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# Applicability of approximate Green's functions in MPIE model of a horizontal wire conductor in lossy soil

***Abstract**. The paper presents a study of the applicability of approximate Green's functions in frequency domain analysis of a horizontal wire conductor above or buried in homogeneous lossy soil. The authors analyze the effects of two distinct approximate formulations of Sommerfeld integrals based on the use of quasi-static image theory and complex image theory. This analysis is focused on the current distribution with respect to conductor length and position, soil parameters and frequency range.*

***Streszczenie.** W artykule przedstawiono badania nad zastosowaniem przybliżonych funkcji Greena w analizie częstotliwościowej poziomych przewodów usytuowanych nad lub zakopanych w jednorodnej stratnej ziemi. Autorzy przeanalizowali efekty dwóch przybliżonych sformułowań całek Sommerfelda w oparciu o kwasistatyczną teorię odbić i kompleksową teorię odbić. Skupiono się na analizie rozkładu prądu z odniesieniem do długości przewodnika i jego pozycji, parametrów gleby i zakresu częstotliwości. (**Stosowalność przybliżonych funkcji Greena w MPIE modelach poziomych przewodów w stratnej glebie**)*

**Keywords:** Green's functions, MPIE, wire conductor in lossy soil.
**Słowa kluczowe:** funkcje Greena, MPIE, przewód w stratnej glebie.

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## Introduction

The electromagnetic analysis of horizontal wire-conductors buried in finitely conductive earth is often part of complex electromagnetic compatibility studies. Different strategies for modelling have been developed, ranging from transmission line theory to exact approach based on electromagnetic theory [1]. The last one is based on the electromagnetic theory and uses rigorous Sommerfeld formulation for the electric field due to elementary Hertz dipoles sources and thus represents the most accurate solution of the problem. This model involves numerical integration of appropriate Green's functions that take into account the influence of the air/soil half-spaces by rigorous Sommerfeld formulation [2]. However, such rigorous treatment in the electromagnetic model is very demanding in terms of both computer memory and time. For that reason approximate formulations have been intensively studied [3]–[8].

In this paper the authors investigate the effects of the proposed quasi-static approximations of the Green's functions related to the given problem. This study is based on the comparison of the current distribution along a centre fed wire conductor buried in homogeneous lossy soil. The calculations are done in wide frequency range from 0.001 to 10 MHz. A detailed parametric analysis clearly illustrates the validity domain of the proposed approximations with respect to their practical applications. The results will be compared by NEC reflection coefficient method [10].

## Mathematical model

We consider single centre fed horizontal $x$-directed wire conductor of length $L$ located above or buried in homogeneous lossy soil, as shown in Fig. 1.

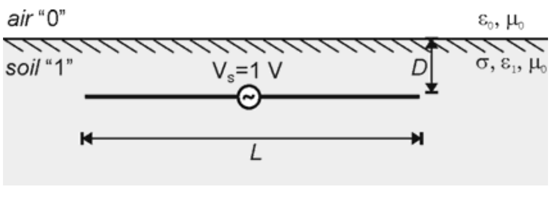


Fig.1. A centre fed horizontal conductor in homogeneous lossy soil

The central fed energization is assumed by a harmonic voltage generator $V_S$ in frequency range from 1 kHz to 10 MHz, the time variation $e^{j\omega t}$ is assumed and suppressed. The air (medium "0") occupies the upper half-space ($z > 0$), whereas the soil (medium "1") occupies the lower half-space ($z < 0$). Both mediums are characterized by permeability $\mu_0$, air permittivity $\varepsilon_0$ and soil permittivity $\varepsilon_1 = \varepsilon_0\varepsilon_r$. The conductivity of the soil is $\sigma$.

## Electromagnetic model

To solve current distribution we use mathematical model that is based on full-wave theory (denoted as EM model) in MPIE formulation for the electric field $E_x$ tangential to the conductor [6] due to filament of current $I(x')$ and charge $q(x')$ sources at the conductor axis:

(1) $$E_x = -j\omega\int_\ell G_A^{xx} I(x')dx' - \frac{\partial}{\partial x}\int_\ell G_V q(x')dx'$$

where: $G_A^{xx}$ and $G_V$ are corresponding Green's functions.

