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Directional reflectance and more accurate prediction of tubular light guide efficiency

Abstract. *The main chracteristic of the light guide is the tube's efficiency of the dalylight transport into the building interior. It mainly depends on the reflectance of the inner high-reflective layer. Reflectance of metalic materials is related to their complex refractive index and it depends on the wavelengh of the incident light, on the angle of incidince and on the light polarisation. Using Fresnel equations, we are able to determine directional reflactance of the layer and improve the accuracy of the calculations for the light tube's efficiency in various daylight conditions.*

Streszczenie. *Główną charakterystyką światłowodu jest skuteczność rury wprowadzającej światło dzienne do wnętrza budynku. Zależy ona przede wszystkim od stopnia odbicia strumienia świetlnego wewnętrznej powłoki refleksyjnej. Stopień odbicia powłoki jest powiązany z całkowitym współczynnikiem załamania i zależy od długości fali padającego światła, kąta padania i polaryzacji światła. Wykorzystując równania Fresnela określiliśmy składową odbicia kierunkowego powłoki i poprawiliśmy dokładność obliczeń skuteczności rury świetlnej dla różnych warunków oświetlenia dziennego.* ***(Odbicie kierunkowe i dokładniejsza predykcja skuteczności światłowodu tubularnego)***

Keywords: light tube, directional reflectance, Fresnel equations, refractive index.
Słowa kluczowe: rura świetlna, odbicie kierunkowe, równania Fresnela, współczynnik załamania.

Introduction

Daylight is a commodity which can be used free of charge and helps us save electric energy for illumination in indoor spaces. If the deficiency of the natural daylight occurs in the building interior, several devices and daylight technologies which control and redistribute daylight to desired places can be used. One of the novel technology is the tubular light guide system consisting of a cupola, tube with high reflective inner surface and diffuser. Mirror light tubes are used commercially to transfer daylight from the roof or facade of a building to the deep interiors. Light guides installed in the roofs transmit light under very different daylight conditions, with various sky luminance distribution and sunlight intensity during a day and the whole year. The inner layers of the tubular light guides consists mainly of the thin silver or aluminium films, which ensure the high reflectivity. Light guide manufacturers often declare the inner reflectivity value as the normal reflectivity of the surface. However, light rays inside the tube fall to the inner surface at the oblique angle of incidence. Reflectivity of the opaque metals depends on the angle of incidence including a wavelength dependency, therefore, the recent calculations of the light guide efficiencies are very approximate and directional reflectance has to be taken into account [1]. Manufacturers also declare incomprehensibly high and unreal values of the inner reflectance of the light tubes. Numerical solutions based on the Fresnel equations how that the unreal reflectivity values are reachable for the common materials only under the specific physical conditions.

Theoretical methodology

The optical constants of an isotropic material are the index of refraction n and the extinction coefficient k. They are the real and imaginary components of the complex index of refraction $m = n + ik$. They can be measured at a given wavelength by direct methods or inferred from photometric or polarimetric measurements. Real and imaginary part of the refractive index depend on the wavelength of the incident light. Wavelength dependencies of the refractive indices of silver and aluminium, which are commonly used in the light tube production are presented in Fig. 1 and 2 [2]. The light beam transiting through the light tube is reflecting in accordance with the Snell's law which maps the angle of refracted beam β with the angle of incident beam α through the refractive indices m_1, m_2 of both media:

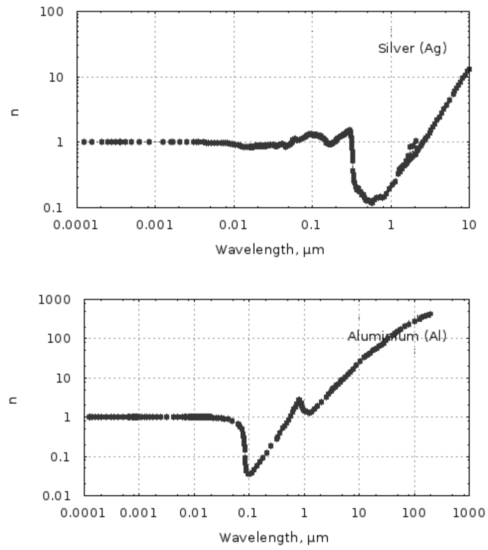

Fig. 1 Dependence of the real part of the refractive index on the wavelength for silver and aluminium material.

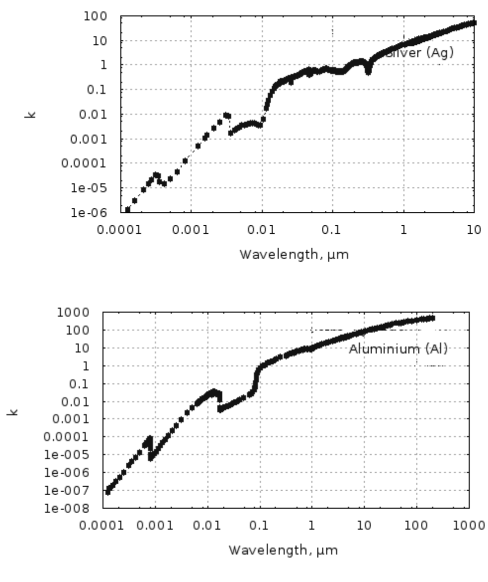

Fig. 2 Dependence of the imaginary part of the refractive index on the wavelength for silver and aluminium material.

