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Determining of complex magnetic permeability of the ferromagnetic material by complex impedance of inductance coil with ferromagnetic core

***Abstract**. The article presents a methodology of determining of complex magnetic permeability of ferromagnetic material by the results of measuring complex impedance of inductance coil with ferromagnetic core. The complex impedance of the inductance coil was measured with immittance meter E7-25.*

*Streszczenie: W pracy przedstawiono metodę wyznaczania zespolonej przenikalności magnetycznej materiału przez pomiar zespolonej indukcyjności cewki z rdzeniem wykonanym z badanego materiału. Impedancja zespolona została zmierzona przy pomocy miernika immitancji E7-25, opisanego w pracy. (**Wyznaczanie zespolonej przenikalności magnetycznej przez pomiar zespolonej indukcyjności**)*

Keywords: Inductance coil with ferromagnetic core, complex impedance, magnetic permeability.
Słowa kluczowe: indukcyjność, cewka, rdzeń ferromagnetyczny, impedancja zespolona, przenikalność magnetyczna.

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Introduction

A laying-on parametric eddy current probe (inductance coil with ferromagnetic core) with U-type core is traditionally used in the process of eddy current defectoscopy for ferromagnetic material with anisotropy properties. This eddy current probe is a coil with ferromagnetic U-type core with winding of number turns w thereon. To determine active resistance and reactance introduced by the tested area of ferromagnetic material it is necessary to know complex magnetic permeability.

Earlier we devoted a number of works to research of interrelation of eddy current probe with ferromagnetic specimen [1]–[3].

Mathematical model of physical processes which occurs in the ferromagnetic core of the coil

The objective of the research is development of methods for determining complex magnetic permeability of ferromagnetic material by the results of measuring of active resistance and reactance of inductance coil with toroidal core (shown in Fig.1).

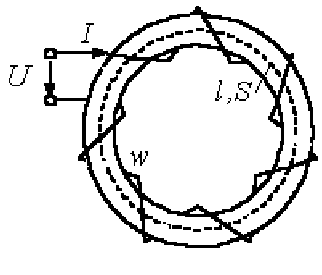

Fig. 1. A coil with toroidal ferromagnetic core

The flux induced in the ferromagnetic core of inductance coil is determined by correlation [4]:

(1) $$U = 4{,}44\, fw\hat{O}_m ,$$

where: U – effective voltage of the winding; f – working frequency; Φ_m – amplitude value of the working magnetic flux, w – number of winding turns.

Amplitude value of the working magnetic flux:

(2) $$\Phi_m = \frac{U}{4{,}44\, fw} .$$

We shall formulate complex power which is absorbed by the ferromagnetic core of coil by means of magnetic loss P_{fer}:

(3) $$\underline{S} = P_{fer} + \mathrm{j}P_{fer} / \operatorname{tg}\delta$$

If we replace hysteresis loop by ellipse, then the losses in the unit of volume of core ferromagnetic material [5]:

(4) $$P_{fer} = \omega\mu_r\mu_0 H^2 V \sin\delta ,$$

where: ω – circular (cyclic) frequency; μ_{r1} – relative magnetic permeability of coil core material; μ_0 – constant (μ_0=4π·10^{-7}); H – magnetic field intensity in the core; δ – magnetic loss angle for coil core material; V=S·l – core volume; S – core cross-section area; l – length medial magnetic line of core.

As is known, magnetic field strength [4]:

(5) $$H = \frac{B}{\mu_r\mu_0} = \frac{\Phi}{\mu_r\mu_0 S} ,$$

where: B – magnetic inductance in coil core; Φ – working magnetic flux; S – core cross-section area.