The rigorous EM model involves exact formulations for the Green's functions of the vector and scalar potentials. Here, $G_A^{xx}$ is the $x$-component of the dyadic Green's function for the magnetic vector potential due to $x$-directed horizontal electric dipole HED in conductive half space. Respectively, $G_V$ is the scalar potential Green's function due to one charge $q$ associated to the HED.

(2) $$G_A^{xx} = \frac{\mu_0}{2}\left[G_{dir} - G_{img} + S_0\left\{\frac{2e^{-u_1|z+z'|}}{u_1+u_0}\right\}\right]$$

(3) $$G_V = \frac{1}{2\underline{\varepsilon}_1}\Bigg[\Big[G_{dir} + K_{10}G_{img} + S_0\left\{\frac{2k_1^2e^{-u_1|z+z'|}}{k_1^2u_0+k_0^2u_1} - \frac{2k_1^2e^{-u_1|z+z'|}}{u_1(k_1^2+k_0^2)}\right\}\Bigg]$$

where: $G_{dir}$ and $G_{img}$ correspond respectively to the so called direct and image term:

(4) $$G_{dir} = \frac{e^{-jk_1\sqrt{\rho^2+|z-z'|^2}}}{\sqrt{\rho^2+|z-z'|^2}} = \frac{e^{-jk_1R_{dir}}}{R_{dir}}$$

(5) $$G_{img} = \frac{\mathrm{e}^{-jk_1\sqrt{\rho^2+|z+z'|^2}}}{\sqrt{\rho^2+|z+z'|^2}} = \frac{e^{-jk_1R_{img}}}{R_{img}}$$

where $R_{dir}$ and $R_{img}$ are the distances between the dipole and the observation point and the image of the dipole and the observation point. The terms $S_0\{\cdot\}$ in (2) and (3) are Sommerfeld-type integrals that we solve by direct numerical integration in a similar way to the approach used by Burke et al. [6]

$$S_0\{\tilde{F}\} = \frac{1}{2\pi}\int_0^\infty \tilde{F}(\lambda) J_0(\lambda\rho)\lambda d\lambda \tag{6}$$

The expressions for the vector and scalar spectral domain Green's functions relative to this problem may be found in [12, 13]. The involve Fresnel reflection coefficients $R_{TE}$ and $R_{TM}$:

$$R_{TE} = \frac{u_1 - u_0}{u_1 + u_0} \qquad R_{TM} = \frac{k_0^2 u_1 - k_1^2 u_0}{k_0^2 u_1 + k_1^2 u_0} \tag{7}$$

$$\begin{aligned} &u_i = \sqrt{\lambda^2 - k_i^2}; \quad i = 0,1; \\ &k_0^2 = \omega^2 \mu_0 \varepsilon_0; \quad k_1^2 = \underline{\varepsilon}_r k_0^2; \\ &\underline{\varepsilon}_r = \varepsilon_r - j\sigma(\omega\varepsilon_0)^{-1} \quad \underline{\varepsilon}_1 = \varepsilon_0 \underline{\varepsilon}_r \end{aligned} \tag{8}$$

## Approximate models

The quasi-static approximations of the Green's functions $G_A^{xx}$ and $G_V$ are based on the exponential approximation of the spectral expressions that arise in $S_0\{\cdot\}$ when frequency tends to zero. The spatial domain Green's functions are later obtained in closed form in terms of Sommerfeld identity. In comparison to the classical quasi-static approach, this image representation involves the propagation effect [5].

