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Reactive Compensation of LTI Loads in Three-Wire Systems at Asymmetrical Voltage

***Abstract**. The paper presents fundamentals of design of reactive compensators for total compensation of the reactive and unbalanced currents of linear time invariant (LTI) loads supplied with asymmetrical sinusoidal voltage in three-wire systems. Theoretical fundamentals for the presented method are provided by the Currents' Physical Components (CPC) based power theory. Development of the reactive compensator equation is the very core of the method presented. A draft of the CPC-based power theory of LTI loads in three-wire systems with asymmetrical, but sinusoidal supply voltage is also presented in the paper.*

***Streszczenie.** Artykuł przedstawia podstawy projektowania kompensatorów reaktancyjnych prądu biernego i niezrównoważenia liniowych i czasowo niezmienniczych (LTI) odbiorników trójfazowych zasilanych sinusoidalnym, lecz niesymetrycznym napięciem. Teoretyczne podstawy kompensacji oparte są na Teorii Mocy Składowych Fizycznych Prądów (ang.: Currents' Physical Components (CPC) – based Power Theory). Głównym elementem przedstawionej metody jest równanie kompensatora reaktancyjnego. Artykuł przedstawia także zarys teorii mocy liniowych czasowo niezmienniczych odbiorników trójfazowych zasilanych niesymetrycznym napięciem sinusoidalnym. (**Kompensacja reaktancyjna w trój-przewodowych układach trójfazowych z niesymetrycznym napięciem zasilania**)*

Keywords: Power theory, unbalanced loads, Currents' Physical Components, CPC, power factor
Słowa kluczowe: Teoria mocy, odbiorniki niezrównoważone, Składowe Fizyczne Prądu, współczynnik mocy

Introduction

The reactive power and three-phase loads imbalance cause degradation of the power factor and an increase in the cost of the electric energy delivery. Therefore, as early as in 1917 there was the first reported attempt [1] of the load balancing and compensation of the reactive power. The first reactive compensator developed by Steinmetz in [1] is known [6, 7] as the Steinmetz's compensator.

The development of power electronics has provided new tools for reactive power compensation along with load balancing and reduction of harmonic distortion, in the form of switching compensator, commonly known under a bit misleading name "a harmonic power filter". Although the power rating of switching compensator has increased substantially with the increase of the switching power of semiconductor switches, only the reactive compensator can meet the power expectations at certain high power applications. These are, for example, high power AC arc furnaces or AC to DC conversion facilities for DC voltage transmission. Moreover, independently of practical needs, the power theory should be able to provide fundamentals not only for switching compensator design and control, but also provide fundamentals for reactive compensators design.

The development of methods of reactive compensators design for three-phase systems focused a considerable attention [2-7], including compensators for systems with nonsinusoidal voltages and currents [3]. The method of a reactive balancing compensator design for systems with nonsinusoidal voltages and currents was eventually solved in [9], on the condition that the supply voltage is symmetrical, however. The power theory at that time was not developed sufficiently to describe power properties of sys-terms with asymmetrical supply voltage, even if the voltage was sinusoidal. This main obstacle was overcome in [10]. A draft of the CPC-based power theory of LTI loads supplied over a three-wire line with asymmetrical sinusoidal voltage is presented in the following section. Details of that theory can be found in [10].

CPC of LTI loads at asymmetrical supply voltage

Three-phase three-wire systems considered in this paper have a structure shown in Fig. 1 and the reactive compensator is to be installed at the primary side of the transformer, where voltages $u_R(t)$, $u_S(t)$, $u_T(t)$ and currents $i_R(t)$, $i_S(t)$ and $i_T(t)$ can be observed and measured. The voltages are referenced to an artificial zero, so that these voltages cannot contain symmetrical component of the zero sequence.

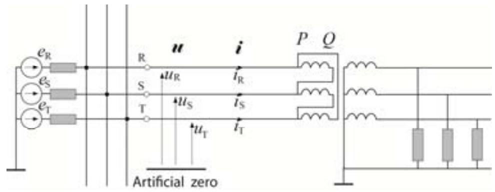

Fig. 1. Static load supplied from a three-wire line

These voltages and currents can be arranged into three-phase voltage and current vectors $\boldsymbol{u}(t)$ and $\boldsymbol{i}(t)$,

$$\boldsymbol{u} \overset{\text{df}}{=} [u_R,\ u_S,\ u_T]^T, \qquad \boldsymbol{i} \overset{\text{df}}{=} [i_R,\ i_S,\ i_T]^T .$$

