

The p-q theory and Compensating Current Calculation for Shunt Active Power Filters: Theoretical Aspects and Practical Implementation

Abstract. The paper presents an analysis of the Akagi's p-q theory and its implementation for active filtering under nonsinusoidal voltage conditions. A modified definition of the active component of the current for such conditions is proposed. Next, our platform for testing control strategies of parallel shunt filters, based on a DSP DS1103 system, is presented. An implementation of the modified p-q theory on the dSpace 1103 DSP system and some experimental results are presented in the second part of the paper.

Streszczenie. Artykuł przedstawia analizę teorii mocy p-q Akagi'ego i jej zastosowań do aktywnej filtracji w warunkach niesinusoidalnych. Proponowana jest też, zmodyfikowana dla takich warunków, definicja składowej czynnej prądu. Artykuł omawia następnie zestaw laboratoryjny do testowania strategii sterowania równoległego filtru aktywnego, oparty na systemie DS1103 cyfrowej obróbki sygnałów (DSP). W części drugiej przedstawiono zastosowanie zmodyfikowanej teorii p-q do sterowania filtru aktywnego z użyciem systemu DSP dSpace 1103 oraz otrzymane wyniki eksperymentalne. (Teoria p-q oraz obliczanie prądu kompensującego równoległych filtrów aktywnych: aspekty teoretyczne i zastosowania praktyczne)

Keywords: p-q theory; active current; nonsinusoidal voltage; active filtering; DSP.

Słowa kluczowe: Teoria p-q, prąd czynny, napięcia niesinusoidalne, filtracja aktywna, cyfrowa obróbka sygnałów.

Introduction

In the control circuit of a shunt active power filter (SAPF), the generation of the reference compensating current to be processed by the controller is the key component that ensures the fulfilment of the compensation task and leads to a high performance of the active filtering system. As a main principle, starting from a distorted load phasor current (\underline{i}_L), the shunt active power filter is able to inject such a compensating phasor current (\underline{i}_F) in the point of common coupling (PCC) so that the current drawn from the network has the desired shape and zero passing (\underline{i}_{des}),

$$(1) \quad \underline{i}_F = \underline{i}_L - \underline{i}_{des}.$$

Obviously, when the three-phase voltage system is balanced and sinusoidal, the global compensation of current harmonics and reactive power leads to an active power flow to the nonlinear load by absorbing a sinusoidal current from the network which is in phase with the supply voltage. However, the operation under nonsinusoidal conditions does not allow achieving simultaneously the two major compensation goals. In this context, the p-q theory of instantaneous reactive power introduced by Nedelcu [1], [2] and developed for active filtering by Akagi [3], [4] was brought into actuality to provide the mathematical foundation in the control of static converters involved in the power quality improvement. Furthermore, the p-q theory is about to become a means of identifying and analyzing the properties of powers in circuits with nonsinusoidal voltages and currents [5], [6].

After introducing the p-q theory concepts in section 2, section 3 presents a development of the authors for active filtering application under nonsinusoidal voltage conditions. Next, sections 4, 5 and 6 are dedicated to a short presentation of a shunt active system platform developed by authors in their laboratory. In the second part, the implementation of the p-q theory under nonsinusoidal voltage condition on the dSpace 1103 DSP system is presented. Next, some experimental results for both balanced and unbalanced loads are illustrated. Finally, some concluding remarks are drawn.

The p-q theory concepts

The first version of the p-q theory for active filtering application was published in 1984 in a prestigious

international journal by professor Akagi and his coauthors Kanazawa and Nabae [3]. It is also known as the instantaneous reactive power theory for three-phase circuits. The first step was to introduce the instantaneous space vectors (\underline{u} and \underline{i}) by transforming the three-phase systems of voltages (u_a, u_b, u_c) and currents (i_a, i_b, i_c) into two-phases orthogonal stationary reference frames (u_α, u_β) and (i_α, i_β). Then, the conventional instantaneous power (p) and the reactive power (q) have been identified as the real and imaginary parts of the instantaneous complex power (\underline{s}).

For the original adopted power invariant a-b-c to α - β transformation,

$$(2) \quad \underline{s} = \underline{u} \cdot \underline{i}^* = p + jq,$$

where

$$(3) \quad \underline{i}^* = i_\alpha - ji_\beta.$$

This way, the obtained expression of p and q are:

$$(4) \quad p = u_\alpha \cdot i_\alpha + u_\beta \cdot i_\beta = u_a \cdot i_a + u_b \cdot i_b + u_c \cdot i_c$$

$$(5) \quad q = u_\beta \cdot i_\alpha - u_\alpha \cdot i_\beta.$$

If the non-power invariant transformation a-b-c to α - β is adopted in order to preserve the magnitude of the instantaneous three-phase quantities, expression (2) becomes

$$(6) \quad \underline{s} = \frac{3}{2} \cdot \underline{u} \cdot \underline{i}^* = p + jq,$$

