Ariel DZWONKOWSKI, Anna GOLIJANEK-JĘDRZEJCZYK

Gdansk University of Technology, Department of Metrology and Information Systems

doi:10.15199/48.2015.10.33

Estimation of the uncertainty of the AC/AC transducer output voltage

***Abstract**. The paper presents a methodology for estimating uncertainty of voltage transducers processing function, on the example of CYVS411D07 voltage transducers. The analysis was conducted using the method based on the GUM Guide propagation of uncertainty law and the determination of the coverage factor based on the effective number of degrees of freedom (t-Student distribution). The paper also presents example evaluation results of measured voltage uncertainty for a few selected voltage and frequency values.*

***Streszczenie.** W referacie zaprezentowano metodologię szacowania niepewności funkcji przetwarzania przetwornika napięciowego, na przykładzie przetworników napięciowych CYVS411D07. Analizę przeprowadzono przy wykorzystaniu metody opartej na Przewodniku GUM z zastosowaniem prawa propagacji niepewności oraz wyznaczeniem współczynnika rozszerzenia na podstawie efektywnej liczby stopni swobody (rozkładu t-Studenta). W artykule przedstawiono także przykładowe wyniki oceny niepewności pomiaru napięcia, dla kilku wybranych wartości napięcia oraz częstotliwości. (**Szacowanie niepewności napięcia wyjściowego przetwornika AC/AC)**.*

Keywords: AC/AC transducer, uncertainty estimation, analytical method
Słowa kluczowe: przetwornik AC/AC, oszacowanie niepewności, metoda analityczna

Introduction

According to science of measurement methodology, it is required to present the measurement result together with the estimated value of the expanded uncertainty [1, 2, 3, 4, 5]. Therefore, by making measurements of various physical quantities in all fields of science and technology [4, 5, 6], it is necessary to estimate the uncertainty of measurement results.

Due to the fact that voltage converters are used in almost every field of science, it is necessary to develop a methodology for estimating uncertainty of these transducers.

The article presents the methodology of estimating the uncertainty of CYVS411D07 voltage transducers.

A traditional approach to determining measurement uncertainty - the analytical method [1] has been presented, which is based on convolution operations of the input distributions using a mathematical model for the size of the input. In this case, the designated measure of uncertainty is the expanded uncertainty calculated as a product, designated on the basis of the effective number of degrees of freedom of the coverage factor k_p and the standard uncertainty [7].

The study

The transducer output voltage is dependent on input voltage u and the transducer conversion ratio k_u.

The voltage is determined by the following measurement function, which is the basis for estimation of voltage measurement uncertainty [1]:

$$u = k_u \cdot u_k \tag{1}$$

where: u - transducer output voltage, k_u – transducer conversion ratio, u_k - transducer input voltage.

The study was aimed at estimating the CYVS411D07 voltage transducers measurement uncertainty.

To estimate the uncertainty of the voltage transducer a measurement system consisting of a Fluke 5500A voltage calibrator and a Keithley 2002 multimeter was designed. All tested converters were powered by stabilized power supply type CY-WYS-3 which was dedicated of the manufacturer.

Several type CYVS411D07 voltage transducers were tested. The tests were conducted for 5 frequency values at 8 different voltage values for each of them.

A block diagram of the measurement system for testing the accuracy of the transducers is shown in Fig. 1.

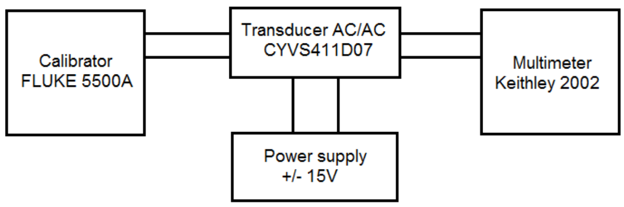

Fig. 1. A block diagram of the measurement system for testing the accuracy of the CYVS411D07 transducers

This paper will present the results for the two transducers coded P1 and P2 respectively.

Experimental results

The paper demonstrated only selected results: the transmitter P1 for the frequency of 50 Hz and transducer P2 for frequency of 50 Hz and 500 Hz, with a calibrator voltage equal to 230 V.

Table 1 shows a sample in a series of $n = 30$ observations obtained in the measurement for transducer P2 for 50 Hz and Table 2 presents the measurement results for transducer P2 and 500 Hz .