(1) $$\frac{\sin\alpha}{\sin\beta} = \frac{m_2}{m_1}$$

For metals and other opaque media the normal incidence reflectance ρ is given by:

(2) $$\rho = \frac{(n_1 - n_2)^2 + k_1^2}{(n_1 + n_2)^2 + k_1^2}$$

where n_2 is the refractive index of the entrance medium (in our case air) and n_1 and k_1 are the real and imaginary parts of the complex refractive index $m_1 = n_1 + ik_1$ of the absorbing medium. The terms n_1 and k_1 are usually known as the refractive index and absorption index respectively.

However, calculation of the reflection coefficient depends also on the polarization of the incident ray and the angle of incidence of the light beam. (see e.g. [3]). If the light is s-polarised, the reflection coefficient is given by:

(3) $$\rho_S = \left(\frac{m_1\cos\alpha - m_2\cos\beta}{m_1\cos\alpha + m_2\cos\beta}\right)^2$$

If the light is p-polarised, the reflection coefficient is given by:

(4) $$\rho_P = \left(\frac{m_1\cos\beta - m_2\cos\alpha}{m_1\cos\beta + m_2\cos\alpha}\right)^2$$

Expressing the absolute value of the complex refractive index, the specular reflectance of a material at an angle of incidence α is related to the index of refraction as follows [2]:

$$\rho_S(\alpha) = \frac{(a - \cos\alpha)^2 + b^2}{(a + \cos\alpha)^2 + b^2}$$

(5)

$$\rho_P(\alpha) = \rho_S(\alpha)\left[\frac{(a - \sin\alpha\tan\alpha)^2 + b^2}{(a + \sin\alpha\tan\alpha)^2 + b^2}\right]$$

Even if the scattered monochromatic radiation shows the polarization features, the natural (visible) light is assumed to be unpolarised, so the reflection coefficient is

(6) $$\rho(\alpha) = \frac{1}{2}[\rho_S(\alpha) + \rho_P(\alpha)]$$

where for the coefficients *a*, *b*, the equations are valid:

$$a^2 = \frac{1}{2}\left[\sqrt{\left(n^2 - k^2 - \sin^2\alpha\right) + 4n^2k^2} + (n^2 - k^2 - \sin^2\alpha)\right]$$

(7)

$$b^2 = \frac{1}{2}\left[\sqrt{\left(n^2 - k^2 - \sin^2\alpha\right) + 4n^2k^2} - (n^2 - k^2 - \sin^2\alpha)\right]$$

Numerical solutions of the Fresnel equations presented in this paper by the Eqs. (6) - (7) are shown in Fig. 3 for the commonly used materials in the light tube construction, aluminium and silver for the incoming radiation 550 nm, which is the value of maximum sensitivity of the human eye. Reflectance at the normal incidence for aluminium is 91,54% and 95,97 % for silver, respectively.

For aluminium, the reflectance for the unpolarised light gradually decreases to the angle of incidence 80^o and thereafter increases rapidly. Reflectance of the silver is more stable, but the variation is also not negligible. This facts clearly indicate that the constant inner reflectance of the light guide often presented by the manufacturers and distributors is a myth.

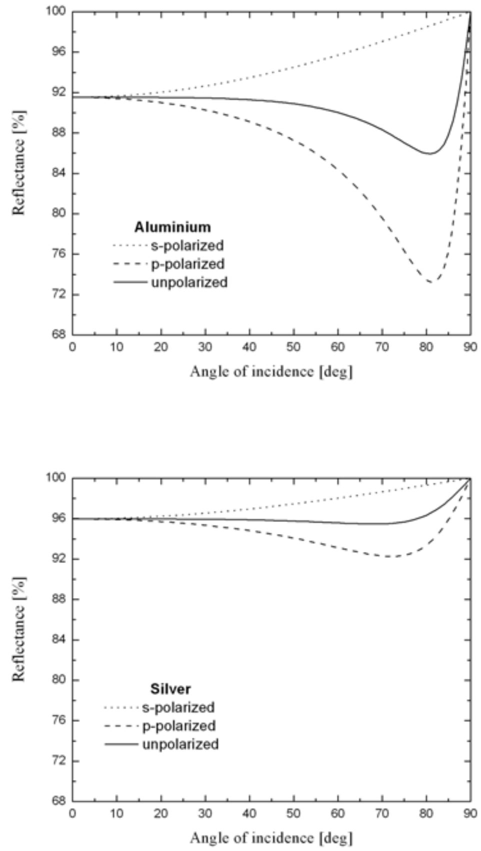

Fig. 3 Directional reflectance of aluminium and silver for 550 nm incoming radiation.

