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Calculations of electromechanical eigenvalues based on instantaneous power waveforms

Abstract. *The paper presents the results of calculations of the eigenvalues (associated with electromechanical phenomena) of the Polish National Power System model on the basis of analysis of the simulated and measured instantaneous power disturbance waveforms of generating units in Łaziska Power Plant. The method for calculations consists in approximation of instantaneous power swing waveforms in particular generating units with use of the waveforms being a superposition of the modal components associated with the searched eigenvalues and their participation factors.*

Streszczenie. *W artykule przedstawiono wyniki obliczeń wartości własnych (związanych ze zjawiskami elektromechanicznymi) modelu Krajowego Systemu Elektroenergetycznego na podstawie analizy symulacyjnych i pomiarowych przebiegów mocy chwilowej zespołów wytwórczych w elektrowni Łaziska. Wykorzystana metoda obliczeń polega na aproksymacji przebiegów odchyłek mocy chwilowej za pomocą przebiegów stanowiących superpozycję składowych modalnych związanych z poszukiwanymi wartościami własnymi i ich czynnikami udziału. (**Obliczenia elektromechanicznych wartości własnych na podstawie przebiegów mocy chwilowej**).*

Keywords: power system, eigenvalues associated with electromechanical phenomena, transient states, angular stability.
Słowa kluczowe: system elektroenergetyczny, wartości własne związane ze zjawiskami elektromechanicznymi, stany nieustalone, stabilność kątowa.

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Introduction

Maintaining the angular stability of a power system (PS) is one of the most important aspects of its work. PS state matrix eigenvalues can be used for assessing the PS angular stability [1]. The eigenvalues can be calculated from the PS state equations, however, the calculation results then depend on the values of the system state matrix elements; they also – indirectly – depend on the assumed system models and their uncertain parameters [2]. The eigenvalues can also be calculated with good accuracy from analysis of the actual disturbance waveforms occurring in the PS after various disturbances [3].

The aim of this paper is to analyse the accuracy of calculating the eigenvalues (associated with electromechanical phenomena) of the state matrix of the Polish Power System (PPS) model interfering in the instantaneous power waveforms of the generating units working in Łaziska Power Plant. The calculations were carried out basing on the analysis of simulation and measurement disturbance waveforms of the instantaneous power in units: no 9 (KOP113), no 10 (KOP123) and no 11 (KOP213) in Łaziska Power Plant.

Linearised model of a power system

The power system model linearised around the working point is described by the state and output equations [4, 5, 6]:

$$(1) \qquad \Delta\dot{\boldsymbol{X}} = \boldsymbol{A}\Delta\boldsymbol{X} + \boldsymbol{B}\Delta\boldsymbol{U}, \quad \Delta\boldsymbol{Y} = \boldsymbol{C}\Delta\boldsymbol{X} + \boldsymbol{D}\Delta\boldsymbol{U},$$

where: $\Delta\boldsymbol{X}$, $\Delta\boldsymbol{U}$, $\Delta\boldsymbol{Y}$ - deviations of the vectors of: state variables, inputs and output variables, respectively. The waveforms of input quantities of the linearised system model can be calculated directly by integrating the state equation, or by using the eigenvalues and eigenvectors of the state matrix $\boldsymbol{A}$ [4, 5, 6].

The waveform of the given output value is a superposition of the modal components which depend on the eigenvalues and eigenvectors of the state matrix. The i-th output value (at $\boldsymbol{D} = \boldsymbol{0}$ and assuming only single eigenvalues) is:

– in the case of a disturbance being a Dirac pulse of the j-th input value $\Delta U_j(t) = \Delta U\delta(t–t_0)$ [5, 6]:

$$(2) \quad \Delta y_i(t) = \sum_{h=1}^{n} F_{ih} e^{\lambda_h (t-t_0)} \Delta U, \quad t \geq t_0, \quad F_{ih} = \boldsymbol{C}_i \boldsymbol{V}_h \boldsymbol{W}_h^{\mathrm{T}} \boldsymbol{B}_j,$$