and the current space vector can be expressed as

$$(7) \quad \underline{i} = \frac{2}{3} \cdot \frac{\underline{u}}{|\underline{u}|^2} \cdot \underline{s}^* = \frac{2}{3} \cdot \frac{\underline{u}}{|\underline{u}|^2} \cdot (P + p_\sim - jQ - jq_\sim),$$

where,

$$(8) \quad |\underline{u}|^2 = u_\alpha^2 + u_\beta^2.$$

In shunt active filtering systems, expression (7) can be used to calculate the reference filter current if the apparent power vector is replaced by the desired apparent power vector of the filter,

$$(9) \quad \underline{i}_{F_ref} = \frac{2}{3} \cdot \frac{\underline{u}}{|\underline{u}|^2} \cdot \underline{s}_F^*.$$

According to the Akagi's p-q theory - based approach, the shunt active filter could compensate the AC

components of the instantaneous active and reactive powers (p_{\sim} and q_{\sim} , for partial compensation) or the instantaneous reactive power q and the AC component (p_{\sim}) of the instantaneous active power p (for total compensation). Thus, the imposed complex apparent power of the active filter is given by

$$(10) \quad \underline{s}_F^* = \begin{cases} -p_{\sim} + jq_{\sim} & \text{or} \\ -p_{\sim} + jq \end{cases}$$

For active filtering applications, our opinion is that we should concentrate on the desired line current, not on the powers compensation. Thus, when total compensation is expected, the reference current requires only the load current and its active component, in accordance with expression (1). So, if the total compensation is proposed, the desired line current is the active load current.

From (7), the active current vector is,

$$(11) \quad \underline{i}_a = \frac{2}{3} \cdot \frac{P}{|\underline{u}|^2} \underline{u}$$

If the partial compensation is proposed (only the harmonics), the desired line current is the active and reactive load currents. So, the desired line current will be

$$(12) \quad \underline{i}_{des} = \frac{2}{3} \cdot \frac{P - jQ}{|\underline{u}|^2} \underline{u}$$

But, according to the opinion of the most specialists in the field (Fryze, Shepherd, Zakikhani, Czarnecki, Wilhems and many others), the active current must have the same shape as the voltage [8, 9, 10, 11, 12]. It means that, in expression (11), the square of voltage space vector magnitude must be constant. However, when the supply voltages are distorted, the magnitude of the voltage space vector is time dependent (Fig. 1) and the calculation of the desired supply

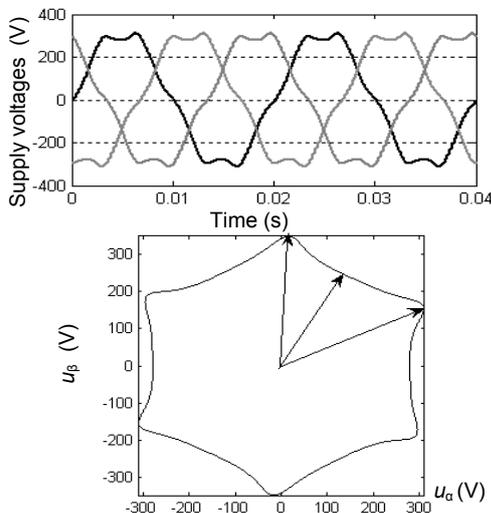


Fig. 1. Example of distorted voltages and associated space vector locus [13]

current by (11) or (12) leads to a nonsinusoidal waveform of this current which has a different distortion level compared to the voltage distortion (Fig. 2).

The active current under nonsinusoidal voltage conditions and the active filtering

In order to obtain an active current whose waveform has the same shape as the supply voltage, in accordance with Fryze's definition, the denominator in (11) must be

constant and equal with the RMS value of the voltage vector magnitude [14],

$$(13) \quad U = \sqrt{\frac{1}{T} \int_{t-T}^t |\underline{u}|^2 dt}$$

Thus, the expression of the true active and reactive currents become

$$(14) \quad \underline{i}_a = \frac{2}{3} \cdot \frac{P}{U^2} \underline{u}; \underline{i}_r = -j \frac{2}{3} \cdot \frac{Q}{U^2} \underline{u}$$

The expression (14) of the active current space vector in the p-q theory is similar to definition proposed by Peng in [15].

Moreover, in the time domain, it is the same as the active current defined by Fryze and other authors [14].

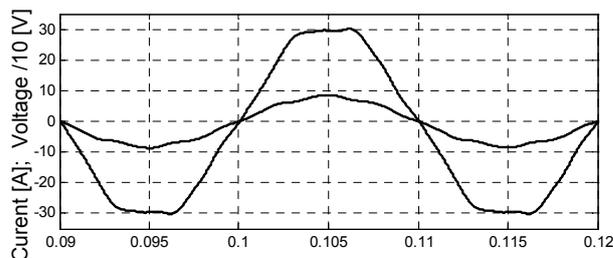


Fig. 2. Distorted supply voltage and reference supply current if reference active filter current is calculated by (14) and (15)

Active filtering system configuration

To study the active filtering techniques, we developed an experimental platform in our laboratory (Fig. 3).

