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Electromagnetic scattering by a moving object

Abstract. *Here the problem is analysed whether a scatterer moving in space with a constant velocity, generates the electromagnetic field distribution which is a moving map of the distribution obtained for the same scatterer being at rest. A perfectly conducting half-plane is chosen as a test scatterer. It is shown that the distributions are different and both geometrical optics and diffracted field are affected.*

Streszczenie. *W niniejszej pracy analizowany jest problem, czy rozkład pola elektromagnetycznego rozproszonego na poruszającym się obiekcie jest przemieszczającym się odwzorowaniem rozkładu tego pola w przypadku, gdy obiekt jest nieruchomy. Jako obiektu testowego użyto doskonale przewodzącej półpłaszczyzny. Pokazano, że te rozkłady różnią się i że zmiany dotyczą zarówno pola optyki geometrycznej, jak i pola ugiętego.**(Rozpraszanie fali elektromagnetycznej przez poruszające się obiekty)***

Keywords: electromagnetic scattering, moving object
Słowa kluczowe: rozpraszanie, fale elektromagnetyczne, poruszające się obiekty

Introduction

The study of electromagnetic (EM) scattering by various objects has a long history. In this class of problems a typical task to be done at the beginning is to determine the resulting field distribution. In other words, to find how the scattering object modifies the incident field. For linear problems the solution is represented as a sum of the incident (undisturbed) field and the scattered field, the latter being a correction introduced by the scatterer. In order to gain a physical interpretation of the solution obtained, the exact solution is often approximated and represented with functions easy to interpret the accompanying phenomena in physical terms. In case of high-frequency problems, where characteristic object dimensions are small compared to the incident field wavelength, a typical procedure consists in asymptotic expansion of the exact solution as the wave number tends to infinity.

While majority of scattering problems involve objects that are at rest in analysed space, recently there appeared practical problems, where the scattering object is in move. Examples come from space transportation, defence and cosmology, to list only a few. A natural question appears, whether for a scatterer that is moving in space with a constant velocity, the resulting field distribution is a moving map of the field distribution obtained for the scatterer at rest. On the grounds of Galilean transformation the positive answer seems justified. However, the postulates of the Special Theory of Relativity predict that the problem is more complex, with distinctions especially seen at relativistic velocities of the object. Various aspects of EM scattering at relativistic velocities are discussed in [1] - [7].

In this paper we address this problem by studying plane wave diffraction by a moving, perfectly conducting half-plane. This choice is motivated by having a peculiar object possessing a cute edge on one hand, and relatively simple analytical description of the problem on the other hand. It is assumed that the plane wave changes harmonically in space and time. Here, three major physical phenomena are involved: the shadowing of the incident field in a portion of space, the creation of the reflected wave, and the creation of the diffracted wave that originates at the scatterer edge. These phenomena are the same in different frames of reference, however it will be seen that they lead to different field distributions in those frames.

Problem formulation

The problem considered is 2D. In the *laboratory* frame of reference $\{x, y, z\}$ the electromagnetic, harmonic in both space and time, plane wave

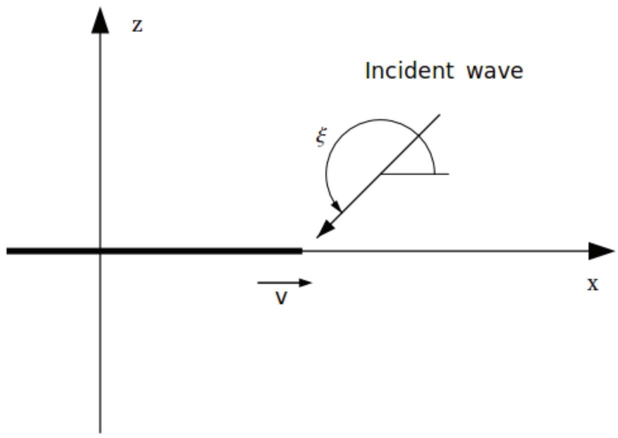

Fig. 1. Geometry of the problem. The incident plane wave is propagating under the angle ξ with respect to the x axis. The half-plane is moving with the velocity $\boldsymbol{v}$ in the x direction.

$$(1) \qquad \boldsymbol{E}^i = \boldsymbol{E}_0\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-ct)} \qquad c\boldsymbol{B}^i = c\boldsymbol{B}_0\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-ct)}$$

is propagating. The incident angle is ξ. The corresponding wave number $\boldsymbol{k}$ and the radius vector are given by

$$(2) \qquad \boldsymbol{k} = [\cos\xi, 0, \sin\xi], \quad \boldsymbol{r} = [x, y, z].$$

This wave is scattered by a perfectly conducting half-plane, which is moving along the x direction with the constant velocity $\boldsymbol{v}$ (see Fig. 1). This velocity may take values from zero to relativistic ones.