By substituting expression (5) into expression (4) we receive:

(6) $$P_{fer} = \frac{\omega V \Phi^2 \sin\delta}{\mu_r\mu_0 S^2} .$$

By substituting expression (6) into expression (3) we receive:

(7) $$\underline{S} = \frac{\omega V \Phi^2 \sin\delta}{\mu_r\mu_0 S^2} + \mathrm{j}\frac{\omega V \Phi^2 \cos\delta}{\mu_r\mu_0 S^2} ,$$

or

(8) $$\underline{S} = \frac{\omega V \Phi^2}{\mu_r\mu_0 S^2}\left(\sin\delta + \mathrm{j}\cos\delta\right).$$

On the other hand, complex power is determined by voltage U and resistance of ferromagnetic core of the coil [4]:

(9) $$\underline{S} = \frac{U^2}{\underline{Z}^*} ,$$

where $\underline{Z}^*$ – conjugate complex impedance of the coil.

From the equation (9) we get the following:

(10) $$\underline{Z}^* = \frac{U^2}{\underline{S}} .$$

If we put equation (8) into equation (10), we get the following:

(11) $$\underline{Z}^* = \frac{\mu_r \mu_0 S^2 U^2}{\omega V \Phi^2 \left(\sin\delta + \mathrm{j}\cos\delta\right)}.$$

Complex impedance of the coil:

(12) $$\underline{Z} = \frac{\mu_r \mu_0 S^2 U^2 \left(\sin\delta + \mathrm{j}\cos\delta\right)}{\omega V \Phi^2}.$$

Singling out real and imaginary parts from equation (12), we shall get expressions for determining active resistance *R* and reactance *X* of the coil:

(13) $$R = \frac{\mu_r \mu_0 S^2 U^2 \sin\delta}{\omega V \Phi^2}.$$

(14) $$X = \frac{\mu_r \mu_0 S^2 U^2 \cos\delta}{\omega V \Phi^2}.$$

Putting the equations $U = 6{,}28\, fw\Phi = \omega w\Phi$ and $V = Sl$ into equations (13) and (14) we shall come to expressions for constituents of coil active resistance *R* and reactance *X*:

(15) $$R = \frac{\mu_r \mu_0 S \omega w^2 \sin\delta}{l}.$$

(16) $$X = \frac{\mu_r \mu_0 S \omega w^2 \cos\delta}{l}.$$

As is known, complex magnetic permeability of the ferromagnetic material [5]:

(17) $$\tilde{\mu}_r = \mu_1 - \mathrm{j}\mu_2 = \mu_r e^{-\mathrm{j}\delta} = \mu_r \cos\delta - \mathrm{j}\mu_r \sin\delta.$$

Components μ_1 and μ_2 of complex magnetic permeability we shall determine from equations (15) and (16):

(18) $$\mu_1 = \mu_r \cos\delta = \frac{Xl}{\mu_0 S \omega w^2}$$

(19) $$\mu_2 = \mu_r \sin\delta = \frac{Rl}{\mu_0 S \omega w^2}.$$

Experimental research

First, before carrying out experimental research we should remember general information about real inductance parameters measuring.

Imperfectness of inductance may be taken into consideration through equivalent circuits.

There are two equivalent inductance circuits: sequential (fig. 2a) and parallel (fig. 2b) circuits. Real resistance of inductance coil winding brings forth sequential resistance and losses in inductance coil core bring forth parallel resistance.

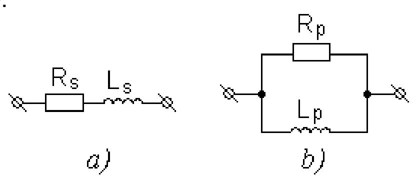

Fig. 2. Equivalent inductance coil circuits

Circuits with real inductance L (with losses) are characterized by full resistance which is called impedance.

Inductance coil impedance is shown in the form of complex number:

(20) $$\underline{Z} = R + \mathrm{j}X_L.$$

There often can be used complex impedance module:

(21) $$Z = \sqrt{R^2 + X_L^2}.$$

Reactive component of impedance *Z* for inductance X_L is determined through equation:

(22) $$X_L = 2\pi f L.$$

The following circuit parameters on alternating current as tangent of loss angle δ and reciprocal called Q-factor are of great importance for inductance coil circuit.