– in the case of a disturbance being a step change of the j-th input value $\Delta U_j(t) = \Delta U\delta(t–t_0)$ [5]:

$$(3) \quad \Delta y_i(t) = \sum_{h=1}^{n} K_{ih} \left(e^{\lambda_h (t-t_0)} - 1\right) \Delta U, \quad t \geq t_0, \quad K_{ih} = F_{ih}\lambda_h^{-1},$$

where: $\lambda_h = \alpha_h + \mathrm{j}\,\nu_h$ - h-th eigenvalue of the state matrix, F_{ih} - participation factor of the h-th eigenvalue in the i-th output waveform, $\boldsymbol{C}_i$ - i-th row of $\boldsymbol{C}$ matrix, $\boldsymbol{V}_h$ - h-th right-side eigenvector of the state matrix, $\boldsymbol{W}_h$ - h-th left-side eigenvector of the state matrix, $\boldsymbol{B}_j$ - j-th column of $\boldsymbol{B}$ matrix, n - dimension of the state matrix $\boldsymbol{A}$. The values λ_h and F_{ih} can be real or complex.

In case of the waveforms of instantaneous power swings in PS, the eigenvalues associated with motion of generating units rotors, called *electromechanical eigenvalues* in the paper, are of decisive significance. The electromechanical eigenvalues intervene in different ways in the instantaneous power waveforms of particular generating units, which is related to the different values of their participation factors.

The method for calculations of electromechanical eigenvalues

For calculations there were used the disturbance waveforms of generating unit instantaneous power deviations which occurred after purposeful introducing a small disturbance to the PS. The assumed disturbance is a square pulse of the voltage regulator reference voltage in one of generating units. The system response to an input in the form of a short square pulse with a suitably selected height and length is close to that to a Dirac pulse [5, 6].

The method for calculations of electromechanical eigenvalues used in investigations consists in approximation of instantaneous power waveforms in particular generating units with use of expression (2) in the case of a pulse disturbance or (3) in the case of a step disturbance. The electromechanical eigenvalues and participation factors of specific modal components are the unknown parameters of this approximation. In the approximation process, these parameters are iteratively selected to minimize the value of the objective function defined as a mean square error between the approximated and approximating waveform:

$$\varepsilon_{\mathrm{w}}(\boldsymbol{\lambda},\boldsymbol{F})=\sum_{i=1}^{N}\left(\Delta P_{i(\mathrm{m})}-\Delta P_{i(\mathrm{a})}(\boldsymbol{\lambda},\boldsymbol{F})\right)^{2}, \tag{4}$$

where: $\boldsymbol{\lambda}$ - vector of electromechanical eigenvalues, $\boldsymbol{F}$ - vector of participation factors, N - number of samples, the index m denotes the approximated waveform, while the index a the approximating waveform of the instantaneous power P, calculated from the searched eigenvalues and participation factors. The objective function (4) is minimized by a hybrid algorithm which is a serial combination of a genetic algorithm with a gradient algorithm [5, 6, 7].

Due to the existence of the objective function local minima in which the optimisation algorithm may freeze, the eigenvalues were calculated repeatedly based on the same waveform. If the objective function values were higher than a certain assumed limit, the results were rejected. The adopted final result of the calculations of real and imaginary parts of the particular eigenvalues were the arithmetic means from the real and imaginary parts, respectively, of the eigenvalues obtained from the results not rejected in further calculations [5, 6].

Calculations based on instantaneous power simulation waveforms

In order to verify the calculation method accuracy, the instantaneous power waveforms obtained from simulations with use of the PPS model (Fig. 1) were employed.

In this model there were taken into account 49 selected generating units working in high and highest voltage networks as well as 8 equivalent generating units representing influence of PSs of neighbouring countries. The analysed PPS model was worked out in Matlab-Simulink environment. It consists of 57 models of generating units as well as the model of the network and loads.