Table 1. The measurement results for transducer P2 for 50 Hz

l.p	*u* [V]	l.p	*u* [V]	l.p	*u* [V]	l.p	*u* [V]
1	2.8759	9	2.8761	17	2.8762	25	2.8762
2	2.876	10	2.8762	18	2.8762	26	2.8761
3	2.876	11	2.8761	19	2.8761	27	2.8763
4	2.8761	12	2.8762	20	2.8762	28	2.8767
5	2.8761	13	2.8762	21	2.8761	29	2.8765
6	2.876	14	2.8762	22	2.8763	30	2.8766
7	2.8761	15	2.8762	23	2.8763		
8	2.8761	16	2.8762	24	2.8763		

Table 2. The measurement results for transducer P2 for 500 Hz

l.p	*u* [V]	l.p	*u* [V]	l.p	*u* [V]	l.p	*u* [V]
1	2.8987	9	2.8987	19	2.8988	27	2.8987
2	2.8986	10	2.8986	20	2.8989	28	2.8984
3	2.8986	11	2.8987	21	2.8986	29	2.8986
4	2.8984	12	2.8986	22	2.8985	30	2.8988
5	2.8987	13	2.8987	23	2.8984	25	2.8985
6	2.8986	14	2.8988	24	2.8987	26	2.8987
7	2.8986	15	2.8989	25	2.8985		
8	2.8986	16	2.8987	26	2.8987		

The uncertainty component – estimated by A method

To determine Type A uncertainty, the probability distribution of values of the observations was examined. Most frequently normal distribution is assumed 'a priori' (especially when the number of measurements is greater than 30). However, the literature [7, 8] indicate the inadequacy effect of assumed probability distribution that can significantly degrade the estimated uncertainty of the measurement.

In order to avoid this error, an analysis of the mathematical model of the probability distribution of observations was performed. On this basis, histograms of obtained from measurements samples were created using k = 5 or 6 classes grouping (according to the Sturges formula) in order to verify the adopted hypothesis of the probability distribution.

For transducer P1 (50 Hz), the width of the intervals was 0.18 mV (Fig. 2).

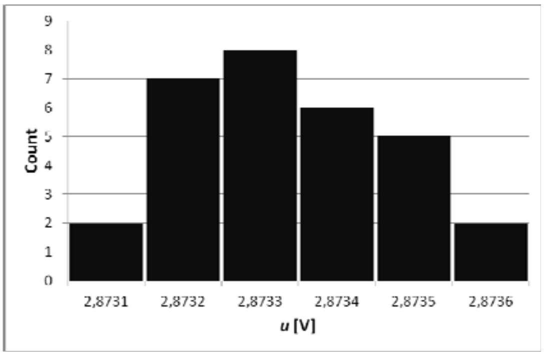

Fig. 2. Histogram for P1

As for results obtained for transducer P2 for a frequency of 50 Hz, the width of the intervals of the histogram was 0.20 mV (Figure 3).

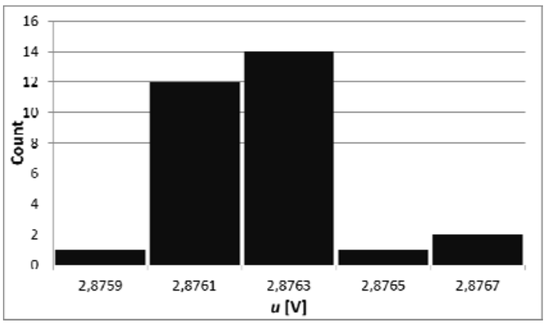

Fig. 3. Histogram for P2, *f*=50 Hz

And for the results obtained for transducer P2 for the frequency of 500 Hz the width of the intervals of the histogram was 0.10 mV (Figure 4).

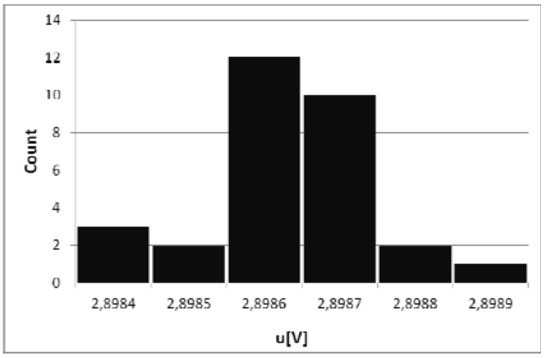

Fig. 4. Histogram for P2, *f*=500 Hz

It was concluded from the shape of the histogram that there is a possibility of acceptance a model for the following distributions: Laplace distribution for transducer P1 and trapezoidal distribution for transducer P2 for both presented frequencies.