Model A: When $\omega \to 0$ the most simple approximation of (6) is obtained by using quasi-static image theory [5]. This approach is based on the assumption $u_1 \sim u_0$ since $\lambda^2 >> k_n^2$ for $n = 0, 1$ that leads to approximation of the Fresnel reflection coefficients:

$$R_{TE}^{10} \to 0 \qquad R_{TM}^{10} = \frac{k_0^2 - k_1^2}{k_0^2 + k_1^2} = -K_{10} \tag{9}$$

and further leads to the following set of approximate expressions of the Green's functions:

$$\begin{aligned} &G_{A\,\omega\to 0}^{xx} \to \frac{\mu_0}{4\pi}\frac{e^{-jk_1 R_{dir}}}{R_{dir}} \\ &G_{V\omega\to 0} \to \frac{1}{4\pi\underline{\varepsilon}_1}\left[\frac{e^{-jk_1 R_{dir}}}{R_{dir}} + K_{10}\frac{e^{-jk_1 R_{img}}}{R_{img}}\right] \end{aligned} \tag{10}$$

Model B: The second approximation valid when $\omega \to 0$ is based on is based on the assumption $u_1 \sim \lambda$, $u_1 \neq u_0$ and Wait-Spies [8] and Bannister's air-earth complex image theory [9]:

$$\frac{2\lambda}{u_1 + \lambda} \approx 1 - e^{-\lambda d} \quad e^{-u_1|z+z'|} = e^{-jk_1|z+z'|a} e^{-\lambda|z+z'|b} \tag{11}$$

where $d = 2/\sqrt{j\omega\mu_0(\sigma + j\omega\varepsilon_1)}$ is complex depth. The constants $a$ and $b$ in (11) get values of 0.4 and 0.96 [3].

This approach leads to the following set of approximate Green's functions:

$$R_{TE}^{10} \to 0 \qquad R_{TM}^{10} = \frac{k_0^2 - k_1^2}{k_0^2 + k_1^2} = -K_{10} \tag{9}$$

$$\begin{aligned} G_{A\omega\to 0}^{xx} \to \frac{\mu_0}{4\pi}\Bigg[&\frac{e^{-jk_1 R_{dir}}}{R_{dir}} + \left(K_{10} - 2\right)\frac{e^{-jk_1 R_{img}}}{R_{img}} \\ &+ \frac{e^{-jk_1 a|z+z'|}}{\sqrt{\rho^2 + (b|z+z'|)^2}} - \frac{e^{-jk_1 a|z+z'|}}{\sqrt{\rho^2 + (d + b|z+z'|)^2}}\Bigg] \end{aligned} \tag{12}$$

$$\begin{aligned} G_{V\omega\to 0} \to \frac{1}{4\pi\underline{\varepsilon}_1}\Bigg[&G_{dir} + \left(K_{10} - 2\right)G_{img} \\ &+ \frac{2e^{-jk_1 a|z+z'|}}{\sqrt{\rho^2 + (b|z+z'|)^2}}\Bigg] \end{aligned} \tag{13}$$

## Numerical results

To determine the domain of applicability of proposed approximate expressions for the Green's functions we compare the current along a horizontal conductor in homogeneous lossy soil. The studied cases are: $L$ = 10-m (short conductor) and $L$ = 100-m (long conductor), with radius $a$ = 0.007 m positioned at depth $D$: 0.5 m, 1 m and 1.5 m. Three values for the soil conductivity $\sigma$ are assumed: 0.001 S/m, 0.01 S/m and 0.1 S/m. The relative permittivity of the soil is fixed at $\varepsilon_r$ = 10. The excitation is central feed by a harmonic voltage source of 1 V in frequency range from 1 kHz to 10 MHz.

In order to obtain the accuracy, we calculate the current rms error [11] by accumulating all differences in the current distribution along the conductor:

$$\varepsilon_{\mathrm{rms}} = \left[\frac{\sum_{i=1}^{N}\left|\underline{I}_{EMi} - \underline{I}_{approxi}\right|^2}{\sum_{i=1}^{N}\left|\underline{I}_{EMi}\right|^2}\right]^{1/2}. \tag{14}$$

Here, $\underline{I}_{EMi}$ and $\underline{I}_{approxi}$ are phasors of the current samples along the wire computed by the EM model and by using the approximate models A and B, and $N$ is number of samples.

