Iurii A. SIROTIN
National Technical University “KPI”, Ukraine

Non-pulsed mode of supply in a three-phase system at asymmetrical voltage

Abstract*. Two orthogonal decompositions of a total three-phase current are considered for a three-phase system with asymmetric sinusoidal voltage at the connection point of asymmetrical loading. In the first one the total current is represented as an orthogonal sum of balanced and non-balanced current components, the balanced component containing Fryze's active current (active power). In the second decomposition the total current is represented as an orthogonal sum of pulsed and non-pulsed current components. Both decompositions are used to create the optimal mode of supply with a constant instantaneous power at asymmetrical voltage.*

Streszczenie. *Analizowano ortogonalna dekompozycję prądów trójfazowych w systemie z napięciem asymetrycznym dołączonym do asymetrycznego obciążenia. Prąd całkowity jest sumą składowej zrównoważonej i niezrównoważonej. (**Bezimpulsowe zasilanie w systemie trójfazowym asymetrycznym**)*

Keywords: three-phase system, instantaneous power; active and reactive, complex, apparent power; pulsation power, unbalanced load, power equation, power factor, unbalanced mode, asymmetrical voltage, compensation
Słowa kluczowe: sieć trójfazowa, moce chwilowe, składowa czynna I bierna mocy chwilowej

Introduction

The energy transfer from a source with symmetrical sinusoidal voltage to a symmetric linear load of three-phase system occurs at a constant rate. Instantaneous power (IP) has no oscillatory (pulsating) component and is equal to the average (active) power: the mode is non–pulsed [1]. Normal operation of generators and motors (without fluctuations and vibrations) requires the non–pulsed mode.

In order to balance the asymmetrical load and create the balanced (non-pulsed) mode under symmetrical voltage the method of compensating the pulsating power (PP) is used [1]. However, under *asymmetrical* voltage the concepts of balanced mode [2] and of non-pulsed mode do not coincide [3]. Under asymmetrical voltage the compensation of PP does not provide the balanced mode. After compensation, the IP of source circuit becomes constant, but it is not equal to the total active power of the load. Under *asymmetrical* voltage the PP compensator requires to apply active means for generating the energy of active power. It remains uncertain whether the statement: «…*for general periodic voltages and currents parallel compensation can be designed such that the power delivered by the supply is constant*» [4] is true.

It is possible to create a non-pulsed mode having IP equal to the initial active power by using Fryze’s active current [5]. In terms of energy, Fryze’s active current is determined unambiguously: "it supplies energy for the given voltage with the same (active) power as the total current and with minimal losses". After Fryze’s compensation the power factor (PF) is equal to 1 [6]. Under asymmetrical sinusoidal voltage the Fryze’s compensator eliminates additional losses completely by compensating the unbalanced current and the reactive current of the load; it also reduces the pulsations significantly, but not in full measure (IP is remains non-constant) [6].

Our objective is to create a non-pulsed mode which provides the supply energy with the initial active power along with minimal loss at asymmetrical voltage.

Energy processes in a sinusoidal mode

At the connection point of the consumer loading to the distribution network in a three-wire section $< a,b,c >$ of the three-phase system the voltage and current sinusoidal waveforms

(1) $$\boldsymbol{i}(t) = (i_a(t)\ i_b(t)\ i_c(t))^\top = \sqrt{2}\Re e[\boldsymbol{I}e^{j\omega t}]$$

(2) $$\boldsymbol{u}(t) = (u_a(t)\ \ u_b(t)\ u_c(t))^\top = \sqrt{2}\Re e[\boldsymbol{U}e^{j\omega t}]$$

are completely defined by three-dimensional complex vectors (the voltage 3d-phasor $\boldsymbol{U} = (\dot{U}_a\ \dot{U}_b\ \dot{U}_c)^\top$ and current 3d-phasor $\boldsymbol{I} = (\dot{I}_a\ \dot{I}_b\ \dot{I}_c)^\top$) – complex rms vectors:

(3) $$\boldsymbol{U} = \frac{\sqrt{2}}{T}\int_0^T \boldsymbol{u}(t)e^{-j\omega t}dt\ , \qquad \boldsymbol{I} = \frac{\sqrt{2}}{T}\int_0^T \boldsymbol{i}(t)e^{-j\omega t}dt\ .$$

Here and further $\top$ is the transposition symbol, T denotes the period ($T\omega = 2\pi$). The rms values of (1-2) are equal to the voltage and current 3d-phasor norms

(4) $$I = |\,\boldsymbol{I}\,| = \sqrt{|\,\dot{I}_a\,|^2 + |\,\dot{I}_b\,|^2 + |\,\dot{I}_c\,|^2}\ ,$$

(5) $$U = |\,\boldsymbol{U}\,| = \sqrt{|\,\dot{U}_a\,|^2 + |\,\dot{U}_b\,|^2 + |\,\dot{U}_c\,|^2}\ .$$

In sinusoidal operating mode (under sinusoidal conditions) the instantaneous power (IP)

(6) $$p(t) = u_a(t)i_b(t) + u_b(t)i_b(t) + u_c(t)i_c(t)$$

can be expressed as

(7) $$p(t) = \Re e[\dot{S} + \dot{N}e^{j2\omega t}] = P + \Re e[\dot{N}e^{j2\omega t}]\ .$$

