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Frequency Symbolic Analysis of Linear Periodically Time-Varying Circuits with Many Parametric Elements

***Abstract**. The possibility of application of frequency symbolic method of analysis of linear periodically time-varying (LPTV) circuits with one parametric element to the circuits with many parametric elements is shown. Such use requires the approximation of parametric transfer functions of circuit by Fourier series by the frequency of decomposition, that are equal to the greatest common divisor of basic frequencies of change parametric elements of circuit.*

***Streszczenie**. W artykule zaprezentowano możliwość zastosowania analizy częstotliwościowej w zadaniu wieloparametrowej identyfikacji usterek liniowych obwodów elektrycznych.Takie podejście wymaga wstępnego określenia transmitancji operatorowej obwodu przy wykorzystaniu szeregu Fouriera o częstotliwości bazowej równej największemu wspólnemu dzielnikowi podstawowej częstotliwości przy której zachodzą zmiany parametrów elementów obwodu. **Wieloparametrowa analiza częstotliwościowa liniowych obwodów parametrycznych**.*

Keywords: linear periodically time-variable circuits, frequency symbolic method.
Słowakluczowe: liniowe obwody parametryczne, analiza częstotliwościowa.

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Introduction

Most of parametric circuits that are used in electronic devices contains one parametric element [1]. Efficiency of functioning of such circuits may be achieved at significant depths of modulation of parameter of parametric element. However, increasing the depth of modulation, in practice, limited or can lead to the emergence unwanted harmonic components in the parameter of parametric element and, consequently in output signal. So sometimes it may be advisable introduction in circuit several parametric elements whose parameters vary in a relatively small limits. From the above follows and relevance of the proposed work.

The formation of symbolic parametric transfer function $W(s,t)$ of established mode of LPTV circuit with one parametric element that changes periodically in time with period T by frequency symbolic method described in detail in [2] and is based on the approximation of the transfer function $W(s,t)$ by trigonometric polynomial of Fourier $\hat{W}(s,t)$ with a certain number of harmonic components k of the frequencies $i\Omega$ where s - complex variable, t - time, $\Omega=2\pi/T$- - basic frequency changes of parametric element, $i=0,1,2,\ldots,k$.

Technique of analysis of circuits with many parametric elements

The parametric transfer function $W(s,t)$ of established mode of circuit with many parametric elements that change periodically in time, should be approximated by trigonometric polynomial of Fourier $\hat{W}(s,t)$ that contains harmonic components with frequencies $i\Omega$ where Ω is equal the greatest common divisor *(GCD)* values of the basic frequencies for which is vary the parametric elements of circuit, respectively. For example, if in circuit: a) n parametric elements, and they are changing with the basic frequencies, $\Omega_1,\Omega_2,\ldots,\Omega_n$ the function $\hat{W}(s,t)$ should decompose in frequency $\Omega=GCD(\Omega_1,\Omega_2,\ldots,\Omega_n)$, b) all elements are parametric, but are changing with one basic frequency Ω, then the function $\hat{W}(s,t)$ should be decompose in frequency $\Omega=GCD(\Omega_1,\Omega_2)$, c) $GCD(\Omega_1,\Omega_2,\ldots,\Omega_n)$, different from 1, does not exist, so such circuit has no established mode, and therefore can not be analyzed by frequency symbolic methof. Thus, the difficulty of task is determined not by the number of parametric elements in the circuit but the number of different basic frequencies their changes.
In another frequency symbolic analysis method of LPTV circuits with many parametric elements with proper choice of values k of no additional changes are not provide.

Example 1. To investigate the RLC series circuit (Fig. 1) with two parametric elements $c(t)$ and $L(t)$ by the FSM.