For sequential inductance measuring circuit loss angle tangent *tg*(δ) and Q-factor are determined by equation:

(23) $$Q_L = \frac{1}{\operatorname{tg}\delta} = \frac{|X_s|}{R_s} = \frac{2\pi fL}{R_s}.$$

For parallel inductance measuring circuit loss angle tangent *tg*(δ) and Q-factor are determined by equation:

(24) $$Q_L = \frac{1}{\operatorname{tg}\delta} = \frac{|X_p|}{R_p} = \frac{2\pi fL}{R_p}.$$

Immittance measuring devices nowadays are considered to be the best devices for measuring active resistance *R*, capacity *C*, inductance *L*, complex impedance *Z*, complex conductivity *Y*, Q-factor.

When carrying out experimental research we measured active resistance and reactance of eddy current probe by immittance meter E7-25. Measurements were made on frequencies 100Hz, 200Hz, 500Hz, 1kHz, 2kHz, 5kHz, 10kHz, 20kHz, 50kHz, 100kHz, 200kHz.

The structure chart of an immittance meter E7-25 is shown in fig. 3 [6].

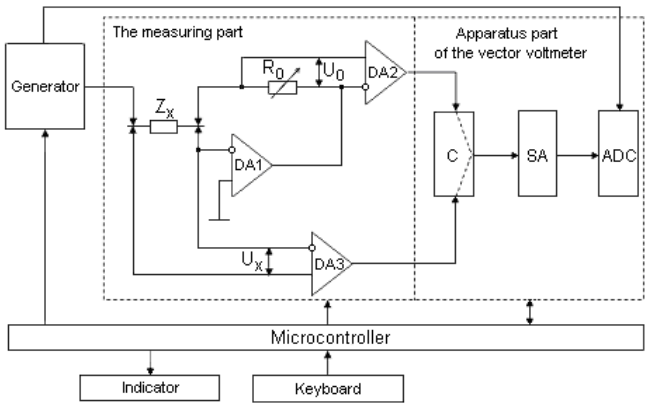

Fig. 3. The structure chart of an immittance meter E7-25

The measuring part of the device E7-25 consists of measured object Z_x, amplifier *DA1*, measure of active resistance R_0 and differential amplifiers *DA2, DA3*. Amplifier *DA1* sustains voltage at the inverting input with near zero value (virtual zero).

Under the action of generator voltage, one and the same current goes through the measured object and interior measure resistance R_0. It creates two voltages on these resistances U_X and U_0, correspondingly.

According to Ohm Law, correlation of these voltages will be equal to correlation of resistances [6]:

(25) $$Z_X = R_0 \frac{U_X}{U_0}.$$

Measuring voltages U_X and U_0 (see fig. 3) are realized by hardware/software vector voltmeter. Apparatus part of the vector voltmeter consists of commutator (*C*), scaling amplifier (*SA*) and analog-digital converter (*ADC*).

Results of measuring by analog-digital converter coming into the microcontroller, which according to the formula (25), calculates measurements. The results of measuring are displayed on a matrix display. The data about number of dimensions of measurements, working frequency, etc are also displayed thereon.

The operator controls the measuring process using keyboard.

Immittance meter E7-25 measures parameters of complex impedance Z on eight tenth of measuring limit (see fig.3). For transference from one measuring limit to another one, in the device there automatically changes inner resistance of R_0 in feedback circuit of scale amplifier *DA1* and thus there changes the coefficient of its amplification.