Fig. 2 shows respectively the current magnitude and phase calculated by using rigorous electromagnetic model, and the two approximate models A and B. The results are obtained for $f$ = 1 MHz, $\sigma$ = 0.01 S/m, $D$ = 1m.

Fig. 3 shows the current distribution error obtained when using Model A and Model B with respect to EM model. The studied cases are: short conductor ($L$ = 10-m) and long conductor ($L$ = 100-m), buried in lossy soil with depth $D$ = 1 m. The soil conductivity $\sigma_1$ is 0.01 S/m with $\varepsilon_1 = 10\varepsilon_0$. The excitation is central feed by a harmonic voltage source of 1 V in frequency range from 1 kHz to 10 MHz.

The $\varepsilon_{\mathrm{rms}}$ error (14) given in Fig. 3 shows that the accuracy of model A and model B is frequency dependent. The maximal $\varepsilon_{\mathrm{rms}}$ error is around 5-6% calculated at the resonant frequency. However, we may observe that the application of the reflection coefficient approach with NEC code (denoted by NEC-rc) introduces much higher error applied for long conductors.

Fig. 4 shows the accuracy of the approximate models with respect to the soil conductivity. As may be observed, when the soil conductivity is low (0.001 S/m) the □rms error tends to increase significantly, particularly at higher frequencies above 1 MHz. In case of very large value of the soil conductivity (0.1 S/m) the accuracy of all models is much better, i.e. the $\varepsilon_{\mathrm{rms}}$ error is within 5%.

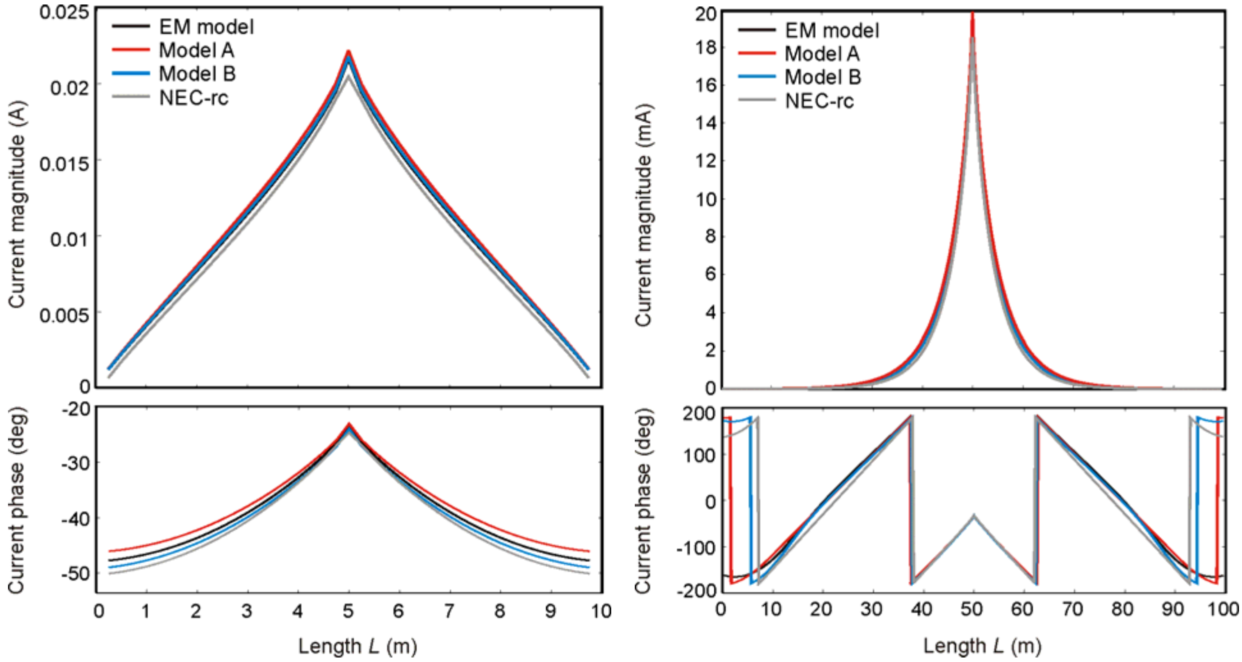


Fig.2. Current magnitude and phase along a 10-m conductor (left) and 100-m conductor (right) at 1 MHz obtained by different models ($D$ = 1 m, $\sigma$ = 0.01 S/m)