The standard complex power (SCP) of the sinusoidal operating mode

(8) $$\dot{S} = \dot{U}_a I_a^* + \dot{U}_b I_b^* + \dot{U}_c I_c^* = \boldsymbol{U}^\top \boldsymbol{I}^*$$

is equal to the complex scalar product of the voltage and current 3d-phasors [3]

(9) $$\dot{S} = \boldsymbol{U}^\top \boldsymbol{I}^* = (\boldsymbol{U}, \boldsymbol{I})\ .$$

Here, the asterisk (*) denotes the complex conjugation operation. SCP is the complex number $\dot{S} = P + jQ$. The real part of SCP is the average (active) power during the interval of observation $[\tau, \tau + T]$

(10) $$\Re e\dot{S} = P = \frac{1}{T}\int_\tau^{\tau+T} p(t)dt\ .$$

The imaginary part of the SCP is equal to the reactive power $\Im m\dot{S} = Q$.

The complex amplitude of pulsing power -pulsation power (PP)

(11) $\dot{N} = \dot{U}_a \dot{I}_a + \dot{U}_b \dot{I}_b + \dot{U}_c \dot{I}_c = \boldsymbol{I}^\top \boldsymbol{U} = \boldsymbol{I}^\top (\boldsymbol{U}^*)^* = (\boldsymbol{I}, \boldsymbol{U}^*)$

is the complex inner product [3] the current vector and $\boldsymbol{U}^* = \begin{bmatrix} U_a^* & U_b^* & U_c^* \end{bmatrix}^\top$ - the complex-conjugated (CC) voltage vector. The following statements are equivalent $p(t) \equiv P \quad \Leftrightarrow \quad N = 0$.

Buchholz's apparent power and geometric power are defined as [2]

(12) $S_B = U \cdot I = |\boldsymbol{U}||\boldsymbol{I}|$, $\quad S_G = |\dot{S}| = \sqrt{P^2 + Q^2}$

The Cauchy - Schwarz inequality

(13) $|(\boldsymbol{U}, \boldsymbol{I})| \le |\boldsymbol{U}||\boldsymbol{I}|$, $\qquad S_G \le S_B$

provides the estimate for PF $\lambda = P/S_B \le 1$.

Balanced and unbalanced mode

In (13) equality is attained only when the total current vector (CV) is (*complex*) proportional to the voltage ($\boldsymbol{I} \| \boldsymbol{U}$)

(14) $\boldsymbol{I} = \dot{Y}_S \boldsymbol{U} \quad \Leftrightarrow \quad \dfrac{\dot{I}_a}{\dot{U}_a} = \dfrac{\dot{I}_b}{\dot{U}_b} = \dfrac{\dot{I}_c}{\dot{U}_c} = \dot{Y}_S$, ($\dot{Y}_S = |\dot{Y}_S| e^{j\varphi_S}$).

Condition (14) defines a balanced mode [2, 3]. If the load is symmetric, the mode is balanced. For a 4-wire circuit the concepts of balance and load symmetry are equivalent.
In the 3-wire circuit the definition of a balanced mode implies that the measured voltages do not contain 0-sequence (e.g., line voltages are measured with respect to an artificial grounding point [2].) In a 3-wire circuit, the mode can be balanced under asymmetrical load too. Thus, the Steinmetz's symmetrization scheme [7] has an asymmetric load, but provides a balanced mode (power factor $\lambda = P/S_B = 1$).

Balanced mode is characterized completely by the complex power ($S_G = |\dot{S}| = S_B$). SCP provides the abbreviated power equation and PF is calculated by the phase shift between voltage and current vectors (the voltage and current 3d-phasor)

(15) $S_B^2 = P^2 + Q^2 \quad \Rightarrow \quad \lambda = \dfrac{P}{S_B} = \dfrac{P}{\sqrt{P^2 + Q^2}} = \cos\varphi_S$.

If the mode is *unbalanced*, then the orthogonal projection of the total current vector (CV) on the voltage vector (VV) determines the balanced CV

(16) $\boldsymbol{I_b} = \dfrac{\boldsymbol{I}^\top \boldsymbol{U}^*}{|\boldsymbol{U}|^2} \boldsymbol{U} = \underbrace{(S^*/U^2)}_{\dot{Y}_S} \boldsymbol{U} = \dot{Y}_S \boldsymbol{U}$.

The *unbalanced* CV $\boldsymbol{I}_u$ is the orthogonal complement of the balanced CV on the total CV. We have the orthogonal decomposition of the current

(17) $\boldsymbol{I} = \boldsymbol{I}_b + \underbrace{(\boldsymbol{I} - \boldsymbol{I_b})}_{I_u} = \boldsymbol{I}_b + \boldsymbol{I}_u$.

Orthogonal decomposition of the total current (17) gives us the Pythagorean theorem for currents and the equivalent power equation for the unbalanced mode

(18) $|\boldsymbol{I}|^2 = |\boldsymbol{I_b}|^2 + |\boldsymbol{I_u}|^2$. $\Leftrightarrow \quad S_B^2 = S_G^2 + D_u^2$,

because [3]

(19) $|\boldsymbol{I_b}|^2 |\boldsymbol{U}|^2 = |\dot{S}|^2 = S_G^2$, $\qquad |\boldsymbol{I_u}|^2 |\boldsymbol{U}|^2 = D_u^2$.