The apparent power in balanced three-phase systems with sinusoidal voltages and current is equal to the magnitude of the complex apparent power $\boldsymbol{S}$, which is commonly defined as

$$\boldsymbol{S} = S e^{j\varphi} = P + jQ. \tag{1}$$

When the load is unbalanced and/or voltages are asymmetrical then the apparent power S is no longer the magnitude of the complex apparent power $\boldsymbol{S}$. Unfortunately, similarity of symbols for both powers may cause confusion and may even lead to errors. Since it is a very common custom of denoting the apparent power by S, a clearly different symbol is used in this paper for the power calculated as

$$\boldsymbol{U}^T \boldsymbol{I}^* = P + jQ \overset{\text{df}}{=} \boldsymbol{C} = C e^{j\varphi} \tag{2}$$

where,

$$\boldsymbol{U} \overset{\text{df}}{=} [\boldsymbol{U}_R\ \ \boldsymbol{U}_S\ \ \boldsymbol{U}_T]^T, \qquad \boldsymbol{I} \overset{\text{df}}{=} [\boldsymbol{I}_R\ \ \boldsymbol{I}_S\ \ \boldsymbol{I}_T]^T .$$

are three-phase vectors of complex rms (crms) values of line-to-artificial zero voltages and line current. Also the adjective “apparent” will not be used. The power denoted as $\boldsymbol{C}$ will be referred to as the ***complex power***.

With respect to active and reactive powers P and Q at the supply voltage $\boldsymbol{u}$, the unbalanced load shown in Fig. 1 is equivalent to a balanced load shown in Fig. 2 on the condition that its phase admittance is equal to

(3) $$\boldsymbol{Y}_{\rm b} \overset{\rm df}{=} G_{\rm b} + jB_{\rm b} \overset{\rm df}{=} \frac{P - jQ}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{C}^*}{||\boldsymbol{u}||^2}\ .$$

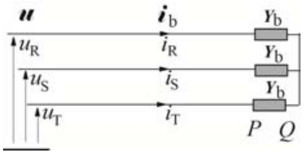

Fig.2. Balanced load equivalent to original load with respect to the active and reactive powers P and Q.

Indeed, the complex power of such a balanced load is

(4) $$\boldsymbol{C}_{\rm b} = \boldsymbol{U}^{\rm T}\boldsymbol{I}_{\rm b}^* = \boldsymbol{U}^{\rm T}(\boldsymbol{Y}_{\rm b}\boldsymbol{U})^* = \boldsymbol{Y}_{\rm b}^*||\boldsymbol{u}||^2 = P + jQ = \boldsymbol{C}.$$

Since $\boldsymbol{Y}_{\rm b}$ is the admittance of a balanced load which is equivalent to the original load with respect to the active and reactive powers, it will be referred to as the ***equivalent balanced admittance***. The equivalent balanced load draws the current

(5) $$\boldsymbol{i}_{\rm b} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} = \sqrt{2}\,{\rm Re}\{\boldsymbol{I}_{\rm b}\,{\rm e}^{j\omega t}\} = \sqrt{2}\,{\rm Re}\{\boldsymbol{Y}_{\rm b}\boldsymbol{U}\,{\rm e}^{j\omega t}\}\ .$$

It is composed of the active current

(6) $$\boldsymbol{i}_{\rm a} \overset{\rm df}{=} G_{\rm b}\,\boldsymbol{u} = \sqrt{2}\,{\rm Re}\{G_{\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}^{\rm n})e^{j\omega t}\}$$

where $\boldsymbol{U}^{\rm p}$ and $\boldsymbol{U}^{\rm n}$ denotes the crms value of the load voltage symmetrical components of the positive and the negative sequence. Symbol $\mathbf{1}^{\rm p}$ denotes a symmetrical unit vector of the positive sequence, while $\mathbf{1}^{\rm n}$ denotes a symmetrical unit vector of the negative sequence, defined as

(7) $$\mathbf{1}^{\rm p} \overset{\rm df}{=} \begin{bmatrix} 1 \\ \alpha^* \\ \alpha \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{-j2\pi/3} \\ 1e^{j2\pi/3} \end{bmatrix}, \quad \mathbf{1}^{\rm n} \overset{\rm df}{=} \begin{bmatrix} 1 \\ \alpha \\ \alpha^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1e^{j2\pi/3} \\ 1e^{-j2\pi/3} \end{bmatrix}$$

and shown in Fig. 3.

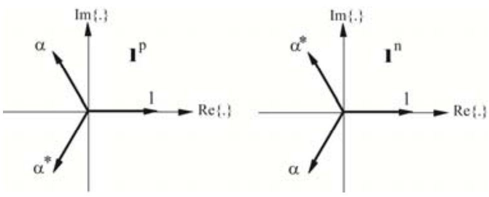

Fig. 3. Three-phase symmetrical unit vectors $\mathbf{1}^{\rm p}$ and $\mathbf{1}^{\rm n}$.