We also introduce the *stationary* frame of reference $\{x', y', z'\}$, wherein the half-plane is at rest. In this frame the half-plane is described by $x' \leq 0, z' = 0$. Our goal is to find the resulting EM field in the laboratory frame.

Solving procedure

It is convenient to make transformation between laboratory and stationary frames of reference, as the solution in the latter frame is already known. Thus,

- We represent the incident field in the laboratory frame of reference as a sum of fields of type E and type H with respect to the y axis. Since in 2D geometry those fields are disconnected, further analysis can be carried out for each independently [8].
- With the use of *Lorentz transformation* and the demand of equal phases in both frames of reference [9] we transform the incident fields to the laboratory frame.
- Given the incident field, we find total field in this frame,

i.e. the solution of the scattering problem wherein the scatterer is at rest.
- We finally transform the total field to the laboratory frame.

This procedure was first used by Einstein [10].

Lorentz transformation

For the geometry considered here the Lorentz transformation formulas can be simplified to the following form:
- for the coordinates in both frames of reference:

$$x' = \gamma(x - \beta ct), \quad ct' = \gamma(ct - \beta x), \quad y' = y, \quad z' = z, \quad \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{3}$$

- for the electromagnetic fields:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}, \quad c\mathbf{B}'_{\parallel} = c\mathbf{B}_{\parallel}, \quad \mathbf{E}'_{\perp} = \gamma(\mathbf{E}_{\perp} + \boldsymbol{\beta}\times c\mathbf{B}_{\perp}), \quad c\mathbf{B}'_{\perp} = \gamma(c\mathbf{B}_{\perp} - \boldsymbol{\beta}\times\mathbf{E}_{\perp}), \tag{4}$$

where the symbols $\parallel$ i $\perp$ denote, respectively, the field components in the direction of the half-plane movement, and in the plane perpendicular to that direction. The vector $\boldsymbol{\beta}$ is defined as $\boldsymbol{\beta} = \hat{\mathbf{x}}\beta$,

where $\hat{\mathbf{x}}$ is a unit vector in the x direction.

Diffraction in the stationary frame of reference

We shall stick to the convention that the quantities in the laboratory frame are not primed and in the stationary frame are primed.

Type E field : The incident field in the stationary frame of reference takes the form

$$\boldsymbol{E}^{\boldsymbol{i}\prime} = \hat{\mathbf{y}}A'_1\, e^{i(\boldsymbol{k}'\cdot\boldsymbol{r}' - ct')}, \quad c\boldsymbol{B}^{\boldsymbol{i}\prime} = \frac{1}{ik'}\nabla' \times \boldsymbol{E}^{\boldsymbol{i}\prime}, \tag{5}$$

and results from Lorentz transformation of the incident field

$$\boldsymbol{E}^{\boldsymbol{i}} = \hat{\mathbf{y}}A_1\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - ct)}, \qquad c\boldsymbol{B}^{\boldsymbol{i}} = \frac{1}{ik}\nabla \times \boldsymbol{E}^{\boldsymbol{i}}. \tag{6}$$

in the laboratory frame of reference. Here, $A'_1 = A_1\gamma(1-\beta\cos\xi)$. Thus the plane wave transforms from the laboratory to the stationary frame of reference also as a plane wave, but with modified amplitude and the direction of propagation.

The total field resulting from diffraction of $\boldsymbol{E}^{\boldsymbol{i}\prime}$ on the motionless half-plane was found by Sommerfeld, and it takes the form

$$\boldsymbol{E}' = \hat{\mathbf{y}}[u(\rho',\phi') - u(\rho', 2\pi - \phi')], \tag{7}$$

where

$$u(\rho',\phi') = A_1 e^{i(k'\rho' - ct')}G(w'), \tag{8}$$

$\hat{\mathbf{y}}$ is the unit vector along the y' axis, and ρ', ϕ' are polar coordinates in the stationary frame of reference. The function $G(a)$ is expressed via the Fresnel integral

$$G(a) = e^{-ia^2}\int_a^{\infty} e^{-i\tau^2}d\tau \tag{9}$$

and

$$w' = \sqrt{2k'\rho'}\sin\frac{\phi' - \xi'}{2}. \tag{10}$$

The angle ξ' is related to ξ through

$$\cos\xi' = \frac{\cos\xi - \beta}{1 - \beta\cos\xi}, \qquad \sin\xi' = \frac{\sin\xi}{\gamma(1-\beta\cos\xi)}. \tag{11}$$