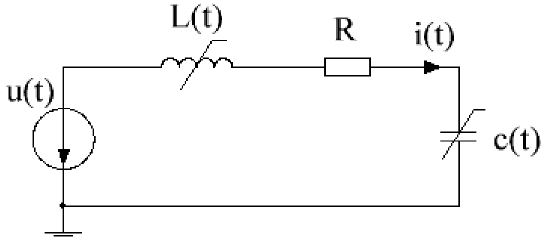

Fig.1. RLC series circuit with two parametric elements
$c(t)=c_0[1+m_c\cos(\Omega_c\cdot t)]$ i $L(t)=L_0[1+m_L\cos(\Omega_L\cdot t)]$

The differential equation of circuit relative to the input signal $u(t)$ and current $i(t)$ is following:

$$\begin{aligned} & c(t)L(t)\cdot i''+[c(t)R+c'(t)L(t)+2c(t)L'(t)]\cdot i'+ \\ (1)\quad & +[1+c'(t)R+c'(t)L'(t)+c(t)\cdot L''(t)]\cdot i= \\ & =c(t)\cdot u'+c'(t)\cdot u \end{aligned}$$

From equation (1) for parametric transfer function $Y(s,t)=I(s,t)/U(s)$ follows the equation L.A. Zadeh:

$$\begin{aligned} (2)\quad & b_2Y''(s,t)+(2b_2s+b_1)\cdot Y'(s,t)+ \\ & +(b_2s^2+b_1s+b_0)\cdot Y(s,t)=a_1s-a_0 \end{aligned}$$

where: $a_0=c'(t)$, $a_1=c(t)$, $b_2=c(t)L(t)$, $b_0=1+c'(t)R+$ $+c'(t)L'(t)+c(t)L'(t)$, $b_1=c(t)R+c'(t)L(t)+2c(t)L'(t)$.

Case 1. According the FSM to determine $i(t)$ when $\Omega_c=4$ *rad/s*, $\Omega_L=4$ *rad/s*, $m_c=0,1$, $m_l=0,05$, $u(t)=1\cdot\cos(t)$, $Y=1/R=1$ *S*, $c_0=1$ *F*, $L_0=1$ *H*.
To decompose $\hat{Y}(s,t)$ by frequency $GCD[4,8]=4$ *rad/s*. Table 1 shows the results of calculations of current $i(t)$ by the FSM from equation (2): $i(t)=Re[\hat{Y}(s,t)\cdot U]$ and by the program Micro-Cap 7.0. (MC7) for circuit from figure 1 when $U=\exp(s\cdot t)$, $s=j\omega$, $\omega=1$ *rad/s* and different the number of harmonic components k in $\hat{Y}(s,t)$. Column 2 presents the results of the program Micro-Cap 7.0.

Table 1. The value of current of RLC series circuit from fig.1

t, s	i(t) by MC7, A	i(t) by the FSM, A			
		k =1	k =2	k =5	k =7
1000	4,8070	4,9646	4,8088	4,8070	4,8070
1001	-4,4749	-4,1543	-4,4760	-4,4748	-4,4749
1002	-9,8817	-9,9647	-9,8929	-9,8816	-9,8817
1003	-5,8202	-6,2405	-5,8104	-5,8203	-5,8202
1004	3,0691	2,8689	3,0711	3,0692	3,0691

Case 2. According the FSM to determine $i(t)$ when $\Omega_c=3$ rad/s, $\Omega_L=2$ rad/s, $m_c=0{,}05$, $m_l=0{,}15$, $u(t)=1\cdot\cos(t)$, $Y=1/R=10$ S, $c_0=1$ F, $L_0=1$ H.

To decompose $\hat{Y}(s,t)$ by frequency $GCD[4,8]=4$ rad/s. In Table 2 shows the results of calculations of current $i(t)=Re[\hat{Y}(s,t)\cdot U]$ by the FSM and by the program Micro-Cap7.0. (MC7) for circuit from Figure 1 when $U=\exp(s\cdot t)$, $s=j\omega$, $\omega=1$ rad/s and a different number of harmonic components k in. Column 2 presents the results of the program Micro-Cap7.0.