The adopted structure consists of a three-phase three-wire active filtering system composed of a two-level VSI which is connected to PCC through an inductive coupling filter to prevent the high order switching harmonics from propagating into the power supply, an inductive distorted current source and an industrial PC.

The VSI based on SKM100GB123D IGBTs power modules ($I_C=100$ A, $V_{CES}=1200$ V), having a DC-capacitor of 1100 μ F and an interfacing reactor of 4.4 mH, acts as SAPF to generating the compensating currents. The line-to-line supply voltage is 380 V rms and the apparent power of VSI is 15 kVA.

The acquisition system based on LEM sensors measures two line-to-line supply voltages, two load line currents, two inverter line currents and the DC-link voltage.

The industrial PC is equipped with a dSPACE 1103 DSP board to control and monitor the entire SAPF system.

The PowerPC 750GX processor of the control board is running at CPU clock of 1 GHz for fast floating-point calculation.

The cascaded control loops, which include the optimal DC-link voltage loop outside the inner current loop, are first designed in a Matlab/Simulink model. The dSPACE Real-Time Interface (RTI) together with Real-Time Workshop (RTW) automatically generate real-time code.

Thus, through the interface between Simulink and DSP, the controller board is fully programmable from the Simulink block diagram environment.

To obtain high switching frequencies, the programmable digital I/O channels are used to generate the required six IGBTs' gate signals.

Control system

In addition of the computed reference current in the methods discussed above, an additional component (i_{Fu}) is required to cope for losses in the power circuit and keep the DC-capacitor voltage at its set value. A PI controller was chosen to generate the amplitude of this additional reference current, whereas a specific circuit provides its shape (u_{Fu}) based on the supply voltage.

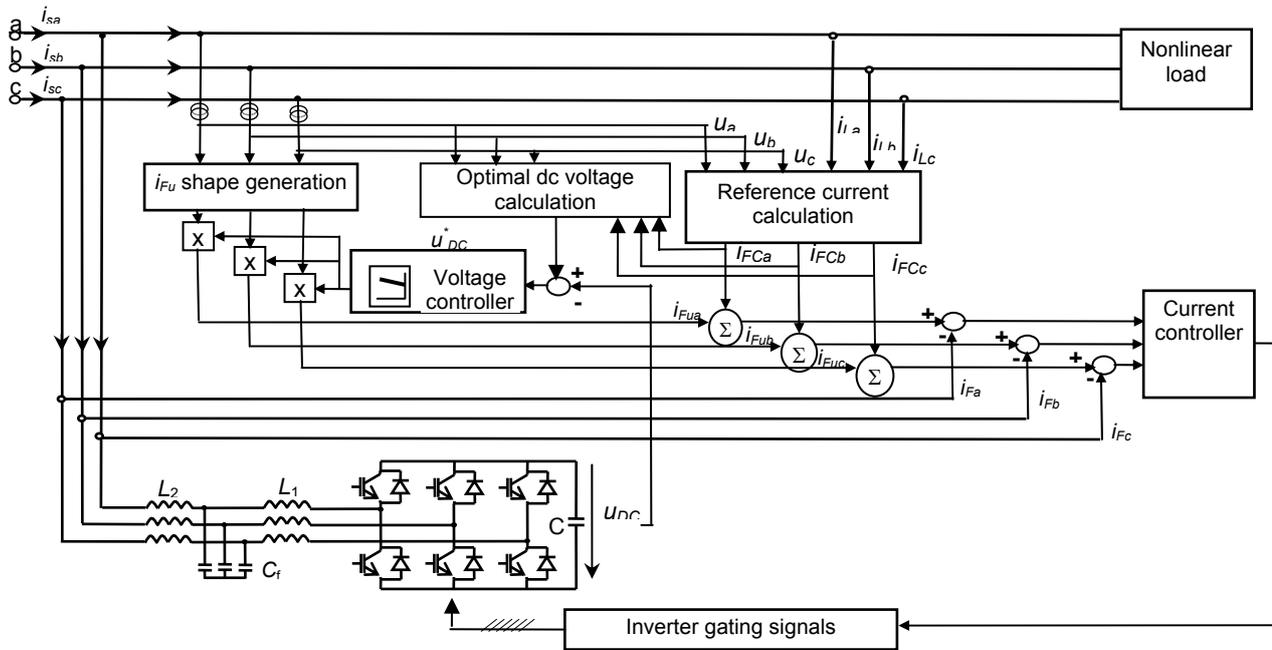


Fig. 3 The structure of active filtering system

In order to obtain this signal, we used specific phase-locked loop (PLL) techniques and circuits [6], [16], [17]. The synchronous reference frame-based PLL introduced in [16] uses only one phase voltage and the PLL control loop does not make use of a PI controller. In [17], a multiple-complex coefficient-filter-based synchronization technique is used to estimate the fundamental positive and negative components of the distorted and unbalanced supply voltages.