Then, using the criterion χ^2 at the significance level α=0.05 (that is, with the inaccuracy of not more than 5%) the hypothesis of enlarged distributions was tested.

Value χ^2 was designated from formula [2]:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \tag{2}$$

where: O_i – number of results received in the experiment and placed in the respective interval, E_i – the expected number of results within a given range, which was calculated based on the assumed distribution.

Table 3 summarizes the results of probability distribution tests for criterion χ^2.

Table 3. Results of the distribution test for criterion χ^2

Transducer	*f* [Hz]	Criterion χ^2
P1	50	for Laplace'a distribution: χ^2=0.861<*k*=6 result positive
P2	50	for trapezoidal distribution: χ^2=1.667<*k*=5 result positive
	500	for trapezoidal distribution: χ^2=2.067<*k*=6 result positive

Based on the results obtained for criterion χ^2 the previously created models of the probability distribution were confirmed. For the applied probability distributions, static parameters of measurement results of each transducer were determined. For transducer P1, for which the results of the observation are described by Laplace distribution, the estimate value of measurand u was defined as median u_{med}, [7]:

$$u = u_{med} \tag{3}$$

The standard deviation s_u of single observation was determined from the dependency:

$$s_u = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^{n} (u_i - \bar{u})^2} \tag{4}$$

whereas, standard uncertainty u_A of Type A of the measurement result (median) was calculated as:

(5) $$u_A(u_{med})=\frac{s_u}{\sqrt{2n}}$$

For transducer P2 observation results were described by trapezoidal distribution for frequencies of 50 Hz and 500 Hz (the ratio base factor is less than 0.35 [8]). The measurand estimator value was determined as [8, 9]:

(6) $$u=\frac{1}{n}\sum_{i=1}^{n}u_i$$

The standard deviation of single observation s_u for transducer P2 was determined from equation (4), whereas, the standard uncertainty u_A of the measurement result was evaluated as [8, 9]:

(7) $$u_A(u)=\frac{s_u}{\sqrt{n}}$$

Finally, the calculated statistical observation parameters for both transducers P1 and P2 (taking into account the sensitivity coefficients) are summarized in Table 4.

Table 4. Uncertainty - Type A

Transducer	*f* [Hz]	*u* [V]	s_u [mV]	u_A [mV]
P1	50	2.8733	0.1377	2.0109
P2	50	2.8762	0.1701	2.4839
	500	2.8986	0.1149	1.6785

The uncertainty component – estimated by B method

To estimate the uncertainty of type B for voltage measurement CYVS411D07 transducers, the formula (1) was used, which defines the function of voltage measurement. The uncertainty voltage measurement $u_B(u)$ was defined by the following relationship [5]:

(8) $$u_B(u)=\sqrt{\left(\frac{\partial u}{\partial k_u}\right)^2 u^2(k_u)+\left(\frac{\partial u}{\partial u_k}\right)^2 u^2(u_k)}$$

where: $u_B(u)$ – transducer output voltage measurement uncertainty, $u(k_u)$ – transducer conversion ratio estimation uncertainty, $u(u_k)$ - transducer input voltage measurement uncertainty.

The specification data from the manufacturers of the calibrator and the multimeter determines the maximum limit on individual errors of measured ranges.

Thus, in order to estimate the uncertainty of Type B voltage measurement $u(u)$, the uncertainty estimation of the transmission voltage transducer $u(k_u)$ and the uncertainty of the output voltage pattern $u(u_k)$ must be estimated.

To determine the uncertainty $u(k_u)$ it is necessary to estimate the uncertainty of the output voltage pattern $u(u_k)$ and voltage measurement uncertainty resulting from error limit of the multimeter $u(u)$ [5]:

(9) $$u(k_u)=\sqrt{\left(\frac{1}{\overline{u_k}}\right)^2 u^2(\overline{u})+\left(-\frac{k_u}{\overline{u_k}}\right)^2 u^2(\overline{u_k})}$$

where: $u(\overline{u})$ - the uncertainty of the average voltage, measured by the Keithely 2002 multimeter, $u(\overline{u_k})$ – the uncertainty of the average voltage at the transducer input terminals.