Table 2. The value of current of RLC series circuit from fig.1

t, c	*i*(*t*) by MC7, A	*i*(*t*) by the FSM, A				
		k =1	*k* =2	*k* =5	*k* =7	*k* =10
1000	26,3061	5,6193	24,4855	26,4071	26,3127	26,3061
1001	3,3097	-3,9266	3,1227	3,3585	3,3160	3,3096
1002	-18,0921	-9,8470	-15,8341	-18,14 $\hat{Y}(s,t)$08	-18,0876	-18,0921
1003	-26,8931	-6,7371	-25,8371	-26,9473	-26,8903	-26,8931
1004	-8,3862	2,5706	-6,3823	-8,3495	-8,3887	-8,3862

Case 3. According the FSM to determine $i(t)$ when $\Omega_c=2$ rad/s, $\Omega_L=2$ rad/s, $m_c=0{,}05$, $m_l=0{,}05$, $u(t)=1\cdot\cos(t)$, $Y=1/R=10$ S, $c_0=1$ F, $L_0=1$ H.

To decompose $\hat{Y}(s,t)$ by frequency $GCD[2,2]=2$ rad/s. In Table 3 shows the results of calculations of current $i(t)=Re[\hat{Y}(s,t)\cdot U]$ by the FSM and by the program Micro-Cap7.0. for circuit from figure1 when $U=\exp(s\cdot t)$, $s=j\omega$, $\omega=1$ rad/s and a different number of harmonic components k in $\hat{Y}(s,t)$. In column 2 presents the results of the program Micro-Cap7.0.

Table 3. The value of current of RLC series circuit from fig.1

t, c	i(t) by MC7, A	i(t) by the FSM, A				
		k =1	k =2	k =5	k =7	k =10
1000	5,7809	5,7779	5,7812	5,7809	5,7809	5,7809
1001	-4,5217	-4,5055	-4,5215	-4,5217	-4,5217	-4,5217
1002	-9,5584	-9,5527	-9,5584	-9,5584	-9,5584	-9,5584
1004	3,1211	3,1037	3,1207	3,1211	3,1211	3,1211
1005	9,3489	9,3491	9,3488	9,3489	9,3489	9,3489

Analysis of Table 1, Table 2 and Table 3 shows the gradual convergence of values $i(t)$ obtained by the FSM by increasing the number of harmonic components k in the function $\hat{Y}(s,t)$ and almost complete coincidence for $k = 10$ of this function with calculations by the program Micro-Cap7.0

Example 2. Show that the circuit of Figure 1 for two parametric elements $c(t)$ and $L(t)$ with depth of modulation m_c, m_L respectively, can provide the same amplification of signal as the circuit with one parametric element (when parametric is only element $c(t)=c_0[1+m\cos(\Omega\cdot t+\varphi)]$ and the inductance L is constant) on condition $m_c, m_L < m$.

The first experiment. Given $L=1H$ is constant and $m=0{,}44$, $\varphi=43°$, $R=0{,}3Ohm$, $\Omega=2rad/s$, $u(t)=1\cdot\cos(t)$. The time dependence of output current calculated by the program Micro-Cap7.0. shown in Figure 2.

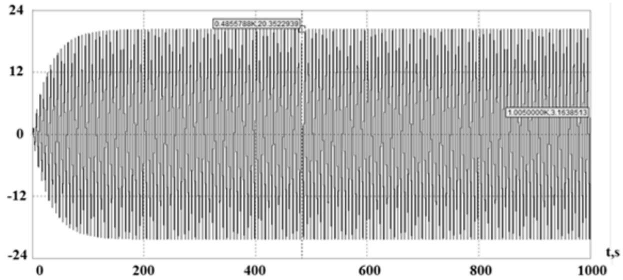

Fig. 2. The current $i(t)$ of RLC series circuit for the case of constant inductance L

The second experiment. Given a parametric capacitance $c(t)=c_0[1+m_c\cos(\Omega\cdot t+\Phi_c)]$ and inductance $L(t)=L_0[1+m_L\cos(\Psi\cdot t+\Phi_L)]$. In this case by changing m_c, m_L, we achieve a amplification of output signal as in the first case. To do this by the FSM we determined the parametric transfer function $\hat{Y}(s,t)=I(s,t)/U(s)$ of circuit from figure 1 with parameters m_c, m_L, in symbols:

(3) $$\hat{Y}(m_c,m_L,s,t)= -10.\cdot s\cdot(.30e30\cdot s+.67e29\cdot s\cdot mL\cdot mc^3-.30e29\cdot s^3\cdot mc^2+.37e30\cdot mL\cdot mc^3+.11e31\cdot mc\cdot mL+.84e21\cdot mc^2\cdot mL^2\cdot s+.10e31\cdot s^2+.40e30\cdot mc^2\cdot mL^2-.50e29\cdot s^4\cdot mc^2+.10e30\cdot s^4-.62e30\cdot mc^2+.13e30\cdot mc^2\cdot mL^2\cdot s^2+.62e28\cdot s^4\cdot mc^2\cdot mL^2-.22e21\cdot s^3\cdot mc^2\cdot mL^2+.60e29\cdot s^3+.10e29\cdot s^4\cdot mL\cdot mc^3-.45e30\cdot s^2\cdot mc^2+.17e29\cdot s^3\cdot mL\cdot mc^3+.13e30\cdot s^2\cdot mL\cdot mc^3-.42e29\cdot s^4\cdot mL\cdot mc-.13e30\cdot s\cdot mc^2-.67e29\cdot s^3\cdot mL\cdot mc-.58e30\cdot s^2\cdot mc\cdot mL-.43e30\cdot s\cdot mc\cdot mL+.94e30)/(-.58e31\cdot s+.27e31\cdot s^2\cdot mL^3\cdot mc^3-.11e31\cdot s\cdot mL\cdot mc^3-.10e30\cdot s^6\cdot mL^3\cdot mc-.14e30\cdot s^5\cdot mL^3\cdot mc+.27e31\cdot s^3\cdot mc^2+.11e32\cdot mc\cdot mL-.80e31\cdot mc^2\cdot mL^2\cdot s-.20e32\cdot s^2-.40e31\cdot mc^2\cdot mL^2+.46e31\cdot s^4\cdot mc^2-.11e32\cdot s^4-.10e32\cdot mc^2\cdot mL^2\cdot s^2-.43e31\cdot s^4\cdot mc^2\cdot mL^2-.56e31\cdot s^3\cdot mc^2\cdot mL^2-.66e31\cdot s^3-.14e31\cdot s^4\cdot mL\cdot mc^3+.66e31\cdot s^2\cdot mc^2-.11e31\cdot s^3\cdot mL\cdot mc^3-.39e31\cdot s^2\cdot mL\cdot mc^3+.72e31\cdot s^4\cdot mL\cdot mc+.19e31\cdot s\cdot mc^2+.70e31\cdot s^3\cdot mL\cdot mc+.23e32\cdot s^2\cdot mc\cdot mL+.87e31\cdot s\cdot mc\cdot mL+.50e30\cdot s^6\cdot mc^2+.60e30\cdot s^3\cdot mL^2-.10e31\cdot s^6-.90e30\cdot s^5+.45e31\cdot s^4\cdot mL^2+.60e31\cdot s^2\cdot mL^2+.15e30\cdot s^5\cdot mL^2+.45e30\cdot s^5\cdot mc^2+.48e29\cdot s^6\cdot mL^3\cdot mc^3-.36e30\cdot s^6\cdot mc^2\cdot mL^2-.10e30\cdot s^6\cdot mL\cdot mc^3+.42e30\cdot s^6\cdot mc\cdot mL+.25e30\cdot s^5\cdot mL^3\cdot mc^3-.57e30\cdot s^5\cdot mc^2\cdot mL^2-.20e30\cdot s^5\cdot mL\cdot mc^3+.80e30\cdot s^5\cdot mc\cdot mL-.13e31\cdot s^4\cdot mL^3\cdot mc+.86e30\cdot s^4\cdot mL^3\cdot mc^3+.50e30\cdot s^6\cdot mL^2-.54e30\cdot s^3\cdot mL^3\cdot mc+.16e31\cdot s^3\cdot mL^3\cdot mc^3-.34e31\cdot s^2\cdot mL^3\cdot mc+.22e31\cdot s\cdot mL^3\cdot mc^3-.94e31)-1.\cdot(.26e31\cdot s\cdot mL-.44e31\cdot s\cdot mL^2\cdot mc+.40e28\cdot s^4\cdot mL+.63e21\cdot s^4\cdot mc-.14e30\cdot s^5\cdot mL^2\cdot mc-.17e31\cdot s^3\cdot mL^2\cdot mc+.17e31\cdot s^2\cdot mL^2\cdot mc-.20e28\cdot s^4\cdot mc^2\cdot mL+.56e30\cdot s^3\cdot mc-.82e30\cdot s^3\cdot mc^2\cdot mL+.96e29\cdot s^5\cdot mL^2\cdot mc^3-.16e30\cdot s^2\cdot mL^2\cdot mc^3+.42e30\cdot s^4\cdot mL^2\cdot mc-.41e29\cdot s^4\cdot mL^2\cdot mc^3+.16e31\cdot mc^2\cdot mL-.20e21\cdot s^4\cdot mc^3-.43e31\cdot mc-.31e31\cdot s\cdot mc^2\cdot mL-.38e21\cdot s\cdot mc^3-.86e30\cdot mc^3\cdot mL^2\cdot s+.28e31\cdot s\cdot mc-.66e28\cdot s^5\cdot mc^2\cdot mL-.15e31\cdot s^2\cdot mc+.40e31\cdot s^2\cdot mL+.11e22\cdot s^2\cdot mc^3+.46e21\cdot s^3\cdot mc^3+.13e29\cdot s^5\cdot mL-.61e30\cdot s^2\cdot mc^2\cdot mL+.72e30\cdot s^3\cdot mL+.17e30\cdot s^3\cdot mL^2\cdot mc^3)/(-.58e31\cdot s+.27e31\cdot s^2\cdot mL^3\cdot mc^3-.11e31\cdot s\cdot mL\cdot mc^3-.10e30\cdot s^6\cdot mL^3\cdot mc-.14e30\cdot s^5\cdot mL^3\cdot mc+.27e31\cdot s^3\cdot mc^2+.11e32\cdot mc\cdot mL-.80e31\cdot mc^2\cdot mL^2\cdot s-.20e32\cdot s^2-$$