Here $D_u = |\boldsymbol{D}|$ is the norm of imbalanced power vector $\boldsymbol{D} = \boldsymbol{U} \times \boldsymbol{I}$ (the vector cross product of the total current on the voltage); $\times$ - is the sign of vector product.

Fryze' active current in a sinusoidal mode

Balanced current vector is complex collinear to the ort of the VV $\boldsymbol{m}^\top = \boldsymbol{U}/U = U^{-1} \begin{bmatrix} \dot{U}_a & \dot{U}_b & \dot{U}_c \end{bmatrix}^\top$

(20) $\boldsymbol{I_b} = \dfrac{\boldsymbol{I}^\top \boldsymbol{U}^*}{|\boldsymbol{U}|^2} \boldsymbol{U} = \underbrace{(S^*/U)}_{\dot{I}_b} \underbrace{\boldsymbol{U}/U}_{\boldsymbol{m}} = \dot{I}_b \boldsymbol{m}$

The complex collinearity coefficient

(21) $\dot{I}_b = \dfrac{S^*}{|\boldsymbol{U}|} = \underbrace{P/U}_{I_a} + j \underbrace{(-Q/U)}_{I_r} = I_a + jI_r$,

defines active and reactive CV

(22) $\boldsymbol{I_{aF}} = I_a \boldsymbol{m} = \dfrac{P}{|\boldsymbol{U}|^2} \boldsymbol{U}$, $\quad \boldsymbol{I_r} = jI_r \boldsymbol{m} = \dfrac{Qe^{-j\pi/2}}{|\boldsymbol{U}|^2} \boldsymbol{U}$

and provides a complete decomposition of the total CV to the Fryze's active and inactive current vectors

(23) $\boldsymbol{I} = \boldsymbol{I_{aF}} + \underbrace{\boldsymbol{I_r} + \boldsymbol{I_u}}_{\text{inactive curent Fryze}} = \boldsymbol{I_{aF}} + \boldsymbol{I_F}$, $\qquad \boldsymbol{I_F} = \boldsymbol{I_r} + \boldsymbol{I_u}$.

In the unbalanced mode, the power equation has three components, and PF is calculated in terms of components of the Fryze' expansion (23) [2]

(24) $S_B^2 = P^2 + Q^2 + D_u^2$,

(25) $\lambda = \dfrac{P}{S_B} = \dfrac{P}{\sqrt{P^2 + Q^2 + D_u^2}} = \dfrac{|\boldsymbol{I_{aF}}|}{\sqrt{|\boldsymbol{I_{aF}}|^2 + |\boldsymbol{I_F}|^2}}$.

Fryze's active current is the solution of a conditional extremum problem with the objective function being equal to the heat loss (by 1 ohm) $F(\boldsymbol{I}) = |\boldsymbol{I}|^2$ with constraint $\Re e(\boldsymbol{U}, \boldsymbol{I}) = \Re e(\boldsymbol{U}^\top \boldsymbol{I}^*) = P$. Active current $\boldsymbol{I}_{aF}$ supplies energy with minimal losses, its active power equals that of the total current.

Pulsing mode and pulsed current

We get another orthogonal decomposition of the total CV, which is associated with IP pulsations. It follows from (11) that in the case when the total CV is orthogonal to the CC vector $\boldsymbol{U}^*$, the pulsations are absent

(26) $\boldsymbol{I} \perp \boldsymbol{U}^* \qquad \Leftrightarrow \qquad \dot{N} = (\boldsymbol{I}, \boldsymbol{U}^*) = 0$.

This mode is called non-pulsed. If the mode is pulsed ($\dot{N} \ne 0$), then the orthogonal projection of the total CV on the CC voltage vector defines the 3d-component of the current

(27) $\boldsymbol{I_p} = \dfrac{(\boldsymbol{I}, \boldsymbol{U}^*)}{(\boldsymbol{U}, \boldsymbol{U}^*)} \boldsymbol{U}^* = \dfrac{\dot{N}}{|\boldsymbol{U}|^2} \boldsymbol{U}^* = \underbrace{(\dot{N}/U)}_{\dot{I}_p} \underbrace{\boldsymbol{U}^*/U}_{\boldsymbol{m}^*} = \dot{I}_p \boldsymbol{m}^*$.

The current component thus introduced has the same PP as the total current

(28) $(\boldsymbol{I_p}, \boldsymbol{U}^*) = \dfrac{\dot{N}}{U^2} (\boldsymbol{U}^*)^\top \boldsymbol{U} = \dfrac{\dot{N}}{U^2} \underbrace{\boldsymbol{U}^\top \boldsymbol{U}^*}_{U^2} = \dot{N}$.

So, the current component (27) (being the orthogonal projection) has the minimal norm (the rms value) among all currents, which have the same PP as the total CV at the given voltage. Component (27) is called *pulsed current* [3]. The orthogonal complement to the total current $\boldsymbol{I}_n = \boldsymbol{I} - \boldsymbol{I}_p$ of the introduced current components (27) does not cause pulsation and is called *non-pulsed current*. It is clear that $(\boldsymbol{I}_n, \boldsymbol{U}^*) = \boldsymbol{I}_n^\top \boldsymbol{U} = 0$. We have the orthogonal decomposition of the total current

(29) $$\boldsymbol{I} = \boldsymbol{I}_p + \boldsymbol{I}_n = \boldsymbol{I}_p + (\boldsymbol{I} - \boldsymbol{I}_p) .$$

The results obtained are valid both for 3-wire and 4-wire circuits. At asymmetric voltage the balance and non-pulsed modes do not match.