The current of the equivalent balanced load, defined by (5) contains also the reactive current

(8) $$\boldsymbol{i}_{\rm r} \overset{\rm df}{=} B_{\rm b}\,\boldsymbol{u}\,(t+T/4) = \sqrt{2}\,{\rm Re}\{jB_{\rm b}\,(\mathbf{1}^{\rm p}\boldsymbol{U}^{\rm p} + \mathbf{1}^{\rm n}\boldsymbol{U}^{\rm n})e^{j\omega t}\}\ .$$

The remaining current of the load, after the current of the balanced load is subtracted, is caused by the load imbalance. It is equal to

(9) $$\boldsymbol{i} - \boldsymbol{i}_{\rm b} = \sqrt{2}\,{\rm Re}\{(\boldsymbol{I} - \boldsymbol{I}_{\rm b}){\rm e}^{j\omega t}\} = \boldsymbol{i}_{\rm u} = \sqrt{2}\,{\rm Re}\{\boldsymbol{I}_{\rm u}\,{\rm e}^{j\omega t}\}.$$

Consequently, the load current is decomposed into the active, reactive and unbalanced currents such that

(10) $$\boldsymbol{i} = \boldsymbol{i}_{\rm a} + \boldsymbol{i}_{\rm r} + \boldsymbol{i}_{\rm u}.$$

As it was proven in [10] the active, reactive and unbalanced currents are mutually orthogonal and consequently, their three-phase rms values satisfy the relationship

(11) $$||\boldsymbol{i}\,||^2 = ||\boldsymbol{i}_{\rm a}||^2 + ||\boldsymbol{i}_{\rm r}||^2 + ||\boldsymbol{i}_{\rm u}||^2$$

with,

(12) $$||\boldsymbol{i}_{\rm a}|| = G_{\rm b}\,||\boldsymbol{u}||, \qquad ||\boldsymbol{i}_{\rm r}|| = |B_{\rm b}|\,||\boldsymbol{u}||$$

Owing to the mutual orthogonality of the current components, the three-phase rms value of the unbalanced current can be expressed as

(13) $$||\boldsymbol{i}_{\rm u}|| = \sqrt{||\boldsymbol{i}\,||^2 - ||\boldsymbol{i}_{\rm a}||^2 - ||\boldsymbol{i}_{\rm r}||^2}$$

To make the decomposition (10) useful for reactive compensator design, we have to know how the reactive and unbalanced currents $\boldsymbol{i}_{\rm r}(t)$ and $\boldsymbol{i}_{\rm u}(t)$ depend on the circuit parameters.

At symmetrical voltage the active and reactive currents are expressed in terms of equivalent admittance of the load, $\boldsymbol{Y}_{\rm e}$, defined in [9] as

(14) $$\boldsymbol{Y}_{\rm e} \overset{\rm df}{=} G_{\rm e} + jB_{\rm e} = \boldsymbol{Y}_{\rm ST} + \boldsymbol{Y}_{\rm TR} + \boldsymbol{Y}_{\rm RS}$$

while at asymmetrical voltage these two currents are expressed in terms of the equivalent balanced admittance $\boldsymbol{Y}_{\rm b}$,

(15) $$\boldsymbol{Y}_{\rm b} = G_{\rm b} + jB_{\rm b} = \frac{\boldsymbol{C}^*}{||\boldsymbol{u}||^2} = \frac{\boldsymbol{Y}_{\rm RS}\,U_{\rm RS}^2 + \boldsymbol{Y}_{\rm ST}\,U_{\rm ST}^2 + \boldsymbol{Y}_{\rm TR}\,U_{\rm TR}^2}{||\boldsymbol{u}||^2}$$

When the supply voltage is asymmetrical and this asymmetry is specified by a ***complex coefficient of the supply voltage asymmetry***, defined as

(16) $$\frac{\boldsymbol{U}^{\rm n}}{\boldsymbol{U}^{\rm p}} = \frac{U^{\rm n}\,e^{j\varphi}}{U^{\rm p}\,e^{j\phi}} = \frac{U^{\rm n}}{U^{\rm p}}e^{j(\varphi-\phi)} \overset{\rm df}{=} \boldsymbol{a} = a\,e^{j\psi}$$

then, as shown in [10], the equivalent and balanced admittances differ mutually by admittance $\boldsymbol{Y}_{\rm d}$, namely

(17) $$\boldsymbol{Y}_{\rm d} \overset{\rm df}{=} \boldsymbol{Y}_{\rm e} - \boldsymbol{Y}_{\rm b}$$

and this difference is equal to

(18) $$\boldsymbol{Y}_{\rm d} = \frac{2a}{1+a^2}[\boldsymbol{Y}_{\rm ST}\cos\psi + \boldsymbol{Y}_{\rm TR}\cos(\psi - \frac{2\pi}{3}) + \boldsymbol{Y}_{\rm RS}\cos(\psi + \frac{2\pi}{3})]\ .$$

Admittance $\boldsymbol{Y}_{\rm d}$ depends not only on the load line-to-line admittances, but also on the supply voltage asymmetry. When the load is balanced, i.e.,

$$\boldsymbol{Y}_{\rm RS} = \boldsymbol{Y}_{\rm TR} = \boldsymbol{Y}_{\rm RS} = \boldsymbol{Y}_{\rm e}\,/\,3, \text{then}\boldsymbol{Y}_{\rm d} = 0$$

independently of the supply voltage asymmetry. When the supply voltage is symmetrical and consequently, asymmetry coefficient $\boldsymbol{a}$ = 0, then $\boldsymbol{Y}_{\rm d}$ = 0, independently of the load imbalance. Therefore, admittance $\boldsymbol{Y}_{\rm d}$ is referred to as an ***asymmetry dependent unbalanced admittance*** in this paper. Admittance $\boldsymbol{Y}_{\rm d}$ can have a non-zero value only if, simultaneously, the load is unbalanced and the supply voltage is asymmetrical.