.40e31·mc^2·mL^2+.46e31·s^4·mc^2-.11e32·s^4-
.10e32·mc^2·mL^2·s^2-.43e31·s^4·mc^2·mL^2-
.56e31·s^3·mc^2·mL^2-.66e31·s^3-
.14e31·s^4·mL·mc^3+.66e31·s^2·mc^2-.11e31·s^3·mL·mc^3-
.39e31·s^2·mL·mc^3+72e31·s^4·mL·mc+.19e31·s·mc^2+.70e31·s
^3·mL·mc+.23e32·s^2·mc·mL+.87e31·s·mc·mL+.50e30·s^6·mc^2
+.60e30·s^3·mL^2-.10e31·s^6-
.90e30·s^5+.45e31·s^4·mL^2+.60e31·s^2·mL^2+.15e30·s^5·mL^2
+.45e30·s^5·mc^2+.48e29·s^6·mL^3·mc^3-
.36e30·s^6·mc^2·mL^2-
.10e30·s^6·mL·mc^3+.42e30·s^6·mc·mL+.25e30·s^5·mL^3·mc^3-
.57e30·s^5·mc^2·mL^2-.20e30·s^5·mL·mc^3+.80e30·s^5·mc·mL-
.13e31·s^4·mL^3·mc+.86e30·s^4·mL^3·mc^3+.50e30·s^6·mL^2-
.54e30·s^3·mL^3·mc+.16e31·s^3·mL^3·mc^3-
.34e31·s^2·mL^3·mc+.22e31·s·mL^3·mc^3-.94e31)·cos(2.·t)-
1.·(.12e32·s·mL-
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.21e30·s^5·mL^2·mc-.25e31·s^3·mL^2·mc-.11e31·s^2·mL^2·mc-
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.50e30·s^5·mc^2·mL+.14e31·s^2·mc+.11e31·s^2·mL+.16e22·s^2·
mc^3-.88e21·s^3·mc^3+.10e31·s^5·mL-
.15e31·s^2·mc^2·mL+.90e31·s^3·mL+.67e30·s^3·mL^2·mc^3)/(-
.58e31·s+.27e31·s^2·mL^3·mc^3-.11e31·s·mL·mc^3-
.10e30·s^6·mL^3·mc-
.14e30·s^5·mL^3·mc+.27e31·s^3·mc^2+.11e32·mc·mL-
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.56e31·s^3·mc^2·mL^2-.66e31·s^3-
.14e31·s^4·mL·mc^3+.66e31·s^2·mc^2-.11e31·s^3·mL·mc^3-
.39e31·s^2·mL·mc^3+.72e31·s^4·mL·mc+.19e31·s·mc^2+.70e31·s
^3·mL·mc+.23e32·s^2·mc·mL+.87e31·s·mc·mL+.50e30·s^6·mc^2
+.60e30·s^3·mL^2-.10e31·s^6-
.90e30·s^5+.45e31·s^4·mL^2+.60e31·s^2·mL^2+.15e30·s^5·mL^2
+.45e30·s^5·mc^2+.48e29·s^6·mL^3·mc^3-
.36e30·s^6·mc^2·mL^2-
.10e30·s^6·mL·mc^3+.42e30·s^6·mc·mL+.25e30·s^5·mL^3·mc^3-
.57e30·s^5·mc^2·mL^2-.20e30·s^5·mL·mc^3+.80e30·s^5·mc·mL-
.13e31·s^4·mL^3·mc+.86e30·s^4·mL^3·mc^3+.50e30·s^6·mL^2-
.54e30·s^3·mL^3·mc+.16e31·s^3·mL^3·mc^3-
.34e31·s^2·mL^3·mc+.22e31·s·mL^3·mc^3-.94e31)·sin(2.·t)