Efficiency of the tubular light guide as the main parameter depends on the reflectance of the layer and on the number of reflections inside the tube. Every reflection decreases the efficiency of the light guide. Total number of reflections N can be calculated for the overcast situation as follows [4]:

(8) $$N = 1 + Int\left[\frac{\left(\frac{H\tan Z}{R} - \frac{\sin\phi_1}{\sin\phi}\right)}{2\left[\frac{2\sin\phi}{\sin\phi_1}(1 - \frac{r_0}{R}\cos\phi_1) + \cos(\phi + \phi_1)\right]}\right]$$

where H is a height of the tube, R is a radius of the tube, r_0 is a distance between the elementary area at the bottom of the light tube and its axis of the symmetry and ϕ is the azimuth of the reflected light beam in the tube. The operand Int returns a truncated whole number from any real number because the amount of reflections must be an integer. The angle ϕ_1 is defined as it was published in [5]:

(9) $$\phi_1 = -\phi + \pi - \arcsin\left(\frac{r_0}{R}\sin\phi\right)$$

The Eq. (8) is usable only if at least one reflection is occurred inside the tube. In general, efficiency of the light transmission through the light tube η can be expressed as:

(10) $$\eta = \frac{\Phi_i}{\Phi_e} \times 100\ [\%]$$

where Φ_i is the luminous flux at the bottom and Φ_e is the luminous flux at the top of the tube. For the overcast unit sky, we can write for the tube's efficiency in accordance with the Eqs. (6) - (9):

$$(11) \qquad \eta = \int_{r_0=0}^{R} \int_{\phi=0}^{2\pi} \int_{\alpha=0}^{\pi/2} \rho(\alpha)^{N(\alpha,\phi,r_0)} \, d\alpha \, d\phi \, dr_0$$

where $\rho(\alpha)$ is a directional reflectance of the mirror layer given by the Eq. (6), α is the angle of incidence of the light beam, for simplicity, zenith angle. Integrating the expression $\rho(\alpha)^{N(\alpha,\ \phi,\ r_0)}$ over the all possible directions of the light ray, we obtain total transmission efficiency of the tube.

Important results

Two important results come out from the theoretical background mentioned above for more accurate prediction of the light guide efficiency:

1. Limited reflectance of the single-layer materials.

Some light guide producers declare the interior reflectance of the tube as a high unreal value close to 100%. In accordance with the Fresnel equations, metallic material without any absorption under the standard physical conditions does not exist. Every opaque metal absorbs some energy from the incoming electromagnetic radiation. The absorption loss of the material is quantified as a complex part of its refractive index.

As an example, we can mention aluminium material with the reflectance $\rho \geq 98\%$ declared on one manufacturer's web page. Aluminium material with $n = 1.015$ would have $k > 14$ to achieve the reflectance 98%. In reality, aluminium's complex part of the refractive index is 6.627, what is less than half of the assumed value (see Fig. 2). Therefore, expensive multilayer materials or surface coatings must be applied in the construction of the tubes. It rapidly increases total prize of the light guide systems or the reflectance of the material is purposely overestimated by the producer.

2. Directional reflectance equals more accurate predictions.

To confirm, that the constant inner reflectance of the light tube is not fully correct approach for the computations, straight 1.8 m long tubular light guide with radius 0.16 m installed under the artificial sky at ICA SAS in Bratislava was used. Measurements of the light guide transmission efficiency were compared with the theoretically computed value for the unit sky situation. Recommendations for measurement of tube transmission efficiency are described in [6]. Results of the measurement are presented in [4]. Numerical solutions are presented in Tab. 1 for the the constant normal reflectance $\rho = 0.92$ declared by the manufacturer and for the directional reflectance considering refractive index of aluminium. Efficiencies were computed in accordance with the Eqs. (6) – (11).

Tab. 1 Measured and calculated light tube efficiencies

Sky type	Measured	ρ = 0.92	ρ (directional)
Unit sky	(55.78 ± 6.55) %	53.12 %	55.62 %

As it is shown in Tab. 1, considering directional reflectance in the calculations yields more accurate prediction of the tubular light guide efficiency. However, tube's inner reflectance ρ depends on the light ray's angle of incidence, therefore this dependence have to be considered in physically correct calculations. For simplicity, polarization of light was neglected. Determining a degree of light polarization inside the light tube is complicated as mathematically as experimentally.

Conclusions

The knowledge of reliable value of the tube's inner reflectance ρ is important for effective usage of tubular light guides. Theoretical prediction and measurements under the artificial sky demonstrated that theories with constant inner reflectance of the pipe are not sufficient for accurate prediction of light guide transmission efficiency. However it was necessary to use a variable reflectance in order to obtain the correct magnitude of the transmission (see also [7]). Next result of this paper follows from material constants of metals described by the complex refractive index. It is shown that reflectance of metals under the standard physical conditions has certain limits given by the absorption loss of the electromagnetic radiation passing through the material. To exceed this limits, special multilayer surfaces have to be used or change the properties of the metallic material by coating with a thin layer of a specific features.

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