Type H field:

$$c\boldsymbol{B}^{\boldsymbol{i}\prime} = \hat{\mathbf{y}}A'_2\, e^{i(\boldsymbol{k}'\cdot\boldsymbol{r}' - ct')}, \quad \boldsymbol{E}^{\boldsymbol{i}\prime} = -\frac{1}{ik'}\nabla' \times c\boldsymbol{B}^{\boldsymbol{i}\prime}, \tag{12}$$

and the total field is equal

$$c\boldsymbol{B}' = \hat{\mathbf{y}}[u(\rho',\phi') + u(\rho', 2\pi - \phi')]. \tag{13}$$

Since the angular coordinate ϕ' in the second term in square brackets in (7) and (13) is simply replaced by $2\pi - \phi'$, it is just enough to examine the transform of $u(\rho',\phi')$ to the laboratory frame of reference.

Exact solution in the laboratory frame of reference

The Lorentz transformation of the total field from the stationary to the laboratory frame of reference yields

$$\boldsymbol{E} = \hat{\mathbf{y}}\gamma\left(E'_{y'} + \frac{\beta}{ik'}\frac{\partial E'_{y'}}{\partial x'}\right) \tag{14}$$

or[1]

$$\boldsymbol{E} = \hat{\mathbf{y}}\gamma B'_1\, e^{ik'(\rho' - ct')}\left[(1+\beta\cos\xi')G(w') - \frac{\beta}{2iw'}(\cos\phi' - \cos\xi')\right], \tag{15}$$

where $B'_1 = A'_1\, e^{-i\pi/4}/\sqrt{\pi}$. Corresponding magnetic induction is given by $c\boldsymbol{B} = \frac{1}{ik}\nabla \times \boldsymbol{E}$.

Asymptotic solution in the laboratory frame of reference

The representation of the total field in (15) does not give us a possibility to interpret it physically. However, such a possibility appears if we expand it asymptotically. It is enough to expand the function $G(w')$ for large values of w':

$$G(a) = H(-a)\sqrt{\pi}e^{i\pi/4}e^{-ia^2} + \frac{i}{2a} + O(a^{-2}) \qquad a \to \infty, \tag{16}$$

where $H(\cdot)$ is a Heaviside (unit step) function. Our restriction to large values of w' is equivalent to the assumption, that the asymptotic field will be valid sufficiently away from the edge $\rho' = 0$ of the screen, and from the shadow boundary $\phi' = \xi'$ of the incident wave. The asymptotic solution takes the following form:

$$\boldsymbol{E} \sim \hat{\mathbf{y}}\gamma B'_1 e^{ik'(\rho' - ct')}\left[H(-w')\sqrt{\pi}e^{i\pi/4}e^{i\boldsymbol{k}'\cdot\boldsymbol{r}' - ik'\rho'}(1+\beta\cos\xi') + \frac{i}{2w'}(1+\beta\cos\phi')\right] \tag{17}$$

This asymptotic representation of the field in the laboratory frame is for brevity described in coordinates specific to the stationary frame. The coordinates in both frames are related through (3).

The first term in (16), multiplied by the common factor, describes the *geometrical optics* contribution to the total field:

$$\boldsymbol{E}^{\boldsymbol{i}} = H(-w')\hat{\mathbf{y}}A_1\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r} - ct)}. \tag{18}$$

(Here, we took advantage of the fact, that plane wave phases are equal in both frames of reference.) Equation (18) describes the incident wave, non-vanishing in its illuminated region. The shadow boundary, separating the illuminated and

[1] In accordance with our convention, we include $u(\rho',\phi')$ contribution only.