Function (3) defines the current of capacity $i(t)$ in established mode in the time domain $i(t)=Re[\hat{Y}(m_c,m_L,t)\cdot U]$.

When we choose m_c, m_L in expression (3) we see that the value of $m_c=0,25$, $m_L=0,26$ provides the same current, as shown in Figure 2, and at the same time is less than the value of $m=0,44$ from the first experiment. The time dependence of output current $i(t)$ calculatedby the program Micro-Cap7.0. shown in figure 3.

Thus we see that when we have two parametric elements – capacitor $c(t)$ and inductance $L(t)$ with the depths of modulation m_c, m_L, respectively, can be provided the same amplification of signal as in amplifier with a one parametric element $c(t)=c_0[1+m\cos(\Omega\cdot t+\varphi)]$ and a constant inductance L, provided $m_c,m_L < m$.

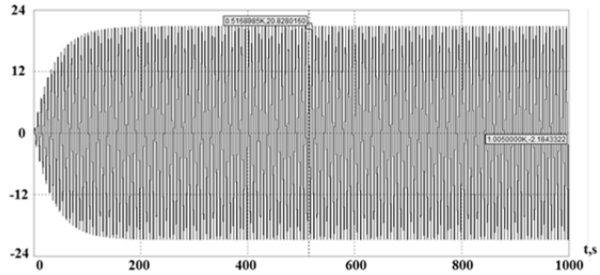

Fig. 3. The current $i(t)$ of RLC series circuit for the case with two parametric elements $c(t)=c_0[1+m_c\cos(\Omega\cdot t+\varphi)]$ and $L(t)=L_0[1+m_L\cos(\Psi\cdot t+\Phi_L)]$

Conclusions

1. Presented in the work examples which are convince in correctness of application of the frequency symbolic method to the analysis of LPTV circuits with many parametric elements.
2. Accuracy of calculations by frequency symbolic method for determined by the number of harmonic components k which included in approximation of the transfer function, and growing with increasing the this number.
3. Difficulty of analysis of LPTV circuit by frequency symbolic method is determined not by a number of parametric elements in the circuit, but the number of different basic frequencies of change of their parameters.
4. If the basic frequencies of changes of parametric elements of circuit have no common divisor (except one), such circuit has no a established mode and can not be analyzed by frequency symbolic method.
5. The results of presented computer experiments are with single-circuit parametric amplifier supporting the possibility of reducing the depth modulation of parametric element with an increase in their number in the circuit.

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