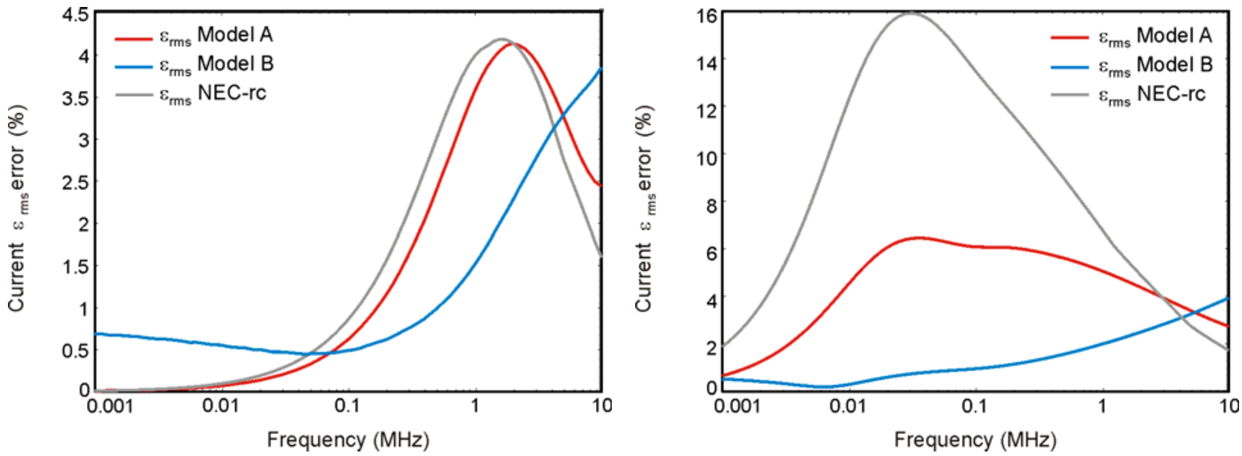


Fig.3. Current rms error obtained by approximate models A and B: conductor length $L$ = 10-m, depth $D$ = 1 m, $\varepsilon$ = 0.01 S/m (left), and $L$ = 100-m, $D$ = 1 m, $\sigma$ = 0.01 S/m (right)

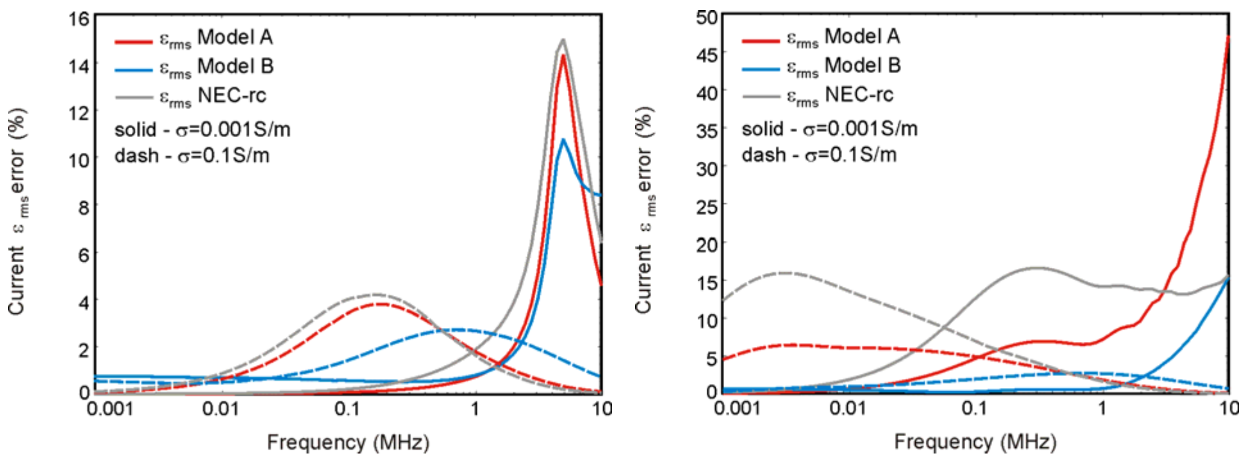


Fig.4. Influence of the conductor depth on the $\varepsilon_{rms}$ error obtained for $D$ = 0.5 m and $D$ = 1.5 m ($L$ = 10-m - left) and ($L$ = 10-m - right)