The vector of crms values of unbalanced currents $\boldsymbol{I}_u$ in the load supply lines can be decomposed, as shown in [10], as follows

$$\boldsymbol{I}_u = \boldsymbol{Y}_d \boldsymbol{U} + \mathbf{1}^n \boldsymbol{A}^p \boldsymbol{U}^p + \mathbf{1}^p \boldsymbol{A}^n \boldsymbol{U}^n \tag{19}$$

where,

$$\boldsymbol{A}^p \stackrel{df}{=} -(\boldsymbol{Y}_{ST} + \alpha \boldsymbol{Y}_{TR} + \alpha^* \boldsymbol{Y}_{RS}) \tag{20}$$

$$\boldsymbol{A}^n \stackrel{df}{=} -(\boldsymbol{Y}_{ST} + \alpha^* \boldsymbol{Y}_{TR} + \alpha \boldsymbol{Y}_{RS}) \tag{21}$$

are unbalanced admittances for the positive and the negative sequence voltages respectively.

Thus, the vectors of the active, reactive and unbalanced currents, $\boldsymbol{i}_a$, $\boldsymbol{i}_r$ and $\boldsymbol{i}_u$ can be specified in terms of four admittances, $\boldsymbol{Y}_b$, $\boldsymbol{Y}_d$, $\boldsymbol{A}^p$ and $\boldsymbol{A}^n$, which can be explicitly expressed in terms of line-to-line admittances $\boldsymbol{Y}_{RS}$, $\boldsymbol{Y}_{ST}$, and $\boldsymbol{Y}_{TR}$, line-to-line supply voltage rms values U_{RS}, U_{ST}, and U_{TR} and the coefficient of the voltage asymmetry, $\boldsymbol{a}$.

Reactive compensator design

Power systems composed of aggregates of single-phase loads, as shown in Fig. 1, can often be balanced, by reconfiguration of the supply of the aggregates of single-phase loads, while sometimes; compensators are needed for that. The load balancing is associated in such a case with compensation of the reactive power.

Compensators can be built as PWM inverter-based switching compensators (known as “active power filters”) or as reactive compensators. Fundamentals of switching compensator control using the Instantaneous Reactive Power (IRP) p-q Theory were developed mainly by Akagi and Nabae in [11] and in numerous other papers. Such compensation based on IRP p-q Theory can fail, however, when the supply voltage is asymmetrical, as it was shown in [12].

Similarly as in LTI systems with symmetrical and sinusoidal supply voltage, the reactive and unbalanced currents, $\boldsymbol{i}_r$ and $\boldsymbol{i}_u$, cause decline of the power factor λ at the supply terminals. These currents can be reduced by a shunt reactive compensator. It can have Δ configuration as shown in Fig. 4. Let us assume that it is built of lossless reactive elements of susceptances T_{RS}, T_{ST} and T_{TR}. The compensated load is specified in terms of four admittances $\boldsymbol{Y}_b$, $\boldsymbol{Y}_d$, $\boldsymbol{A}^p$ and $\boldsymbol{A}^n$. These admittances of the compensator have additional index C.

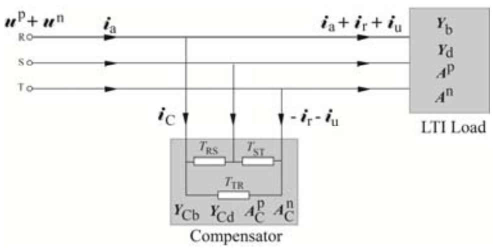

Fig. 4. Three-phase LTI load with reactive compensator

The vector of crms values of the reactive current $\boldsymbol{i}_{Cr}$ of such a compensator is

$$\boldsymbol{I}_{Cr} = jB_{Cb}\boldsymbol{U}$$

where its balanced susceptance is

$$B_{Cb} = \frac{T_{RS} U_{RS}^2 + T_{ST} U_{ST}^2 + T_{TR} U_{TR}^2}{||\boldsymbol{u}||^2} \tag{22}$$