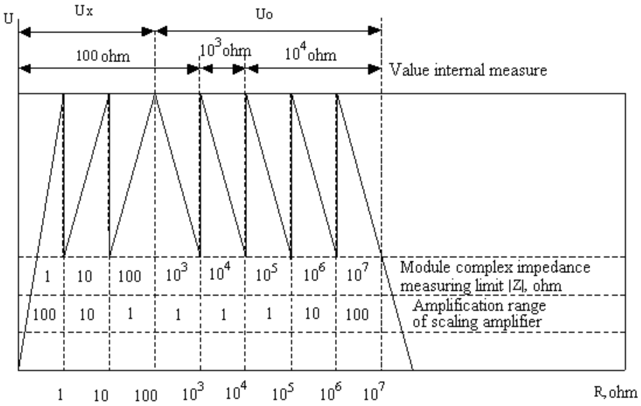

Fig. 3. Measurement limit of module complex impedance $|Z|$ by an immittance meter E7-25

The measuring is carried out on measuring limits of complex impedance Z on 1ohm, 10ohm, 100ohm in preset current mode. On measuring limits of complex impedance Z on 1kilohm, 10kilohm, 100kilohm, 1megohm, the measuring is carried out in preset voltage mode.

By means of interface USB2.0 the data can be transferred from the device to personal computer. And besides one can perform distant programming of all device measuring functions.

Complex resistance was measured for inductance coil with rectangular (U-type) core (see fig. 4).

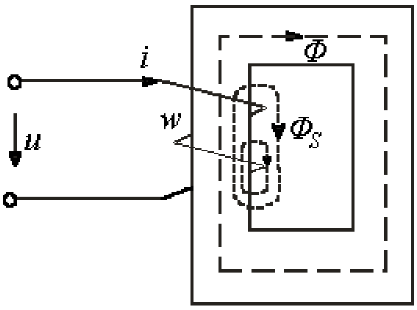

Fig.4. Inductance coil with rectangular core

By results of measurement there were determined components μ_1 and μ_2 of complex magnetic permeability, the values of which are shown in Table I.

Correctness of calculations can be checked by calculating tangent of angle of magnetic losses

(26) $$\delta = \operatorname{arctg}\frac{\mu_2}{\mu_1}$$

and by comparing the obtained result with

(27) $$\delta_1 = \frac{\pi}{2} - \varphi ,$$

where φ - angle determined by device data.

The results of check-up correctness of calculations tangent of angle of magnetic losses are shown in Table II (angles being expressed in degrees).

Table 1. Dependency values components μ_1 and μ_2 of complex magnetic permeability on frequency

f, Hz	100	200	500	10^3	$2{\cdot}10^3$	$5{\cdot}10^3$
μ_1	1482	1353	1224	954	718	445
μ_2	475	371	464	451	447	324
f, Hz	10^4	$2{\cdot}10^4$	$5{\cdot}10^4$	10^5	$2{\cdot}10^5$	-
μ_1	282	207	141	112	98	-
μ_2	165	126	88	69	65	-

Table 2. The results of check-up correctness of calculations tangent of angle of magnetic losses

f, Hz	100	200	500	10^3	$2{\cdot}10^3$	$5{\cdot}10^3$
φ	72.2	74.7	69.3	64.7	58.1	53.9
δ_1	17.8	15.3	20.7	25.3	31.9	36.1
δ	17.8	15.3	20.8	25.3	31.9	36.1
f, Hz	10^4	$2{\cdot}10^4$	$5{\cdot}10^4$	10^5	$2{\cdot}10^5$	-
φ	59.7	58.8	58.2	58.4	56.6	-
δ_1	30.3	31.2	31.8	31.6	33.4	-
δ	30.3	31.2	31.8	31.6	31.4	-

With the aim of improvement of results it is necessary to take into account inductance coil parameters. Therefore, it is necessary to have two coils of the same core size, one core being made of ferromagnetic material and other being made, for example, of cardboard paper.

Conclusion

In the result of our research we obtained formulas for expression of active and reactive resistance for inductance coil with ferromagnetic core. These formulas enable to determine complex magnetic permeability components by the measured coil active resistance and reactance.

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