The two-dimensional subspace of the energy processes in the three-wire circuit

In a three-wire system the voltage measurements relative to artificial grounding point, together with the Kirchhoff's first law, give [2]

(30) $$\dot{U}_a + \dot{U}_b + \dot{U}_c = 0 , \qquad \dot{I}_a + \dot{I}_b + \dot{I}_c = 0 .$$

Thus, the sinusoidal energy processes in the three-wire section $<a,b,c>$ of the 3-wire circuit, characterized by 3d-phasors $\boldsymbol{X}$, which are orthogonal to the zero-sequence ort $\boldsymbol{e_0} = (1,1,1)^\top / \sqrt{3}$:

(31) $$(\boldsymbol{X}, \boldsymbol{e_0}) = \boldsymbol{X}^\top \boldsymbol{e_0} = (\dot{X}_a + \dot{X}_b + \dot{X}_c)/\sqrt{3} = 0 .$$

These 3d-phasors form a two-dimensional subspace. The positive (PS) and negative sequence (NS) orts

(32) $$\boldsymbol{e_1} = \frac{1}{\sqrt{3}}(1, \alpha^*, \alpha)^\top , \qquad \boldsymbol{e_2} = \frac{1}{\sqrt{3}}(1, \alpha, \alpha^*)^\top ,$$
$$(\alpha = e^{j2\pi/3} ,\ 1 + \alpha + \alpha^* = 0 ,\ \alpha^2 = \alpha^* ,\ \alpha\alpha^* = 1)$$

define an orthonormal basis of this subspace [3].

Any vector satisfying (31) is uniquely represented by their symmetrical coordinates in this basis. In particular, the orthogonal decomposition of voltage and current vectors in the basis (32)

(33) $$\boldsymbol{U} = \boldsymbol{U_1} + \boldsymbol{U_2} = \dot{U}_1 \boldsymbol{e_1} + \dot{U}_2 \boldsymbol{e}_2 ,$$

(34) $$\boldsymbol{I} = \boldsymbol{I_1} + \boldsymbol{I_2} = \dot{I}_1 \boldsymbol{e_1} + \dot{I}_2 \boldsymbol{e}_2$$

defines the SCP and PP in symmetric coordinates

(35) $$\dot{S} = \dot{U}_1 I_1^* + \dot{U}_2 I_2^* , \qquad \dot{N} = \dot{U}_1 \dot{I}_2 + \dot{U}_2 \dot{I}_1$$

SCP and PP are defined as the PS and NS currents.

Orthonormal bases of losses and pulsations

The basis of symmetric coordinates is not unique. Let us define two bases, for which we shall analyze the losses and pulsations. The 3d-phasor of interphase voltages $\boldsymbol{U}_\triangledown = \sqrt{3}(\boldsymbol{U} \times \boldsymbol{e_0})$ orthogonal to the CC voltage 3d-phasor

(36) $$(\boldsymbol{U}_\triangledown, \boldsymbol{U}^*) = \boldsymbol{U}_\triangledown^\top \boldsymbol{U} = \sqrt{3}(\boldsymbol{U} \times \boldsymbol{e_0})^\top \boldsymbol{U} = 0 .$$

This allows us to define two orthonormal bases [2]: the basis of losses and CC to it the basis of pulsations

(37) $$\mathcal{B} = \{\boldsymbol{m}, \boldsymbol{m}_\triangledown\} , \qquad \mathcal{B}^* = \{\boldsymbol{m}^*, \boldsymbol{m}_\triangledown{}^*\} .$$

Here $\boldsymbol{m} = \boldsymbol{U}/|\boldsymbol{U}|$ and $\boldsymbol{m}_\triangledown = \boldsymbol{m}^* \times \boldsymbol{e_0}$ are orts of phase and CC interphase voltage vectors; $\boldsymbol{m}^* = \boldsymbol{U}^*/|\boldsymbol{U}^*|$ and $\boldsymbol{m}_\triangledown{}^* = \boldsymbol{m} \times \boldsymbol{e_0}$ are orts of CC phase and interphase VV.

Orts of these bases are related by vector-matrix relations

(38) $$\begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{m}_\triangledown \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{\eta} & \dot{\mu} \\ \mu^* & \dot{\eta} \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} \boldsymbol{m}_\triangledown^* \\ \boldsymbol{m}^* \end{bmatrix} , \quad \begin{bmatrix} \boldsymbol{m}_\triangledown^* \\ \boldsymbol{m}^* \end{bmatrix} = \underbrace{\begin{bmatrix} -\dot{\eta} & \dot{\mu} \\ \mu^* & -\dot{\eta} \end{bmatrix}}_{\mathcal{A}^{-1}} \begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{m}_\triangledown \end{bmatrix} ;$$

where the complex numbers $\dot{\mu} = (\boldsymbol{m}, \boldsymbol{m}^*) = \boldsymbol{m}^\top \boldsymbol{m}$, $\mu^* = \boldsymbol{m}_\triangledown^\top \boldsymbol{m}_\triangledown = (\boldsymbol{m}^\top \boldsymbol{m})^*$, $\dot{\eta} = \boldsymbol{m}_\triangledown^\top \boldsymbol{m} = \boldsymbol{m}^\top \boldsymbol{m}_\triangledown$, and $\eta^* = -\dot{\eta}$ satisfy the condition $|\dot{\mu}|^2 + |\dot{\eta}|^2 = 1$.