Unbalanced admittances of such a compensator are

$$\boldsymbol{A}_C^p = -j(T_{ST} + \alpha T_{TR} + \alpha^* T_{RS}) \tag{23}$$

$$\boldsymbol{A}_C^n = -j(T_{ST} + \alpha^* T_{TR} + \alpha T_{RS}) \tag{24}$$

while the asymmetry dependent unbalanced admittance is

$$\boldsymbol{Y}_{Cd} = j\frac{2a}{1+a^2}[T_{ST}\cos\psi + T_{TR}\cos(\psi - \frac{2\pi}{3}) + T_{RS}\cos(\psi + \frac{2\pi}{3})]\ . \tag{25}$$

The compensator reduces the reactive current to zero at the condition that

$$B_{Cb} + B_b = 0 \tag{26}$$

and it reduces the unbalanced current to zero on the condition that the sum of the unbalanced current of the load, specified by (19), and the compensator are equal to zero. i.e.,

$$(\boldsymbol{Y}_{Cd} + \boldsymbol{Y}_d)\boldsymbol{U} + \mathbf{1}^n(\boldsymbol{A}_C^p + \boldsymbol{A}^p)\boldsymbol{U}^p + \mathbf{1}^p(\boldsymbol{A}_C^n + \boldsymbol{A}^n)\boldsymbol{U}^n = 0\ . \tag{27}$$

Coefficients at the voltage in this equation are identical for each line. Therefore, the unbalanced current is reduced to zero on the condition that for a single line, in particular, for line R

$$(\boldsymbol{Y}_{Cd} + \boldsymbol{Y}_d)\boldsymbol{U}_R + (\boldsymbol{A}_C^p + \boldsymbol{A}^p)\boldsymbol{U}^p + (\boldsymbol{A}_C^n + \boldsymbol{A}^n)\boldsymbol{U}^n = 0\ . \tag{28}$$

Since the condition (28) has to be satisfied both for the real and for the imaginary part of the equation, it provides two equations, namely,

$$\mathrm{Re}\{(\boldsymbol{Y}_{Cd} + \boldsymbol{Y}_d)\boldsymbol{U}_R + (\boldsymbol{A}_C^p + \boldsymbol{A}^p)\boldsymbol{U}^p + (\boldsymbol{A}_C^n + \boldsymbol{A}^n)\boldsymbol{U}^n\} = 0 \tag{29}$$

$$\mathrm{Im}\{(\boldsymbol{Y}_{Cd} + \boldsymbol{Y}_d)\boldsymbol{U}_R + (\boldsymbol{A}_C^p + \boldsymbol{A}^p)\boldsymbol{U}^p + (\boldsymbol{A}_C^n + \boldsymbol{A}^n)\boldsymbol{U}^n\} = 0 \tag{30}$$

Therefore, along with equation (26), there are three independent linear equations for calculating three susceptances T_{RS}, T_{ST} and T_{TR} of the compensator. It means that this problem has to have a solution. Thus, the supply current of any unbalanced LTI load supplied with a sinusoidal asymmetrical voltage can be reduced to its active component, and the power factor can be improved to unity.

Equations (26) (29) and (30), can be rearranged (see Appendix) into the following ***compensator equation***

$$\begin{bmatrix} U_{RS}^2 & U_{ST}^2 & U_{TR}^2 \\ \mathrm{Re}\boldsymbol{F}_1 & \mathrm{Re}\boldsymbol{F}_2 & \mathrm{Re}\boldsymbol{F}_3 \\ \mathrm{Im}\boldsymbol{F}_1 & \mathrm{Im}\boldsymbol{F}_2 & \mathrm{Im}\boldsymbol{F}_3 \end{bmatrix} \begin{bmatrix} T_{RS} \\ T_{ST} \\ T_{TR} \end{bmatrix} = \begin{bmatrix} -B_b ||\boldsymbol{u}||^2 \\ -\mathrm{Re}\boldsymbol{F}_4 \\ -\mathrm{Im}\boldsymbol{F}_4 \end{bmatrix} \tag{31}$$

with complex coefficients $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, $\boldsymbol{F}_3$ and $\boldsymbol{F}_4$ specified in the Appendix.

Numerical illustration. Let us calculate parameters of the compensator for the unbalanced load shown in Fig. 5, assuming that it is supplied with strongly asymmetrical voltage.

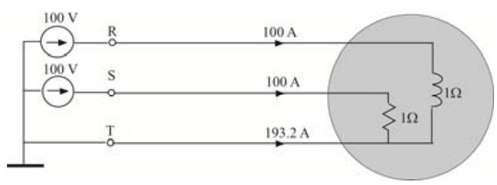

Fig. 5. Example of a circuit with very high load imbalance and very high supply voltage asymmetry.