The uncertainty $u(\overline{u})$ of the average value $\overline{u}$ of the output voltage converter stems from Keithley multimeter measurement uncertainty and is determined by the following dependency:

(10) $$u(\overline{u})=u(u)$$

In this study, the input signal for the transducer was achieved with the Fluke 5500A calibrator. The uncertainty of the mean value of the voltage supplied to the transducer input $u(\overline{u_k})$ is shown by the formula:

(11) $$u(\overline{u_k})=u(u_k)$$

Based on the above dependencies, an estimate of the uncertainty $u_B(u)$ of the measurement of the transducer voltage was performed, for different frequencies. Sample results for two values of a frequency of 50 Hz and 500 Hz are shown in Table 5.

Table 5. Uncertainty - Type B

Transducer	*f* [Hz]	Distribution	$u_B(u)$ [mV]
P1	50	rectangular	3.7509
P2	50	rectangular	3.7509
	500	rectangular	3.7508

Combined uncertainty

Combined standard uncertainty $u_c(u)$ was calculated according with the following dependency:

(12) $$u_c(u)=\sqrt{u_A^2(u)+u_B^2(u)}$$

for results obtained from both transducers (Table 6).

Table 6. Combined uncertainty u_c

Transducer	*f* [Hz]	$u_c(u)$ [mV]
P1	50	4.2559
P2	50	4.4988
	500	4.1095

Expanded uncertainty

According to literature [1, 2, 4] the expanded uncertainty U_p is determined following the formula:

(13) $$U_p(u)=k_p\cdot u_c(u)$$

The coverage factor $k_p=t_p(\nu_{eff})$ is estimated from Student's distribution, where ν_{eff} is effective degrees of freedom and can be obtained from the expanded Welch-Satterthwaite formula, which in the analyzed case is as follows [1]:

(14) $$\nu_{eff}=\frac{u_c^4(u)}{\frac{u_A^4(u)}{\nu_A}+\frac{u_B^4(u)}{\nu_B}}$$

The coefficients ν_A and ν_B are the number of degrees of freedom for the uncertainty Type A and Type B.

The number of degrees of freedom ν_A was designated as [1]:

(15) $$\nu_A=n-1$$

Furthermore, the number of degrees of freedom ν_B of uncertainty Type B was estimated assuming, that the relative uncertainty $\delta_{rel,B}$ of estimation of uncertainty Type B is 20%. Then, according to the following formula [1]:

$$\nu_B = \frac{1}{2 \cdot \delta_{rel,B}^2} \tag{16}$$

After substituting to the equation (16) the results from tables 5, 6 and 7 and the number of degrees of freedom ν_A = 29 and ν_B = 12, the value for ν_{eff} was obtained with corresponding coverage factor k_p values read from t-Student's table, which are presented in Table 7.

Table 7. The parameters necessary to determine the value of the expanded uncertainty U_p as a result of the voltage measurement

Transducer	*f* [Hz]	ν_{eff}	k_p
P1	50	19.2326	2.0930
P2	50	23.0028	2.0678
	500	17.0084	2.1098

The final results of the estimated expanded uncertainty U_p of the voltage u measurement (assuming a 95% confidence level, k_p from Table 7), for transducers P1 and P2 are presented in Table 8.

Table 8. Voltage estimate for transducers P1 and P2

Transducer	*f* [Hz]	Voltage estimation *u* [mV]
P1	50	(2873.30±8.91)
P2	50	(2876.20±9.30)
	500	(2898.60±8.67)

The same data are also presented in graphical form in Figure 5.