Nondimensional complex value

(39) $$\eta = (1 - k_{U2}^2)/(1 + k_{U2}^2)$$

characterizes the degree of asymmetry of VV. Here $k_{U2} = U_1/U_2$ is the voltage unbalance factor (VUF) for NS. If the voltage is symmetrical PS $\boldsymbol{U} = \dot{U}_1 \boldsymbol{e}_1$ then $\dot{\mu} = 0$, $\dot{\eta} = j$, and within a phase factor the entered orts (37) coincide with the orts of PS and NS (32)

(40) $$\boldsymbol{m} = \boldsymbol{e_1} , \quad \boldsymbol{m}_\triangledown = j\boldsymbol{e_2} , \quad \boldsymbol{m}^* = (\boldsymbol{e_1})^* = \boldsymbol{e_2} , \quad \boldsymbol{m}_\triangledown^* = -j\boldsymbol{e_1} .$$

Decomposition of the total current in the bases of losses and pulsations

In the basis of losses $\mathcal{B} = \{\boldsymbol{m}, \boldsymbol{m}_\triangledown\}$ the CV expansion

(41) $$\boldsymbol{I} = \boldsymbol{I_b} + \boldsymbol{I_u} = \dot{I}_b \boldsymbol{m} + \dot{I}_u \boldsymbol{m}_\triangledown = \begin{bmatrix} \dot{I}_b & \dot{I}_u \end{bmatrix} \begin{bmatrix} \boldsymbol{m} \\ \boldsymbol{m}_\triangledown \end{bmatrix} .$$

is defined by the coordinates

(42) $$\dot{I}_b = \boldsymbol{I}^\top \boldsymbol{m}^* = S^*/U , \qquad \dot{I}_u = \boldsymbol{I}^\top \boldsymbol{m}_\triangledown^* = \dot{D}_0/U .$$

Here $\dot{D}_0 = (\boldsymbol{I} \times \boldsymbol{U})^\top \boldsymbol{e_0}$ is the projection of the imbalance power vector $\boldsymbol{D} = \boldsymbol{U} \times \boldsymbol{I}$ on ort $\boldsymbol{e_0}$ [3].

The expansion (41) is characterized by the pair of complex power $S^* = \dot{I}_b U$, $\dot{D}_0 = \dot{I}_u U$, and gives a quadratic expansion of the apparent power (equation of losses)

(43) $$|\boldsymbol{I}|^2 = |\boldsymbol{I_b}|^2 + |\boldsymbol{I_u}|^2 \Leftrightarrow S_B^2 = |S^*|^2 + |\dot{D}_0|^2$$

The basis of pulsations $\mathcal{B}^*$ determines the decomposition of the total current to the non-pulsed and pulsed currents

(44) $$\boldsymbol{I} = \boldsymbol{I_p} + \boldsymbol{I_n} = \dot{I}_p \boldsymbol{m}^* + \dot{I}_n \boldsymbol{m}_\triangledown^* = \begin{bmatrix} \dot{I}_p & \dot{I}_n \end{bmatrix} \begin{bmatrix} \boldsymbol{m}^* \\ \boldsymbol{m}_\triangledown^* \end{bmatrix} .$$

The coordinates of the expansion (44)

(45) $$\dot{I}_p = (\boldsymbol{I}, \boldsymbol{m}^*) = \boldsymbol{I}^\top \boldsymbol{m} , \qquad \dot{I}_n = (\boldsymbol{I}, \boldsymbol{m}_\triangledown^*) = \boldsymbol{I}^\top \boldsymbol{m}_\triangledown$$

determine phasors of pulsed power and non-pulsed power

(46) $$\dot{N} = \dot{I}_p \cdot U , \qquad \dot{K} = \dot{I}_n \cdot U .$$

Orthogonal decomposition of the total current (44) gives the Pythagorean theorem for the currents and the equivalent power equation for the pulsed mode [3]

(47) $$|\boldsymbol{I}|^2 = |\boldsymbol{I_p}|^2 + |\boldsymbol{I_n}|^2 , \Leftrightarrow S_B^2 = |\dot{N}|^2 + |\dot{K}|^2 .$$

Under an asymmetric voltage equations (43) and (47) do not coincide. For the introduced bases (37) the matrices

(48) $$\mathcal{A}^\top = \begin{bmatrix} \dot{\eta} & \mu^* \\ \dot{\mu} & \dot{\eta} \end{bmatrix} \qquad \mathcal{A}^* = \begin{bmatrix} -\dot{\eta} & \mu^* \\ \dot{\mu} & -\dot{\eta} \end{bmatrix}$$

determine the transformations between the coordinates of 3d-phasors without 0–sequence.