The complex rms values of the positive and negative sequence symmetrical components in the system analyzed are equal to

$$\begin{bmatrix} \boldsymbol{U}^{\mathrm{p}} \\ \boldsymbol{U}^{\mathrm{n}} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1, & \alpha, & \alpha^* \\ 1, & \alpha^*, & \alpha \end{bmatrix}\begin{bmatrix} 100 \\ 100e^{-j120^0} \\ 0 \end{bmatrix} = \begin{bmatrix} 66.66 \\ 33.33e^{j60^0} \end{bmatrix}\mathrm{V}.$$

The three-phase rms values of the supply voltage symmetrical components are

$$||\boldsymbol{u}^{\mathrm{p}}|| = \sqrt{3}\,U^{\mathrm{p}} = \sqrt{3}\times 66.66 = 115.47\ \mathrm{V}$$
$$||\boldsymbol{u}^{\mathrm{n}}|| = \sqrt{3}\,U^{\mathrm{n}} = \sqrt{3}\times 33.33 = 57.73\ \mathrm{V}$$

and consequently, three-phase rms value of the supply voltage is

$$||\boldsymbol{u}|| = \sqrt{||\boldsymbol{u}^{\mathrm{p}}||^2 + ||\boldsymbol{u}^{\mathrm{n}}||^2} = \sqrt{115.47^2 + 57.73^2} = 129.1\ \mathrm{V}.$$

Since the active and reactive powers are equal to, respectively, P = 10kW and Q = 10kvar, the equivalent balanced admittance $\boldsymbol{Y}_{\mathrm{b}}$ of the load in the circuit shown in Fig. 5 is equal to

$$\boldsymbol{Y}_{\mathrm{b}} = G_{\mathrm{b}} + jB_{\mathrm{b}} = \frac{P - jQ}{||\boldsymbol{u}||^2} = 0.600 - j\,0.600\ \mathrm{S}$$

The complex coefficient of the supply voltage asymmetry is equal to

$$\boldsymbol{a} = a e^{j\psi} = \frac{\boldsymbol{U}^{\mathrm{n}}}{\boldsymbol{U}^{\mathrm{p}}} = \frac{33.33e^{j60^0}}{66.66} = 0.50e^{j60^0}.$$

The crms values of the supply voltage measured with respect to the artificial zero are

$$\boldsymbol{U}_{\mathrm{R}} = \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}} = 88.18e^{j19.1^0}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{S}} = \alpha^*\boldsymbol{U}^{\mathrm{p}} + \alpha\boldsymbol{U}^{\mathrm{n}} = 88.18e^{-j139.1^0}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{T}} = \alpha\boldsymbol{U}^{\mathrm{p}} + \alpha^*\boldsymbol{U}^{\mathrm{n}} = 33.33e^{j120.0^0}\ \mathrm{V}$$

while the crms values of line-to-line voltages are equal to

$$\boldsymbol{U}_{\mathrm{RS}} = \boldsymbol{U}_{\mathrm{R}} - \boldsymbol{U}_{\mathrm{S}} = 173.21\ e^{j30^\circ}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{ST}} = \boldsymbol{U}_{\mathrm{S}} - \boldsymbol{U}_{\mathrm{T}} = 100\ e^{-j90^\circ}\ \mathrm{V}$$
$$\boldsymbol{U}_{\mathrm{TR}} = \boldsymbol{U}_{\mathrm{T}} - \boldsymbol{U}_{\mathrm{R}} = 100\ e^{j180^\circ}\ \mathrm{V}.$$

The load unbalanced admittances are equal to

$$\boldsymbol{A}^{\mathrm{p}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha\boldsymbol{Y}_{\mathrm{TR}} + \alpha^*\boldsymbol{Y}_{\mathrm{RS}}) = -[1 + \alpha(-j1)] = 1.932e^{-j165^0}\ \mathrm{S}$$
$$\boldsymbol{A}^{\mathrm{n}} = -(\boldsymbol{Y}_{\mathrm{ST}} + \alpha^*\boldsymbol{Y}_{\mathrm{TR}} + \alpha\boldsymbol{Y}_{\mathrm{RS}}) = -[1 + \alpha^*(-j1)] = 0.518e^{-j105^0}\mathrm{S}.$$
$$\boldsymbol{Y}_{\mathrm{d}} = \frac{2a}{1 + a^2}[\boldsymbol{Y}_{\mathrm{ST}}\cos\psi + \boldsymbol{Y}_{\mathrm{TR}}\cos(\psi - \frac{2\pi}{3}) + \boldsymbol{Y}_{\mathrm{RS}}\cos(\psi + \frac{2\pi}{3})]$$
$$= \frac{2\times 0.5}{1 + 0.5^2}[\cos(60^0) - j\cos(60^0 - 120^0)] = 0.566e^{-j45^0}\ \mathrm{S}.$$

With these values of the load admittances, the coefficients:

$$\boldsymbol{c}_1 = j\frac{2a\cos\psi}{1 + a^2} = j\frac{2\times 0.6\cos 60^0}{1 + 0.5^2} = j\,0.4$$
$$\boldsymbol{c}_2 = j\frac{2a\cos(\psi - 120^\circ)}{1 + a^2} = j\,0.4$$
$$\boldsymbol{c}_3 = j\frac{2a\cos(\psi - 240^\circ)}{1 + a^2} = -j\,0.8$$