The resulting reference current is accurately tracked by using a hysteresis-band current control whose main advantages are related to simple hardware implementation, quick current controllability and robustness under load parameters variation [18], [19].

For compensating current calculation, two models were build. The first one is based on expressions (9) and (19) and the second is based on expressions (1), (11) and (12) or (1), (13) and (14).

Optimal DC-Voltage Controller Design

A PI controller is adopted to control the voltage across the capacitor. The PI controller parameters have been tuned according to the Modulus Optimum (MO) criterion for an efficient disturbance rejection [20]. In addition, the pass band frequency (f_p) of the unity feedback system must be imposed.

If the transfer function of the voltage controller is written as

$$(15) \quad G_{Ru}(s) = \frac{1 + \theta_{lu} \cdot s}{\theta_u \cdot s},$$

the following expressions can be used to calculate the two time constants [20]:

Finally, the PI controller parameters are obtained as a function of the pass band frequency – f_p :

$$(16) \quad \theta_{lu} = \frac{0.36}{f_p};$$

$$(17) \quad \theta_u \approx 0.6495 \cdot \frac{3 \cdot K_{Tu} \cdot U_s}{K_{Ti} \cdot C \cdot U_{DC}} \cdot \frac{1}{f_p^2}.$$

In the expressions (16) and (17), the significance of parameters are: K_{Tu} and K_{Ti} - the proportional constants

associated to the voltage and current transducers; U_s - the RMS value of the phase voltage; C – dc circuit capacitor and U_{DC} is average voltage of the dc circuit capacitor.

The implementation of a specific control system for an optimal prescribed DC-voltage is originated by extensive analysis and experimental results on the active filtering system, when the coupling interface and DC-storage circuit are well defined. It has been pointed out that, for each value of the apparent power to be compensated, there is an optimal value of DC-voltage which minimizes the total harmonic distortion factor of the supply current after compensation (Fig. 4) [21].

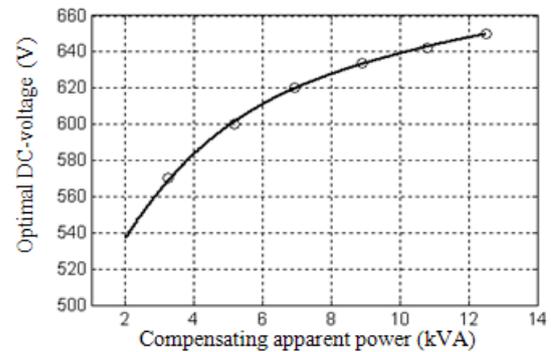


Fig. 4. Optimal DC-voltage versus compensating apparent power

The authors have found an appropriate 4th degree polynomial function for the optimal DC-voltage (U_{DCo}) curve fitting, which is defined as follows:

$$(18) \quad U_{DCo} = -0.0073 \cdot S_C^4 + 0.32 \cdot S_C^3 - 5.5 \cdot S_C^2 + 48.17 \cdot S_C + 460.3$$

where the compensating apparent power (kVA) is:

$$(19) \quad S_C = \sqrt{S_{Load}^2 - P^2}$$

Thus, only the supply voltages and load currents are needed to calculate the optimal DC-voltage [22].