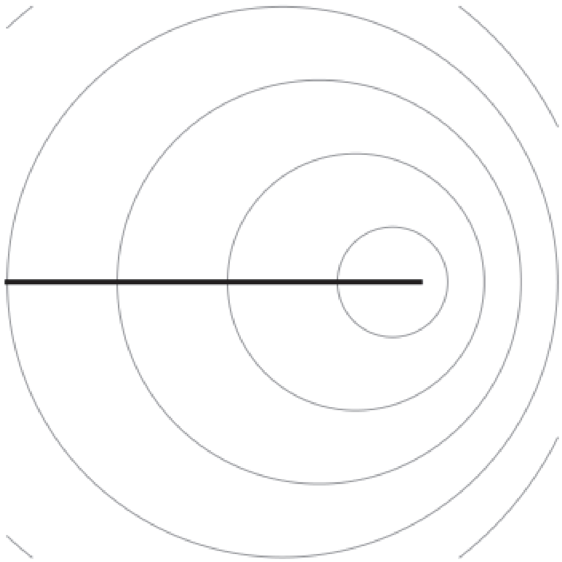

Fig. 3. Surfaces of constant phase as the half-plane is moving in the x direction.

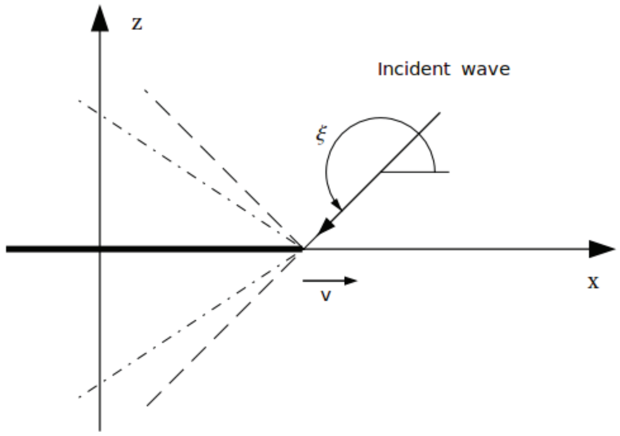

Fig. 2. Location of the shadow boundaries of the geometrical optics field: dash-dash if the half-plane is at rest, dash-dot-dash if the half-plane is in move.

the shadow region, is defined by the equation $\phi' = \xi'$. Unlike the stationary frame of reference, the shadow boundary in the laboratory frame *is not* parallel to the direction of propagation of incident wave rays. Similar conclusion applies to the asymptotic representation of $u(\rho', 2\pi - \phi')$. (In this case the shadow boundary and the illuminated and shadow regions correspond to the reflected wave.) With $u[\rho', 2\pi - \phi']$ included, the location of the shadow boundaries of the incident and reflected wave, for the half-plane being at rest and in move, are depicted in Fig. 2.

The second term in the square bracket of (18) multiplied by the common factor describes the cylindrical wave:

$$\text{(19)} \qquad \boldsymbol{E^d} = \hat{\mathbf{y}}\gamma B_1' e^{ik'(\rho' - ct')} \frac{i}{2w'}(1 + \beta\cos\phi').$$

This wave is recognized as a *diffracted wave* originated at the edge of the half-plane. The surfaces of constant phase are defined in the coordinates specific to the laboratory frame by

$$\text{(20)} \quad ct - \sqrt{(x - \beta ct)^2 + (z/\gamma)^2} - \beta x = C, \quad C = \text{const},$$

or equivalently by:

$$\text{(21)} \qquad (x - \beta\gamma^2 C)^2 + z^2 = (ct - \gamma^2 C)^2.$$

These surfaces are cylinders with centres moving along the x axis (see Fig. 3) with the velocity $\boldsymbol{v}$. The Doppler effect is clearly seen. The amplitude of this wave vanishes with the distance from the edge as ${\rho'}^{-1/2}$. The radiation pattern exhibits singularity at the shadow boundary $\phi' = \xi'$ of the incident wave, and $\phi' = 2\pi - \xi'$ of the reflected wave. These singularities are side effects of the asymptotic approximation of the solution (in both stationary and laboratory frames) and do not appear in the exact solution. The reason is that the assumption of large a in (16) is not satisfied in vicinities of the shadow boundaries.

In addition to the fact that unlike the stationary frame the shadow boundaries in the laboratory frame are not parallel to the rays of incident and reflected wave, the radiation pattern of the diffracted wave in the laboratory frame is also modified by the inclusion in (19) of the term proportional to β.

Conclusions

The problem of plane wave scattering by a half-plane in motion was used to demonstrate that the scattered field distribution in case of the moving object is not a moving map of the corresponding field distribution obtained for the object at rest. The differences appear in both the geometrical optics component and in the edge diffracted component. They affect the illuminated and shadow regions of the geometrical optics field and the radiation pattern of the diffracted field. The differences increase with growing values of the velocity of the object, and are clearly noticed at relativistic velocities.

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