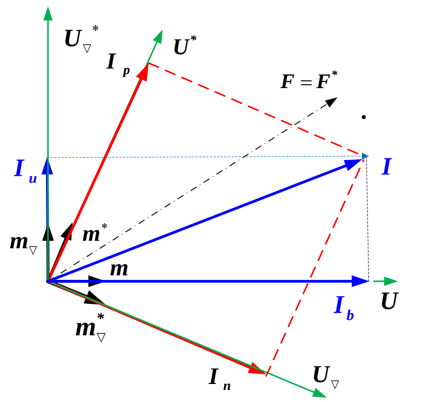

Figure 1. Decomposition of the total CV in the bases of losses and pulsations

In the bases (37) coordinates of the CV are related by vector - matrix relations

(49) $$\begin{bmatrix}\dot I_n\\ \dot I_p\end{bmatrix} = \underbrace{\begin{bmatrix}-\dot\eta & \mu^*\\ \dot\mu & -\dot\eta\end{bmatrix}}_{A^*}\begin{bmatrix}\dot I_b\\ \dot I_u\end{bmatrix}, \quad \begin{bmatrix}\dot I_b\\ \dot I_u\end{bmatrix} = \underbrace{\begin{bmatrix}\dot\eta & \mu^*\\ \dot\mu & \dot\eta\end{bmatrix}}_{A^\top}\begin{bmatrix}\dot I_n\\ \dot I_p\end{bmatrix}.$$

Matrices (48) connect the corresponding pairs of powers (S^*, $\dot D_0$) and ($\dot N$, $\dot K$) too [3].

Straightforward PP compensation

According to the PP method [1] the load current is divided into the current compensator and the current source

(50) $$\boldsymbol{I}_L = \boldsymbol{I}_S + \boldsymbol{I}_C \qquad (\boldsymbol{I}_C = \boldsymbol{I}_p,\ \boldsymbol{I}_S = \boldsymbol{I}_n).$$

in correspondence with the orthogonal decomposition (44). Using the matrix-vector contact (49), we get the representation of the load current (in coordinate form) in the basis of losses as

(51) $$\underbrace{\begin{bmatrix}\dot I_b\\ \dot I_u\end{bmatrix}}_{I_L} = \underbrace{\begin{bmatrix}\eta^*\dot I_n\\ \dot\mu\dot I_n\end{bmatrix}}_{I_S} + \underbrace{\begin{bmatrix}\mu^*\dot I_p\\ \eta^*\dot I_p\end{bmatrix}}_{I_C}$$

Compensator current

(52) $$\boldsymbol{I}_C = \begin{bmatrix}(\boldsymbol{I}_C)_b\\ (\boldsymbol{I}_C)_u\end{bmatrix} = \begin{bmatrix}\mu^*\dot I_p\\ \eta^*\dot I_p\end{bmatrix}$$

contains balanced and unbalanced components

(53) $$(\boldsymbol{I}_C)_b = \mu^*\dot I_p \qquad (\boldsymbol{I}_C)_u = \eta^*\dot I_p.$$

The complex power of compensator current is equal to

(54) $$S_C^* = (\boldsymbol{I}_C)_b\cdot U = \mu^*\underbrace{\dot I_p U}_{\dot N} = \mu^*\cdot\dot N.$$

In the symmetrical components factors (54) are calculated as

$$\mu^* = (\boldsymbol{m}^\top\boldsymbol{m})^* = 2U_1^*U_1^*/U^2, \qquad \dot N = \dot I_2\dot U_1 + \dot I_1\dot U_2,$$

that for the complex power of compensator current gives

(55) $$S_C^* = \underbrace{\frac{2(P_1k_{U2}^2 + P_2)}{1+k_{U2}^2}}_{P_C} - j\underbrace{\frac{2(Q_1k_{U2}^2 + Q_2)}{1+k_{U2}^2}}_{Q_C}$$

Here $\dot S_1 = \dot U_1\dot I_1^* = P_1 + jQ_1$, $\dot S_2 = \dot U_2\dot I_2^* = P_2 + jQ_2$ are the complex powers of the PS and NS of initial mode.

In the circuit considered, active powers must be balanced. In order to provide the initial energy consumption for the load with the active power (in the new mode)

$$P_L = P_1 + P_2,$$

the PP compensator must generate energy with active power

(56) $$P_C = P_L - P_S = \frac{2(P_1k_{U2}^2 + P_2)}{1+k_{U2}^2}$$

in addition to the source energy having the active power

$$P_S = (P_1 - P_2)\eta = (P_1 - P_2)(1-k_{U2}^2)/(1+k_{U2}^2).$$

Optimum PP compensation

Active Fryze current $\boldsymbol{I}_{aF} = I_a\boldsymbol{m}$ supplies energy to the load with the same (active) power as the total current and with minimal losses in source circuit.

However, under asymmetrical voltage IP of active current it contains a pulsating component $(\boldsymbol{I}_{aF})_p = (\boldsymbol{I}_{aF}^\top\boldsymbol{m})\boldsymbol{m}^*$ ($\boldsymbol{I}_{aF}^\top\boldsymbol{m} = \dot\mu P/U \neq 0$) [3].