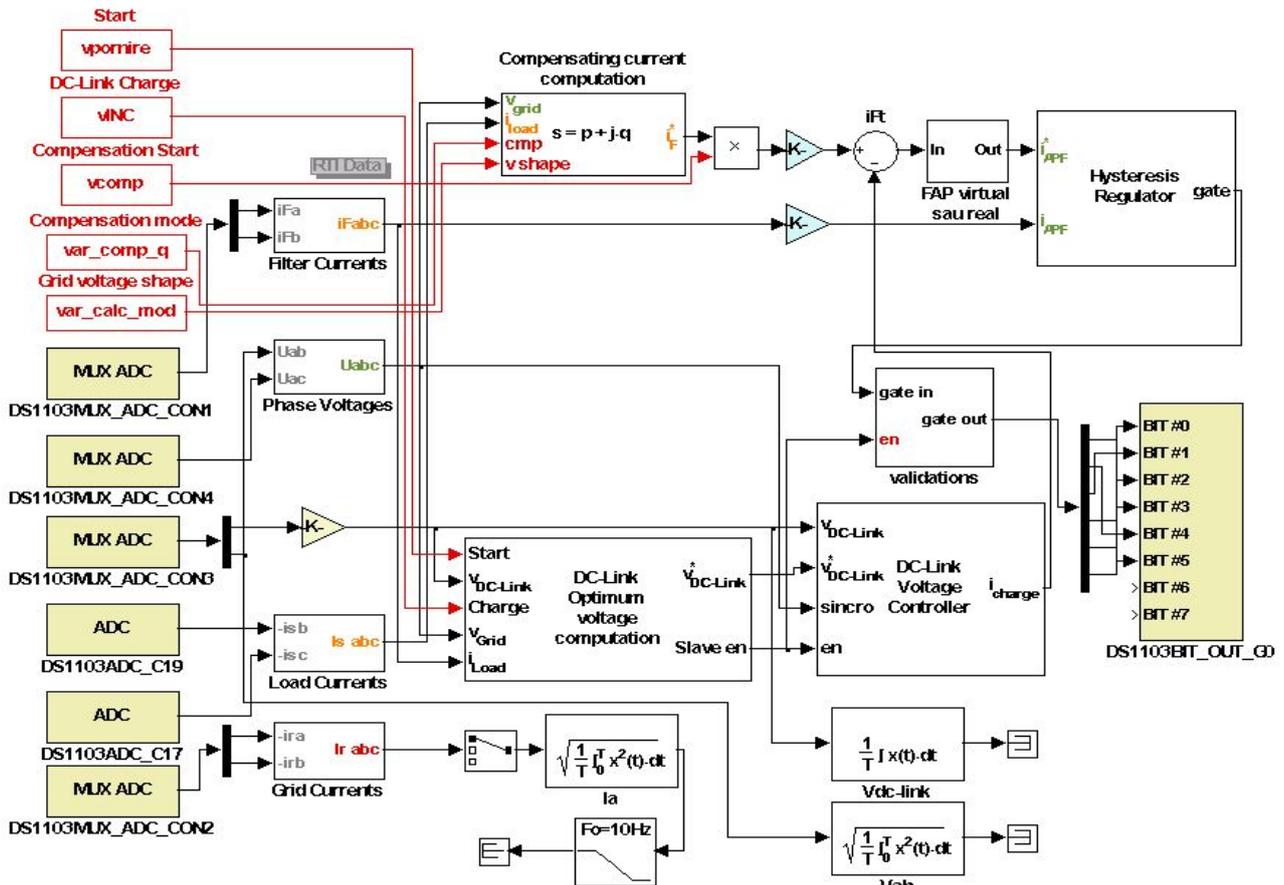


Fig. 5. Compiled Simulink model of the control system.

Control Implementation on dSPACE 1103 system

To perform the real-time control of the active filtering system, the control algorithm previously described has been built under Matlab/Simulink environment combined with the RTI and RTW tools provided by dSPACE 1103 system (Fig. 5). After normalizing, the digital inputs supplied by ADC blocks are used according to the adopted control strategy. Since the hysteresis control has been chosen, the three-phase SLAVE DSP PWM block cannot be used for the IGBT's gating signals. Consequently, two options remain for gating signals transfer to the IGBT's drivers, that is either through digital to analog (D/A) channels or through digital output channels. A detailed experimental analysis on the analog outputs shown that the accurate transfer through D/A channels is guaranteed for signal frequencies up to 3.5 kHz. Therefore, the generated switching signals are taken out of the DS1103 with the help of six digital outputs through the DS1103BIT_OUT block of Master PPC library. A specific block has been created to control the start-up process of the shunt active power filter and the associated DC-capacitor charging.

Two digital to analog converters are used to control two line-contactors (named K1 and K2) which allow a two-stage process of the DC-capacitor charging. In addition, some protections were taken into consideration and validation conditions were used to avoid unexpected behaviours during the system operation.

As far as the sampling time is concerned, it was reduced as much as possible without reaching a critical value associated to overrun errors. Although a sampling time of 20µs allowed the implementation of all control strategies taken into consideration, a decrease of 10 % was possible by reducing the amount of calculation when using expression (1) to calculate the reference line current.

To manage the entire process and display the

waveforms and numerical values, a graphical user interface was created. It facilitates the continuous communication with the control algorithm.

Experimental results

Next, the experimental results are presented for two types of load cases:

Case 1 - partial compensation of a three-phase AC voltage regulator with balanced and unbalanced load;

Case 2 – total compensation of a three-phase controlled rectifier with balanced R-L load.

Case 1 experimental results

If the load is approximately balanced, the experimental waveforms from figure 6 show good performances. So, the line current (is) is nearly sinusoidal and its wave contains the inverter switching noise.