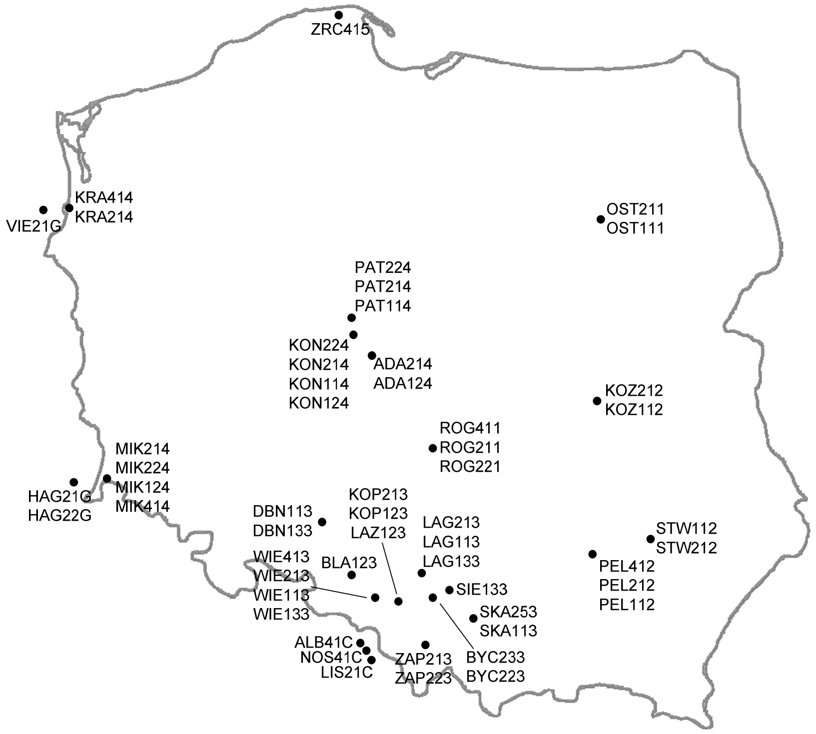


Fig.1. Generating units included in the Polish Power System model

The calculations presented in this paper consider the following models: a synchronous generator GENROU [8, 9], a static or electromachine [8, 9] excitation system operating in the PPS, a steam turbine IEEEG1 [8, 9] or water turbine HYGOV and, optionally, a power system stabilizer PSS3B [8, 9]. For the equivalent generating units representing influence of power systems of the neighbouring countries there was used the simplified model of a synchronous generator (GENCLS [9]).

The eigenvalues (including electromechanical eigenvalues) of the system state matrix can be calculated directly on the basis of the structure and parameters of the PS model in program Matlab-Simulink. These electromechanical eigenvalues are called *original eigenvalues* further in the paper. Comparison of the eigenvalues calculated basing on minimization of the objective function (4) and the original eigenvalues is a measure of the calculation accuracy [5].

The state matrix of the analysed PPS model has 56 complex electromechanical eigenvalues. They were sorted in ascending order with respect to the real parts and numbered from λ_1 to λ_{56}.

In exemplary calculations there were taken into account the waveforms of the instantaneous power deviations in units no 10 (KOP123) and no 11 (KOP213) in Łaziska Power Plant (Fig. 1). There were taken into consideration the instantaneous power waveforms of the unit occurring after introducing a disturbance to the waveform of the voltage regulator reference voltage in that unit. Only one electromechanical eigenvalue λ_{15} = –1.0477±j10.0241 1/s influences the instantaneous power waveform in unit 10 and only one electromechanical eigenvalue λ_8 = –1.1405±j10.6099 1/s in unit 11.

In Tab.1 there are listed the absolute errors $\Delta\lambda_{15}$ and $\Delta\lambda_8$ of calculations of the appropriate eigenvalues. There were taken into account the waveforms occurring after introducing pulse and step disturbances of different amplitudes (V_{ref0} denotes the steady/initial value before disturbing the reference voltage of the generating unit voltage regulator to which the disturbance was introduced).

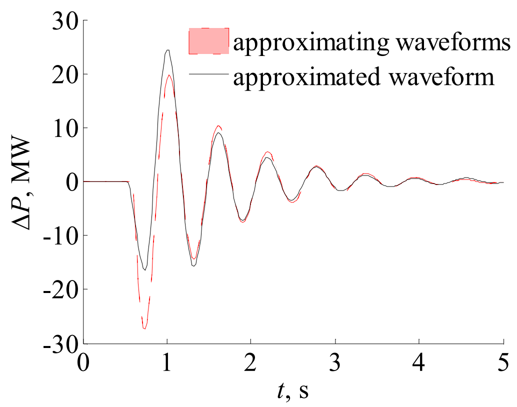
Table 1. Absolute errors of eigenvalue calculations based on instantaneous power simulation waveforms