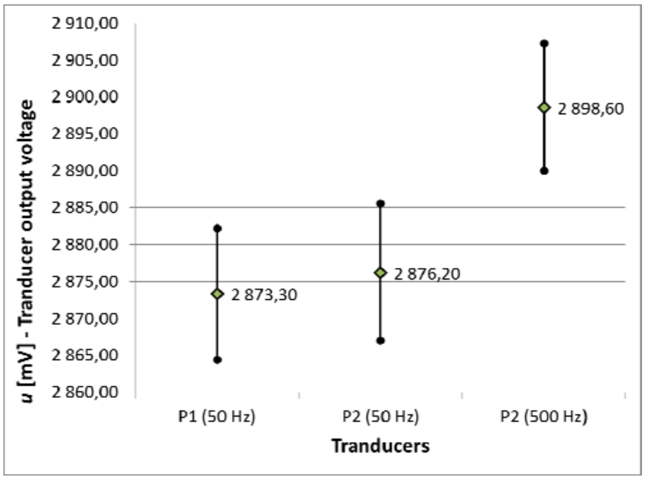

Fig. 5. Voltage estimate with the expanded uncertainty for transducers P1 and P2

In the case of transducers P1 and P2 for frequency of 50 Hz measurand estimate adopts similar values and their difference is about 3 mV. In turn, for different values of the frequency (converter P2) the difference value of measurand estimate is about 22 mV and it is about one order of magnitude higher than for the transducers analyzed for the frequency of 50 Hz.

It can be observed that in case of both transducers for the frequency of 50 Hz and 500 Hz the estimated values of uncertainty are similar and do not exceed 9.5 mV.

Summary

The article presents the problem of uncertainty estimation for voltage transducers CYVS411D07 processing functions using the method based on the GUM recommendation using the law of propagation of uncertainty.

In order to estimate the correct value of uncertainty with no information whether the results of observations are subject to a pre-adjusted probability distribution, it is necessary to perform an analysis. This kind of analysis should be conducted every time to adopt the type of probability distribution that will most faithfully correspond to the results of observation.

It is also important to determine a coverage factor of uncertainty estimated on the effective number of degrees of freedom.

The obtained results of measurements and analyzes show that with increasing frequency of the voltage at the input terminals of respondents transducers the measurand estimate value increases.

The conducted considerations also show that in case of both transducers, for the frequency of 50 Hz and 500 Hz, relative uncertainty of the estimated values are similar and are 0.31%, 0.32% and 0.30% respectively.

The results of performed analyses confirm the validity of their use for voltage measurements when the uncertainty does not exceed 0.35% of the measured voltage value.

REFERENCES

[1] Guide to the Expression of Uncertainty in Measurement (GUM). ISO/IEC/OIML/BIPM, first edition, 1992. last ed. BIPM JCGM 100 (2008)

[2] Taylor John R., Wstęp do analizy błędu pomiarowego, Wydanie 2. PWN 2011 (in Polish)

[3] Praca zbiorowa, Niepewność pomiarów w teorii i praktyce. Główny Urząd Miar, Warszawa 2011

[4] Piotrowski J., Kostyrko K., Wzorcowanie aparatury pomiarowej. Wydanie II zmienione i uaktualnione, PWN, Warszawa 2012

[5] Dzwonkowski A., Swędrowski L., Uncertainty analysis of measuring system for instantaneous power research, *Metrology and Measurement Systems*, Vol. XIX (2012), No. 3, 573-582

[6] Golijanek-Jędrzejczyk A., Uncertainty estimation of loop impedance measurement determined by the vector method, *Pomiary Automatyka i Kontrola,* Vol. 58 (2012), nr 11, 987-991

[7] Dorozhovets M., Warsza Z. L., Upgreading calculating methods of the uncertainty in measurements, *Przegląd Elektrotechniczny,* Vol. 83 (2007), nr 1 – 13 (in Polish)

[8] Warsza Z., Two-component estimators of the measurand value of trapezoidal probability distributions of the data sample, *Pomiary Automatyka i Kontrola,* Vol. 57 (2011), nr 1, 105-108 (in Polish)

[9] Warsza Z. L., Korczynski M. J., Improving of the type A uncertainty evaluation by refining the measurement data from a priori unknown systematic influences. Series: *Advances in Intelligent Systems and Computing* 267, Springer (2014), 727-732

***Authors**: dr inż. Ariel Dzwonkowski, Politechnika Gdańska, Katedra Metrologii i Systemów Informacyjnych, ul. Narutowicza 11/12. 80-233 Gdańsk, e-mail: ariel.dzwonkowski@pg.gda.pl; dr inż. Anna Golijanek-Jędrzejczyk, Politechnika Gdańska, Katedra Metrologii i Systemów Informacyjnych, ul. Narutowicza 11/12. 80-233 Gdańsk, e-mail: anna.golijanek-jedrzejczyk@pg.gda.pl*