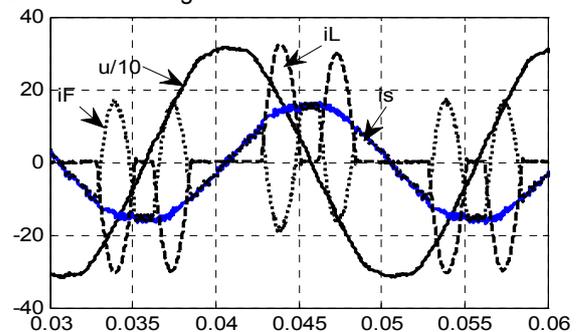


Fig. 6 Experimental waveforms for the proposed current calculation in the load case 1 of partial compensation: line voltage ($u/10$); load current (i_L); active filter current (i_F) and line current after compensation (i_s)

The partial distortion factor (until the harmonic of order 51) of the line current is 90,25% before compensation and

decreases to 3,88% after compensation. This means that the filtering efficiency is 23,26. It must be noted that the line voltage harmonic distortion factor is 2,2%.

Because the compensating current is calculated by the proposed expression, the waveforms of the line voltage and current are the same (Fig. 7).

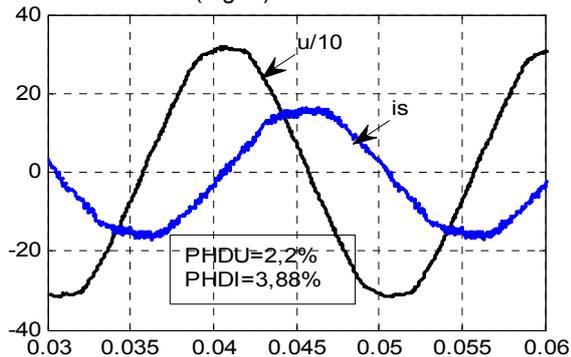


Fig. 7 Experimental waveforms for the proposed current calculation in the load case 1 and partial compensation: line voltage ($u/10$) and line current after compensation (i_s)

On the contrary, when the compensating current is calculated by Akagi's expression, although the line voltage is slightly distorted, the waveforms of the line voltage and current are different (Fig. 8).

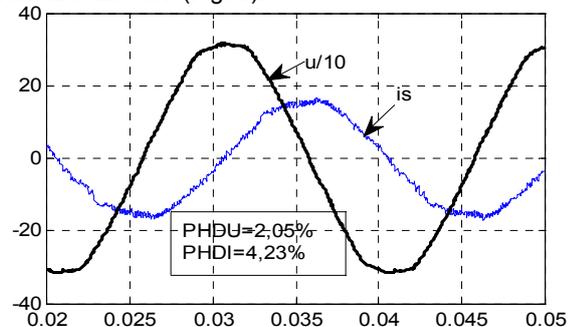


Fig. 8 Experimental waveforms for the classical current calculation in the load case 1 and partial compensation: line voltage ($u/10$) and line current after compensation (i_s)

If the load of the voltage regulator is unbalanced, as it is shown in Fig. 9, the active filter system operates well too (Fig. 10, Fig. 11). The distortion factors of the line currents are 127% on phase-a, 87,5% on phase-b and 116,83 on phase-c, before compensation. After compensation, they decrease to 4,47%, 4,32% and 5,03%, respectively (Fig. 11).

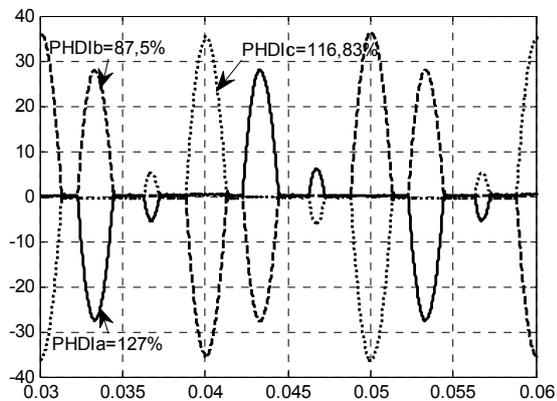


Fig. 9 Experimental waveforms of the load currents in the unbalanced load case 1

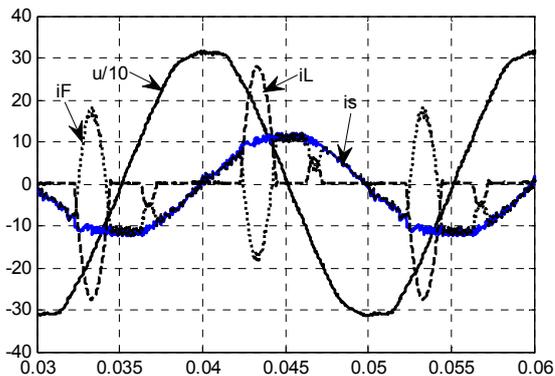


Fig. 10 Experimental waveforms for the classical current calculation in the load case 1 and partial compensation: line voltage ($u/10$); load current (i_L); active filter current (i_F) and line current after compensation (i_s)