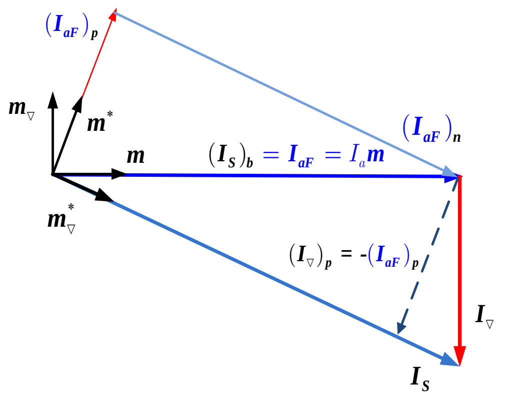

Figure 2. Addition of the unbalanced current to the active current in order to eliminate pulsation

Let us find such current in source circuit $\boldsymbol{I}_S$ that does not contain any pulse current with its current balanced component being equal to the active current (Fig. 2.)

(57) $$(\boldsymbol{I}_S)_p = (\underbrace{\boldsymbol{I}_S^\top\boldsymbol{m}}_{0})\boldsymbol{m}^* = 0, \qquad (\boldsymbol{I}_S)_b = \boldsymbol{I}_{aF}.$$

We denote the unknown unbalanced component of source current, which is to eliminate the pulsating component of active current, as $(\boldsymbol{I}_S)_u = \boldsymbol{I}_{\nabla} = \dot I_{\nabla}\boldsymbol{m}_{\nabla}$.

The decomposition of this source current in the basis of losses is

(58) $$\boldsymbol{I}_S = (\boldsymbol{I}_S)_b + (\boldsymbol{I}_S)_u = \boldsymbol{I}_{aF} + \boldsymbol{I}_{\nabla} = I_a\boldsymbol{m} + \dot I_{\nabla}\boldsymbol{m}_{\nabla}.$$

From (50) and (51) it follows $\boldsymbol{I}_S^\top\boldsymbol{m} = (\boldsymbol{I}_{aF} + \boldsymbol{I}_{\nabla})^\top\boldsymbol{m} = 0$.

It gives

(59) $$(\dot I_a\boldsymbol{m} + \dot I_{\nabla}\boldsymbol{m}_{\nabla})^\top\boldsymbol{m} = I_a\underbrace{(\boldsymbol{m}^\top\boldsymbol{m})}_{\dot\mu} + \dot I_{\nabla}\underbrace{(\boldsymbol{m}_{\nabla}^\top\boldsymbol{m})}_{\dot\eta} = 0$$

and allows to find the unknown phasor of unbalanced current $\dot I_{\nabla} = -(\dot\mu/\dot\eta)I_a$.

Thus (in the source circuit), the required total CV and the corresponding unbalanced current are uniquely determined by the active current value I_a

(60) $$\boldsymbol{I}_S = \dot I_S\boldsymbol{m}_{\nabla}^* = -(I_a/\dot\eta)\boldsymbol{m}_{\nabla}^*, \quad |\boldsymbol{I}_S| = I_a/\eta;$$

(61) $$\boldsymbol{I}_{\nabla} = \dot I_{\nabla}\boldsymbol{m}_{\nabla} = -\dot I_a(\dot\mu/\dot\eta)\boldsymbol{m}_{\nabla}, \quad |\boldsymbol{I}_{\nabla}| = I_a\,\mu/\eta.$$

The pulsating component of the unbalanced current $(\boldsymbol{I}_{\nabla})_{\boldsymbol{p}}$ we have found is directed oppositely to the pulsating component of the active current

(62) $(\boldsymbol{I}_{\nabla})_{\boldsymbol{p}} = \dot{I}_{\nabla}(\boldsymbol{m}_{\nabla}^{\top}\boldsymbol{m})\boldsymbol{m}^{*} = -\dot{\mu}\dot{I}_{a}\boldsymbol{m}^{*} = -(\boldsymbol{I}_{\boldsymbol{aF}})_{\boldsymbol{p}}$.

The active current $\boldsymbol{I}_{\boldsymbol{aF}} = (\boldsymbol{I}_{\boldsymbol{S}}, \boldsymbol{m})\boldsymbol{m}$ is the orthogonal projection of the non-pulsed source current we have found $\boldsymbol{I}_{\boldsymbol{S}}$ to the ort $\boldsymbol{m}$.

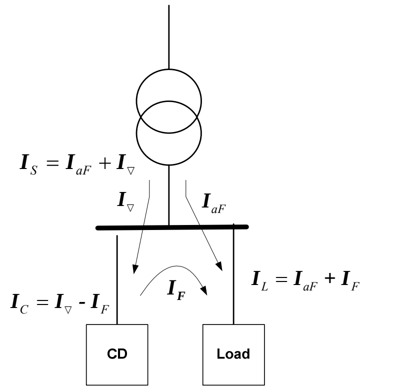

Figure 3. Optimal non-pulsed mode at asymmetrical voltage

The source current contains the active current and additional unbalanced current, which compensates pulsation caused by the active current (Fig. 3).

It follows from (51) that the source circuit current can be decomposed as

(63) $\boldsymbol{I}_{S} = \boldsymbol{I}_{aF} + \boldsymbol{I}_{\nabla} = \underbrace{\boldsymbol{I}_{aF} + \boldsymbol{I}_{\boldsymbol{F}}}_{\boldsymbol{I}_L} + \underbrace{(-\boldsymbol{I}_{\boldsymbol{F}} + \boldsymbol{I}_{\nabla})}_{\boldsymbol{I}_C} = \boldsymbol{I}_{L} + \boldsymbol{I}_{C}$.