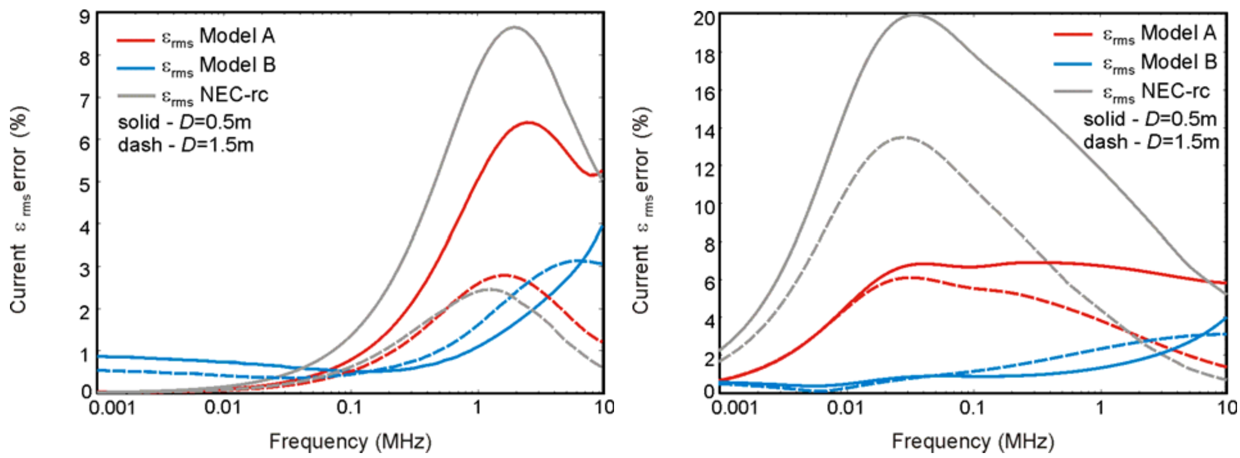


Fig.5. Influence of the soil conductivity on the $\varepsilon_{rms}$ error obtained for $\sigma$ = 0.001 S/m and $\sigma$ = 0.1 S/m ($L$ = 10-m - left) and ($L$ = 10-m - right)

**Conclusion**

In the paper, the authors derive two approximate models of a horizontal wire conductor buried in homogeneous lossy soil. The two models are based on quasi-static image and complex image theory applied to the Green's functions related to this problem. The results of a detailed wide frequency range numerical analysis of a 10-m (short) and a 100-m (long) wire by using various soil parameters may be summarized in:

- The approximate models (A and B) based on image theory introduce differences when calculating the current distribution which is dependent on the frequency. The maximal errors are obtained at the resonant frequency.
- Model A, based on the quasi-static image theory represents generally good approximation with respect to the rigorous EM model in all studied frequency range. However, this model shows limitations due to higher $\varepsilon_{rms}$ error when in cases of low conductivity of the soil and when the conductor is located close to the soil surface. Similarly as NEC reflection coefficient model, the accuracy of this model shows significant dependence on the resonant frequency.
- Model B, based on the quasi-static complex image theory represents very good approximation with respect to the rigorous EM model in wide frequency range. The high accuracy of this model with max 5% $\varepsilon_{rms}$ error is confirmed for all studied cases. The only exception concerns the case of low conductive soil (here $\sigma$ = 0.001 S/m) when the $\varepsilon_{rms}$ error exceeds 5% at very high frequencies, above few MHz.

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