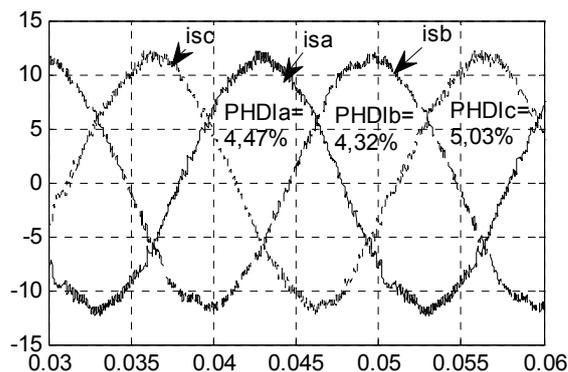


Fig. 11 Experimental waveforms of the line currents in the unbalanced load case 1 and partial compensation

Case 2 experimental results

In this case, only the total compensation results are presented (Fig. 12). The load line current has a typical form and its distortion factor is 28,89%. The line voltage is slightly distorted (2,08%) and the distortion factor of line current decreases to 2,94% after compensation (9,8 filtering efficiency). The phase shift of line voltage and line current after compensation is zero, because the control corresponds to the total compensation (distortion and reactive current). This means that the line current will be only the active current.

The difference between the line currents after filtering through the classical and proposed compensating current calculation modes is shown in Fig. 13 (dotted and black line for Akagi's expression and solid-line for the proposed expression). As it can be seen, the line current obtained by

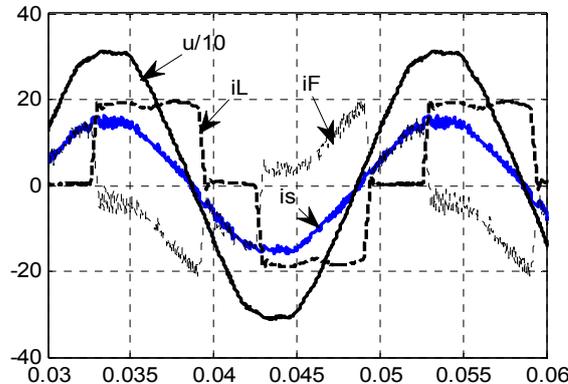


Fig. 12 Experimental waveforms for the proposed current calculation in the load case 2 and total compensation: line voltage ($u/10$); load current (i_L); active filter current (i_F) and line current after compensation (i_s)

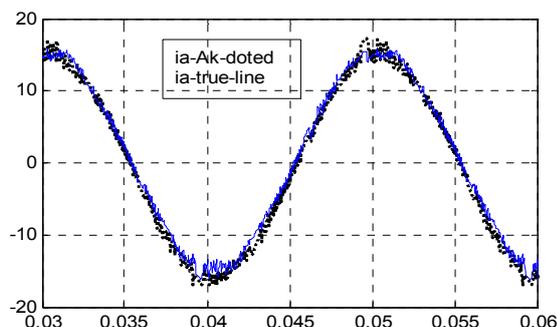


Fig. 13 Experimental waveforms of the line current after compensation in the load case 2 for total compensation: classical calculation-dotted line; proposed calculation-solid line

Akagi's method has not the same waveform as the line voltage and it does not represent the active current.

Conclusions

The goal of the paper is the presentation of the active current extraction in three-phase three-wire systems based on the so called "p-q theory". On this basis, a method for compensating current calculation in three phased, three wire shunt active filter is proposed.

The method is implemented on a dSPACE 1103 DSP board that operates into a laboratory system. The main implementation aspects are described (the compensating current calculation, the tuning of the DC voltage regulator and the optimal DC voltage calculation).

Finally, the experimental results for two types of load cases (partial compensation of a three-phase AC voltage regulator with balanced and unbalanced load and total compensation of a three-phase controlled rectifier with balanced R-L load).

The experimental results demonstrate very good performances of the laboratory platform. The obtained values of the filtering efficiency are 24,4 for the three-phase AC voltage regulator (balanced and unbalanced load) and 9,8 for the three-phase controlled rectifier with balanced R-L load.

Even if the line voltage is slightly distorted, the experimental waveforms show the differences between the line current obtained by Akagi's and proposed method. Thus, for total compensation (harmonics and reactive power) through the proposed method, the waveforms and phases of the line current after compensation and the line voltage are the same. This means that the line current is only the active current.

On the contrary, the line current obtained through the compensating current computed in accordance with the Akagi's method has not the same waveform as the line voltage and it does not represent the active current.

REFERENCES

- [1] Nedelcu V.N., Die einheitliche leistungstheorie der unsymmetrischen und mehrwelligen mehrphasensysteme, *ETZ-A*, 84, 5, p9. 153-157, 1963.
- [2] Nedelcu V.N., Electromechanical conversion theory, *Technical Ed., Bucharest*, 1978. (In Romanian)
- [3] Akagi H., Kanazawa Y., and Nabae A., Generalized theory of the instantaneous reactive power in three-phase circuits, *Int. Power Electronics Conf.*, Tokyo, Japan, 1983, p. 1375-1386.
- [4] Akagi H., New trends in active filters for power conditioning, *IEEE Trans. Ind. Appl.* 1996, 32, (6), pp. 1312-1322.
- [5] Akagi, H., Modern active filters and traditional passive filters, *Bulletin of the Polish Academy of Sciences, Technical Sciences*, Vol. 54, No. 3, pp. 255-269, 2006.