Unit no 10 (KOP123)			
Kind of disturbance	ΔV_{ref}	t_{imp}	$\Delta\lambda_{15}$
	% V_{ref0}	s	1/s
pulse	–5	200	0.0182±j0.1013
	5	200	–0.2309±j0.0572
step	–5	-	–0.0374±j0.2137
	5	-	–0.1179±j0.5611
Unit no 11 (KOP213)			
Kind of disturbance	ΔV_{ref}	t_{imp}	$\Delta\lambda_{8}$
	% V_{ref0}	s	1/s
pulse	–5	200	0.0420±j0.1233
	5	200	–0.3679±j0.1688
step	–5	-	0.0047±j0.0943
	5	-	0.1031±j0.7627

From Tab. 1 it follows that the absolute errors of eigenvalue calculations are in general much larger in the case of disturbances of positive amplitude than in the case of disturbances of negative amplitude. It is caused by stronger influence of nonlinearites and constraints occurring in the PS on the instantaneous power waveforms in the case of introducing a disurbance of positive amplitude. The eigenvalue calculation accuracy is generally comparable in the cases of pulse and step disturbances of negative amplitude.

Fig. 2 shows exemplary simulation waveforms of the instantaneous power deviation in the case of the pulse and step disturbances of negative amplitude in unit 11 in Łaziska Power Plant as well as the bands of the approximating waveforms corresponding to the non-rejected calculation results. The band of the approximating waveforms determines the range of the instantaneous power changes in which „there are" all approximating waveforms corresponding to particular calculation results.

a)



b)

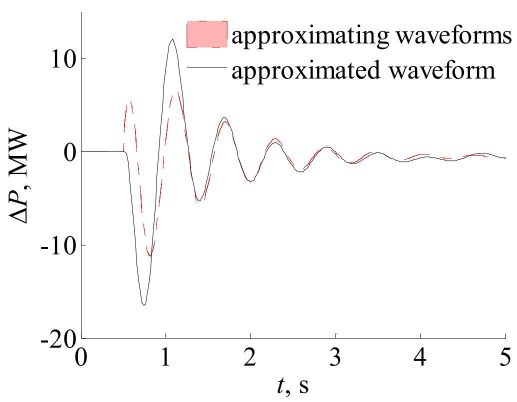


Fig. 2. Waveforms of the instantaneous power deviation in unit no 11 in Łaziska Power Plant in case of pulse (a) and step (b) disturbance of amplitude ΔV_{ref} = –5% V_{ref0}

From Fig. 2 it follows that the approximation quality of the instantaneous power simulation waveforms is satisfactory in the time interval after decay of strongly damped modal components which do not influence the calculation results. In almost all cases the bands of the approximating waveforms were very narrow.

Calculations based on instantaneous power measurement waveforms

In this subsection there is presented the comparison of the calculation results of electromechanical eigenvalues for different sign of the amplitude ΔV_{ref} of the step disturbance of the voltage regulator reference voltage. There were taken into account the instantaneous power measurement waveforms of units no 9 (KOP113), 10 (KOP123) and 11 (KOP213) in Łaziska Power Plant.

Filtering of the measurement waveforms was made with a third order digital Butterworth low pass filter with the cut-off frequency equal to 10 Hz [10]. The zero phase filtering was used [10]. It allowed eliminating the phase delays and distortions of the signals introduced by the filter.

In the case of calculating the PPS electromechanical eigenvalues based on the instantaneous power measurement waveforms, the complete estimation of the calculation accuracy was not possible due to the lack of access to accurate and reliable enough calculation results of these eigenvalues obtained with use of other methods.

The instantaneous power waveforms recorded in units no 9, 10 and 11 in Łaziska Power Plant contain only one significant modal component associated with the electromechanical eigenvalues. Unit no 9 was not included in the PPS model. The eigenvalue influencing significantly the instantaneous power waveform of this unit was denoted by λ_x (calculation of this eigenvalue is not possible on the basis of the structure and parameters of the PPS model).

In Tab. 2 there are listed the results of calculations of the eigenvalues based on the measured instantaneous power waveforms.

