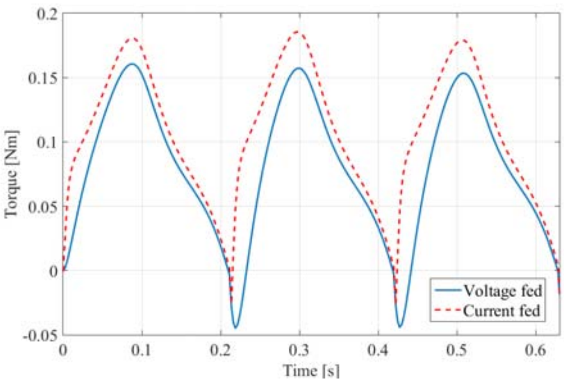

Fig.9. Electromagnetic torque waveforms obtained the voltage fed model and current fed model.

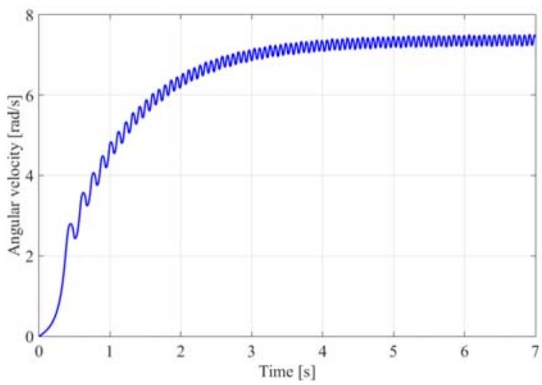

Fig.10. The angular velocity variation in time.

Fig. 9 shows the electromagnetic torque versus time. The difference between the two curves caused by the inductance of the windings and the end winding inductance. Fig. 10 shows the results of the speed and position based closed loop control of the SRM, i.e. the transient waveform of angular velocity.

Conclusions

This paper presents the field-circuit coupling finite element model embedded in closed loop control for analysis of control behavior of the magnetic system. The transient behavior of the problems is analyzed by the open loop and by the closed loop control.

It can be concluded, that the accuracy of the model is very important in the controller design. The closed loop performance indicators of the system are depend from the parameters of the controller, as it can be show in Fig. 7 – Fig. 8. The significant advance of using the finite element model in the feedback control loop in the Matlab-Simulink environment is that the numerical field analysis can be easily interconnected with different control strategies or linked external electric circuits to the windings. Thanks to this environment flexibility, allowing the analysis of a 6/4 switched reluctance motor coupled to the speed and position based control logic.

The aim of future research is decreased the computation time of the control system with FEM model by the help of parallelization.

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