- [6] Akagi H., Watanabe H., and Aredes M., "Shunt active filters," in *Instantaneous power theory and applications to power conditioning*, Wiley-IEEE Press, 2007, pp. 109-220.
- [7] Sozanski K. P., Control circuit for active power filter with an instantaneous reactive power control algorithm modification, *Przegląd Elektrotechniczny*, vol. 2011, no. 1, pp. 96-100, 2011.
- [8] Emanuel A.E., Powers in nonsinusoidal situations: A review of definitions and physical meaning, *IEEE Trans. on Power Delivery*, 5 (1990), No. 3, 1377-1389
- [9] Slonim M.A. and Van Wyk J.D., Power components in a system with sinusoidal and nonsinusoidal voltages and/or currents, *Proc. IEE*, 135 (1988) Issue 2, 76-84
- [10] Willems J.L., Current compensation in three-phase power systems, *European Trans. on Electrical Power*, 3 (1993), Issue 1, 61 - 66
- [11] Czarnecki L.S., On some misinterpretations of the instantaneous reactive power p-q theory, *IEEE Trans. on Power Electronics*, 19 (2004), No. 3, 828-836
- [12] Czarnecki L.S., Orthogonal decomposition of the current in a three phase nonlinear asymmetrical circuit with nonsinusoidal voltage, *IEEE Trans. on Instrum. Meas.*, IM-37 (1988), No. 1, 30-34
- [13] Popescu Mihaela, Bitoleanu A., Suru V., P-Q theory and total active filtering compensation under distorted supply voltage, *World Energy Systems Conf.*, June 28-30, 2012, Suceava, Romania.
- [14] Bitoleanu A. and Popescu Mihaela, How can the IRP p-q theory be applied for active filtering under nonsinusoidal voltage operation?, *Przegląd Elektrot.*, vol. 2011, no. 1, pp. 67-71, 2011.
- [15] Peng F.Z. and Tolbert L.M., Compensation of nonactive current in power systems - definitions from a compensation standpoint, *IEEE Power Engineering Society Summer Meeting*, July 15-20 (2000), Seattle, WA, 983-987.â
- [16] Da Silva C. H., R. R. Pereira, L. E. B. Da Silva, G. Lambert-Torres, B. K. Bose, and S. U. Ahn, A digital PLL scheme for three-phase system using modified synchronous reference frame, *IEEE Trans. Ind. Electron.*, vol. 57, no. 11, pp. 3814-3821, Nov 2010.
- [17] Guo X., Wu W. and Chen Z., Multiple-complex coefficient-filter-based phase-locked loop and synchronization technique for three-phase grid-interfaced converters in distributed utility networks, *IEEE Trans. Ind. Electron.*, vol. 58, no. 4, pp. 1194-1204, April 2011.
- [18] George S. and Agarwal V., Optimum control of selective and total harmonic distortion in current and voltage under nonsinusoidal conditions, *IEEE Trans. Power Del.*, vol. 23, no. 2, pp. 937-944, Apr. 2008.
- [19] Salmeron P. and Litran S. P., A control strategy for hybrid power filter to compensate four-wires three-phase systems, *IEEE Trans. Power Electron.*, vol. 25, no. 7, pp. 1923-1931, July 2010.
- [20] Popescu Mihaela, Bitoleanu A., Control loops design and harmonic distortion minimization in active filtering-based compensation power systems, *Internat. Review Modelling and Simulations*, vol. 3, no. 4, pp. 581-589, Aug. 2010.
- [21] Bitoleanu A., Popescu Mihaela, M. Dobriceanu and F.Nastasoiu, DC-bus voltage optimum control of three-phase shunt active filter system, in *Proc. 12th Int. Conf. Optimization of Electrical and Electronic Equipment*, Brasov Romania, 2010, pp. 538-543.
- [22] Popescu Mihaela, Bitoleanu A., V. Suru, A DSP-Based Implementation of the p-q Theory in Active Power Filtering under Nonideal Voltage Conditions, *IEEE Transaction on Industrial Informatics*, Volume 9, Issue 2 May, 2013, pp. 880-889.

Authors: prof. dr eng. Alexandru Bitoleanu, prof. dr eng. Mihaela Popescu, Electric Drives and Industrial Informatics Department, Faculty for Electrical Engineering, University of Craiova, Romania, Decebal Bd. 107, 200440 Craiova, E-mail: alex.bitoleanu@em.ucv.ro; mpopescu@em.ucv.ro