Table 2. Eigenvalue calculation results based on instantaneous power measurement waveforms of units no 9, 10 and 11 in Łaziska Power Plant

Unit nr	ΔV_{ref}	λ	
	% V_{ref0}	1/s	
9 (KOP113)	–3	λ_x	0.8823±j7.7264
	3	λ_x	–1.0176±j8.0020
10 (KOP123)	–3	λ_{15}	–1.2011±j7.6645
	3	λ_{15}	–1.0695±j7.7341
11 (KOP213)	–3	λ_8	–0.8603±j8.2593
	3	λ_8	–1.0140±j8.5071

From Tab. 2 it follows that the calculation results of the real parts of the eigenvalues are significantly different depending on the sign of the introduced disturbance amplitude. The calculation results of the imaginary parts of these eigenvalues differ relatively little depending on the sign of the introduced disturbance amplitude. Basing on the simulation investigations performed one can conclude that this difference can be caused by stronger influence of nonlinearities and constraints occurring in the PS on the instantaneous power waveforms in the case of introducing a disturbance of positive amplitude.

Fig. 3 shows exemplary waveforms of the instantaneous power deviation in the case of the step disturbance of positive and negative amplitude in unit 11 in Łaziska Power Plant as well as the bands of the approximating waveforms corresponding to the non-rejected calculation results.

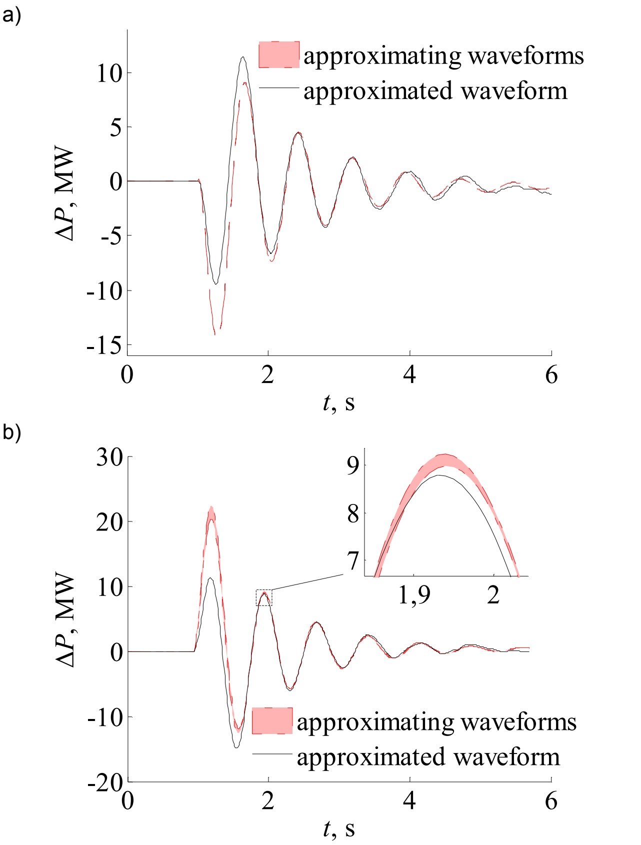


Fig.3. Waveforms of the instantaneous power deviation in unit no 11 in Łaziska Power Plant in case of step disturbance of amplitude $\Delta V_{\text{ref}} = -3\%\ V_{\text{ref0}}$ (a) and $\Delta V_{\text{ref}} = 3\%\ V_{\text{ref0}}$ (b)

From Fig. 3 it follows that the approximation quality of the instantaneous power measurement waveforms is also satisfactory in the time interval after decay of strongly damped modal components. In almost all cases the bands of the approximating waveforms were also very narrow.

Conclusions

The investigations performed allow to draw the following conclusions:

– The simulation investigations prove that it is possible to determine electromechanical eigenvalues with a good accuracy based on the analysis of instantaneous power waveforms under disturbance conditions occurring after introducing a rectangular pulse or a step change of negative amplitude to the voltage regulator reference voltage. In the case of disturbances of positive amplitude there were obtained worse results of eigenvalue calculations, which could be caused by a stronger influence of nonlinearities and constraints occurring in the PS on the instantaneous power waveforms.

– The eigenvalue calculation accuracy based on instantaneous power simulation waveforms was comparable in the case of pulse and step disturbances of negative amplitude. The eigenvalue calculation results in the case of the positive amplitude of a disturbance differ depending on the kind of the disturbance assumed.

– The calculation results of electromechanical eigenvalues based on the measured instantaneous power waveforms recorded in Łaziska Power Plant are different depending on the sign of the introduced step disturbance amplitude. Direct estimation of the accuracy of these calculations is not possible, however on the basis of the simulation calculations carried out, one can conclude that the result obtained in the case of the disturbance of a negative amplitude is more accurate.

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