Hence

$$\boldsymbol{F}_1 = \boldsymbol{c}_3(1 + ae^{j\psi}) - j(\alpha^* + \alpha a e^{j\psi}) = 0.52$$
$$\boldsymbol{F}_2 = \boldsymbol{c}_1(1 + ae^{j\psi}) - j(1 + ae^{j\psi}) = 0.26 - j\,0.75$$
$$\boldsymbol{F}_3 = \boldsymbol{c}_2(1 + ae^{j\psi}) - j(\alpha + \alpha^* ae^{j\psi}) = 0.26 + j\,0.75$$
$$\boldsymbol{F}_4 = (1 + ae^{j\psi})\boldsymbol{Y}_{\mathrm{d}} + \boldsymbol{A}^{\mathrm{p}} + (1 + ae^{j\psi})\boldsymbol{A}^{\mathrm{n}} = -1.01 - j1.01.$$

With such coefficients the compensator equation has the form

$$\begin{bmatrix} 3\times 10^4 & 10^4 & 10^4 \\ -0.52 & 0.26 & 0.26 \\ 0 & -0.75 & 0.75 \end{bmatrix}\begin{bmatrix} T_{\mathrm{RS}} \\ T_{\mathrm{ST}} \\ T_{\mathrm{TR}} \end{bmatrix} = \begin{bmatrix} 10^4 \\ 1.01 \\ 1.01 \end{bmatrix}.$$

With respect to the compensator branch susceptances this equation has the solution :

$$T_{\mathrm{RS}} = -0.58\ \mathrm{S},\ \ T_{\mathrm{ST}} = 0.69\ \mathrm{S},\ \ T_{\mathrm{TR}} = 2.04\ \mathrm{S}$$

Assuming that the supply voltage frequency is f = 50 Hz, thus ω = 314 rad/s, then the compensator parameters are

$$L_{\mathrm{RS}} = -\frac{1}{\omega T_{\mathrm{RS}}} = 5.49\,\mathrm{mH},$$
$$C_{\mathrm{ST}} = \frac{T_{\mathrm{ST}}}{\omega} = 2.20\ \mathrm{mF},\qquad C_{\mathrm{TR}} = \frac{T_{\mathrm{TR}}}{\omega} = 6.50\,\mathrm{mF}$$

The circuit with the balancing compensator and compensation results is shown in Fig. 6.

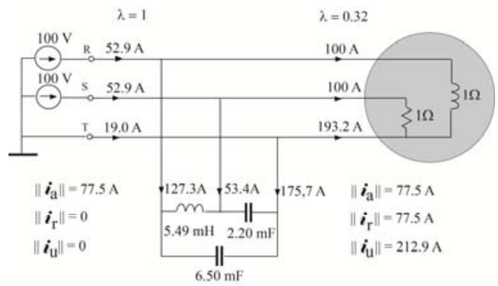

Fig. 6. Unbalanced load with reactive compensator and compensation results.

The reactive compensator with such parameters compensates entirely the reactive and unbalanced currents, reducing the three-phase rms value of the supply current from $||\boldsymbol{i}||$ = 239.4 A to $||\boldsymbol{i}||$ = 77.5 A, which increases the power factor from λ=0.32 to λ=1. The load with the compensator is equivalent to a balanced resistive load of conductance G_{b}. Observe that the supply current remains asymmetrical, however, because of the supply voltage asymmetry.

Conclusion

The paper demonstrates that LTI loads supplied with asymmetrical, but sinusoidal voltages can be compensated by reactive compensators to unity power factor. The presented method of total compensation of the reactive and unbalanced currents requires that equivalent balanced and unbalanced admittances of the load are calculated. This can be done by measurement of complex rms values of voltages and currents at the load terminals.

Appendix

Compensator equation

Since the crms value of the line voltage at terminal R is equal to

$$\boldsymbol{U}_{\mathrm{R}} = \boldsymbol{U}^{\mathrm{p}} + \boldsymbol{U}^{\mathrm{n}}$$

equation (28) can be rearranged to the form

$$(\boldsymbol{Y}_{\text{Cd}}+\boldsymbol{Y}_{\text{d}})(\boldsymbol{U}^{\text{p}}+\boldsymbol{U}^{\text{n}})+(\boldsymbol{A}_{\text{C}}^{\text{p}}+\boldsymbol{A}^{\text{p}})\boldsymbol{U}^{\text{p}}+(\boldsymbol{A}_{\text{C}}^{\text{n}}+\boldsymbol{A}^{\text{n}})\boldsymbol{U}^{\text{n}}=0\,.$$

Taking into account the definition of the complex coefficient of the voltage asymmetry (16), this equation can be rearanged as follows

(A1) $$(\boldsymbol{A}_{\text{C}}^{\text{p}}+\boldsymbol{A}^{\text{p}})+(\boldsymbol{A}_{\text{C}}^{\text{n}}+\boldsymbol{A}^{\text{n}})\,a e^{j\psi}+(\boldsymbol{Y}_{\text{Cd}}+\boldsymbol{Y}_{\text{d}})(1+a e^{j\psi})=0.$$