The load current is equal to the source circuit current prior to compensation $\boldsymbol{I}_{L} = \boldsymbol{I}_{aF} + \boldsymbol{I}_{\boldsymbol{F}}$ .The current of compensating device (CD) has two components

(63) $\boldsymbol{I}_{\boldsymbol{C}} = -\boldsymbol{I}_{\boldsymbol{F}} + \boldsymbol{I}_{\nabla}$.

The first component of the compensator current

(64) $-\boldsymbol{I}_{\boldsymbol{F}} = -(\boldsymbol{I}_{\boldsymbol{r}} + \boldsymbol{I}_{\boldsymbol{u}}) = -\dot{I}_{r}\boldsymbol{m} - \dot{I}_{u}\boldsymbol{m}_{\nabla}$

compensates the Fryze's non-active current that appears in the circuit of asymmetrical (unbalanced) load.

The other (unbalanced) component

(65) $\boldsymbol{I}_{\nabla} = \dot{I}_{\nabla}\boldsymbol{m}_{\nabla} = I_{a}(\dot{\mu}/\eta^{*})\boldsymbol{m}_{\nabla}$

is introduced into the source circuit for compensating the active current pulsation of IP in the source circuit.

Among all non-pulsed currents that supply energy with the initial active power $P = U \cdot |\boldsymbol{I}_{\boldsymbol{aF}}| = U \cdot I_{a}$ the non-pulsed current $\boldsymbol{I}_{\boldsymbol{S}}$ we have found has a minimum rms value and gives the solution of the conditional extremum problem

$$\boldsymbol{I}_{\boldsymbol{S}} = \arg\min_{I \in \mathcal{J}} |\boldsymbol{I}|^{2}$$

Admissible region of the conditional extremum problem

(66) $\mathcal{J} = \{\boldsymbol{I} \mid (\Re e[\boldsymbol{U}^{\top}\boldsymbol{I}^{*}] = P) \ \& \ (\boldsymbol{I}^{\top}\boldsymbol{U} = 0)\}$

ensures it.

The compensator does not requires additional energy generation by active means. It follows from (60) that after compensation the PF value in the source circuit

(67) $\lambda_{after} = |\boldsymbol{I}_{\boldsymbol{aF}}| / |\boldsymbol{I}_{\boldsymbol{S}}| = \eta = (1 - k_{U2}^{2}) / (1 + k_{U2}^{2})$

does not depend from the load unbalance and is due only to the degree of voltage asymmetry. The PF value can be pre-calculated using VUF. In the range of change of the VUF $k_{U2} \in [0; 5\%]$ the value η differs from unity in the third decimal place. If the pre-calculated PF value $\lambda_{after} = \eta$ is less than required, the voltages should be symmetrized.

The method for calculating the parameters of the Δ – compensator at LC elements for any compensating current (without active current) is proposed in [6, 8].

Conclusions

The compensation method proposed above eliminates completely, for any unbalanced load, the IP pulsation under asymmetrical voltage.

The proposed compensator provides the non-pulsed energy supply with the highest possible PF and with the same average (active) power as the initial current.

The PF value is independent from load unbalance and is determined only by the degree of voltage asymmetry.

REFERENCES

[1] Shidlovskii A.K., Kuznetsov A.G. Improving of the power quality in electrical networks. - Kiev: Naukova Dumka, 1985. –268p. (Rus.)

[2] Czarnecki L.S. ,Powers of asymmetrically supplied loads in terms of the CPC power theory, Electrical Power Quality and Utilization Journal, vol. XI, (2007) , no.1, 97-104.

[3] Sirotin Iu. A. Current, power, and the equation of pulsations in the three-phase system, Вісник НТУ «ХПІ»,(2012),№ 23,146-159.www.nbuv.gov.ua/portal/natural/vcpi/Ente/2012_23/19.pdf. (Rus.)

[4] Willems J. L., Reflections on Power Theories for Poly-Phase Nonsinusoidal Voltages and Currents, *Przeglad Elektrotechniczny,* (2010), No. 6, 11-21.

[5] Fryze S. Active, reactive and apparent power in circuits with nonsinusoidal voltage and current, Przegląd Elektrotechniczny (1931). No. 7–8.

[6] Sirotin Iu. A. Fryze's compensator and Fortescue transformation, *Przeglad Elektrotechniczny,* (2011), No. 1, 101-106. http://pe.org.pl/abstract_pl.php?nid=4568.

[7] Sirotin Iu. A. Steinmetz's symmetrization scheme as a special case of optimal Fryze's compensator , *Electrica,* №1, (2011),16-21. www.kudrinbi.ru/modules.php?name=Biblio&View=20464

[8] S. J. Jeon, J. L. Willems, Reactive power compensation in a multiline system under sinusoidal unbalanced conditions, Int. Journal of Circuit Theory and Appl., 2011, vol. 39, Issue 3, 211–224

Author*: prof. dr inż. Iurii A. Sirotin,,* National Technical University "KPI", str. Frunze, 21, Kharkiv, 61002, Ukraine, *E-mail:* yuri_sirotin@ukr.net