In this equation

(A2) $$\boldsymbol{A}_{\text{C}}^{\text{p}}+\boldsymbol{A}^{\text{p}}=-j\,(T_{\text{ST}}+\alpha\, T_{\text{TR}}+\alpha^{*}\,T_{\text{RS}})+\boldsymbol{A}^{\text{p}}$$

(A3) $$\boldsymbol{A}_{\text{C}}^{\text{n}}+\boldsymbol{A}^{\text{n}}=-j(T_{\text{ST}}+\alpha^{*}T_{\text{TR}}+\alpha T_{\text{RS}})+\boldsymbol{A}^{\text{n}}.$$

The asymmetry dependent unbalanced admittance, specified for the load by formula (25), for the compensator can be written in the form

(A4) $$\boldsymbol{Y}_{\text{Cd}}=(\boldsymbol{c}_{1}T_{\text{ST}}+\boldsymbol{c}_{2}T_{\text{TR}}+\boldsymbol{c}_{3}T_{\text{RS}})$$

where,

(A5) $$\boldsymbol{c}_{1}\stackrel{\text{df}}{=}j\,\frac{2a\cos\psi}{1+a^{2}}$$

$$\boldsymbol{c}_{2}\stackrel{\text{df}}{=}j\,\frac{2a\cos(\psi-120°)}{1+a^{2}}$$

$$\boldsymbol{c}_{3}\stackrel{\text{df}}{=}j\,\frac{2a\cos(\psi-240°)}{1+a^{2}}.$$

With formulae (A2) – (A4), equation (A1) can be rearranged to the form

(A6) $$\boldsymbol{F}_{1}T_{\text{RS}}+\boldsymbol{F}_{2}T_{\text{ST}}+\boldsymbol{F}_{3}T_{\text{TR}}+\boldsymbol{F}_{4}=0$$

where

(A7) $$\boldsymbol{F}_{1}=\boldsymbol{c}_{3}(1+a e^{j\psi})-j(\alpha^{*}+\alpha\, a e^{j\psi})$$

(A8) $$\boldsymbol{F}_{2}=\boldsymbol{c}_{1}(1+a e^{j\psi})-j(1+a e^{j\psi})$$

(A9) $$\boldsymbol{F}_{3}=\boldsymbol{c}_{2}(1+a e^{j\psi})-j(\alpha+\alpha^{*}a e^{j\psi})$$

(A10) $$\boldsymbol{F}_{4}=(1+a e^{j\psi})\boldsymbol{Y}_{\text{d}}+\boldsymbol{A}^{\text{p}}+(1+a e^{j\psi})\boldsymbol{A}^{\text{n}}$$

Equation (A6) has to be satisfied both for the real and for the imaginary parts, so that

(A11) $$T_{\text{RS}}\,\text{Re}\boldsymbol{F}_{1}+T_{\text{ST}}\,\text{Re}\boldsymbol{F}_{2}+T_{\text{TR}}\,\text{Re}\boldsymbol{F}_{3}+\text{Re}\boldsymbol{F}_{4}=0$$

$$T_{\text{RS}}\,\text{Im}\boldsymbol{F}_{1}+T_{\text{ST}}\,\text{Im}\boldsymbol{F}_{2}+T_{\text{TR}}\,\text{Im}\boldsymbol{F}_{3}+\text{Im}\boldsymbol{F}_{4}=0.$$

When condition (26) is combined with condition (22) the third equation is obtained

(A12) $$T_{\text{RS}}\,U_{\text{RS}}^{2}+T_{\text{ST}}\,U_{\text{ST}}^{2}+T_{\text{TR}}\,U_{\text{TR}}^{2}=-B_{\text{b}}\,||\boldsymbol{u}||^{2}$$

Equations (A11) and (A12) can be expressed in a following matrix form, referred to as a ***compensator equation***

(A13) $$\begin{bmatrix} U_{\text{RS}}^{2} & U_{\text{ST}}^{2} & U_{\text{TR}}^{2} \\ \text{Re}\boldsymbol{F}_{1} & \text{Re}\boldsymbol{F}_{2} & \text{Re}\boldsymbol{F}_{3} \\ \text{Im}\boldsymbol{F}_{1} & \text{Im}\boldsymbol{F}_{2} & \text{Im}\boldsymbol{F}_{3}\end{bmatrix}\begin{bmatrix}T_{\text{RS}}\\ T_{\text{ST}}\\ T_{\text{TR}}\end{bmatrix}=\begin{bmatrix}-B_{\text{b}}\,||\boldsymbol{u}||^{2}\\ -\text{Re}\boldsymbol{F}_{4}\\ -\text{Im}\boldsymbol{F}_{4}\end{bmatrix}$$

with unknown vector of compensator susceptances, T_{RS}, T_{ST} and T_{TR}.

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