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Characteristic Impedance of Power Lines with Ground Wires

***Abstract**. In the paper the characteristic impedance of a power line equipped with shield wires is analysed. The solution to the problem is found by means of a non-symmetric algebraic Riccati equation. Solutions are presented for practical line configurations.*

***Streszczenie.** W artykule przedstawiono analizę impedancji charakterystycznej linii elektroenergetycznej z ekranowanymi kablami w praktycznym zastosowaniu konfiguracyjnym. W analizie wykorzystano niesymetryczne równanie Riccatiego. (**Impedancja charakterystyczna linii elektroenergetycznej z uziemieniem**).*

Keywords: Transmission line, grounding, characteristic impedance, non-symmetric algebraic Riccati equation.
Słowa kluczowe: linia przesyłowa, uziemienie, impedancja charakterystyczna, niesymetryczne równanie algebraiczne Riccatiego.

Introduction

In power lines, shield wires are passive wires which are periodically grounded, and their main role is to intercept direct lightning strokes which could cause overvoltages higher than the line lightning withstand level. They are installed above the phase conductors, and, if properly placed, assure lightning protection. It must be noted that this direct protection is successful on power transmission lines, which are usually characterized by values of critical impulse flashover voltage (CFO) much higher compared to distribution lines. The direct stroke performance of distribution lines is practically unaffected by the presence of shield wires, since, in case of a direct strike, a backflashover will occur in most of the cases due to the ground potential rise. In distribution lines shield wires can still play a role since, thanks to their coupling with phase conductors, can reduce the induced voltages produced by indirect lightning.

Many studies have dealt with the effectiveness of shield wires on medium and low voltage overhead distribution lines (e.g., in [1]-[7]) and the reduction in terms of induced overvoltages, compared to the results obtained for unshielded lines (e.g. [8]-[10]), is significant; the authors too have deeply investigated the problem of lightning induced overvoltages on unshielded lines [11]-[16]. In order to evaluate the induced overvoltages on shielded lines, and to estimate the role of the periodical grounding in the overvoltage mitigation, an important step is the evaluation of the characteristic impedance of the power line. Although periodical grounding of shield wires significantly modifies the characteristic impedance, this aspect is usually underestimated or neglected. In this paper, by making use of the transmission line (TL) approximation [17], we examine the problem of a multiconductor transmission line (MTL), with one conductor periodically grounded, as shown in Fig. 1, and we will show two ways to compute the characteristic impedance. The examined MTL has m-1 non-grounded wires, the m-th wire is the grounded one.

The paper is organized as follows: we will first formulate the problem in terms of a Riccati equation, then two different solution methods will be shown along with some numerical results, finally conclusions will be presented.

Problem Formulation

We define $\overline{\mathbf{V}}(z)$ and $\overline{\mathbf{I}}(z)$ the voltages and currents vectors of the MTL, where z is the distance evaluated from the line origin. The most relevant distances for observing voltages and currents are those corresponding to each grounding point, we will call them $\overline{\mathbf{V}}_n = \overline{\mathbf{V}}(nL)$ and

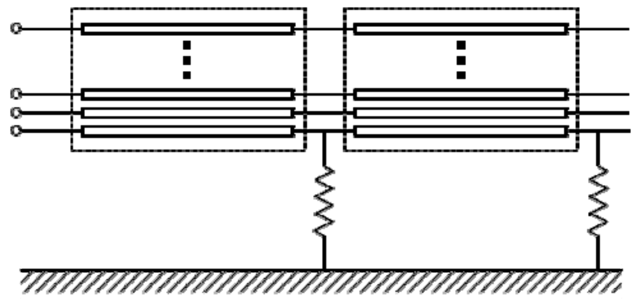

Fig.1. Line configuration

$\overline{\mathbf{I}}_n = \overline{\mathbf{I}}(nL)$, with n = 0, 1, 2, … and where L is the distance between two following grounding points, i.e. the length of an elementary MTL cell. Then we call as $\mathbf{r}$, $\mathbf{l}$ and $\mathbf{c}$ the $m \times m$ per unit length resistance, inductance and capacitance matrices of the MTL, and define $\mathbf{Z} = \mathbf{r} + j\omega\mathbf{l}$ and $\mathbf{Y} = j\omega\mathbf{c}$. The characteristic impedance of a non-grounded MTL is given by $\dot{\mathbf{Z}}_0 = \mathbf{Y}^{-1}\sqrt{\mathbf{YZ}}$.

If we consider a single cell of a non-grounded MTL, its chain matrix can be expressed as [18]

$$(1) \qquad \begin{cases} \overline{\mathbf{V}}_{n+1} = \mathbf{Y}^{-1}\cosh(\mathbf{U})\mathbf{Y}\overline{\mathbf{V}}_n - \dot{\mathbf{Z}}_0\sinh(\mathbf{U})\overline{\mathbf{I}}_n \\ \overline{\mathbf{I}}_{n+1} = -\sinh(\mathbf{U})\dot{\mathbf{Z}}_0^{-1}\overline{\mathbf{V}}_n + \cosh(\mathbf{U})\overline{\mathbf{I}}_n \end{cases}$$

Being $\mathbf{U} = L\sqrt{\mathbf{YZ}}$.

Modal variables are commonly used in MTL theory in order to simplify the analysis. To this purpose we can consider a similarity transformation, in order to diagonalise the product $\mathbf{YZ}$. Since in our case $\mathbf{lc} = 1/c^2$, being c the speed of light in the vacuum, the only product $\mathbf{cr}$ has to be diagonalised. It is possible to introduce the similarity transformation matrix $\mathbf{T}$ so that $\mathbf{T}^{-1}\mathbf{crT} = \mathbf{\Gamma}^2$, being $\mathbf{\Gamma}$ a diagonal matrix. Since $\mathbf{c}$ and $\mathbf{r}$ are real, symmetric and positive definite matrices, then $\mathbf{T}$ and $\mathbf{\Gamma}$ are real matrices. It is then possible to express the product $\mathbf{YZ}$ as

$$(2) \qquad \mathbf{YZ} = j^2\frac{\omega^2}{c^2}\mathbf{T}\left(\mathbf{1} - j\frac{c^2}{\omega^2}\mathbf{\Gamma}^2\right)\mathbf{T}^{-1} = j^2\frac{\omega^2}{c^2}\mathbf{T\Psi}^2\mathbf{T}^{-1},$$

being $\mathbf{1}$ the identity matrix.

It is interesting to note that for the special case of lossless conductors, $\mathbf{\Gamma}$ vanishes and so $\mathbf{\Psi}$ is an identity matrix at every frequency. In this case the best choice for $\mathbf{T}$ is $\mathbf{T} = \sqrt{c\mathbf{l}}^{-1}$.

Then we can define the modal voltages and currents, given respectively by $\widetilde{\mathbf{V}}$ = $-jc/\omega\mathbf{T}^{-1}\mathbf{Y}\,\overline{\mathbf{V}}$ and $\widetilde{\mathbf{I}}$ = $\mathbf{T}^{-1}\,\overline{\mathbf{I}}$, that turn (1) into

$$\text{(3)}\qquad \begin{cases}\widetilde{\mathbf{V}}_{n+1} = \cos(u\mathbf{\Psi})\widetilde{\mathbf{V}}_n - j\mathbf{\Psi}\sin(u\mathbf{\Psi})\widetilde{\mathbf{I}}_n \\ \widetilde{\mathbf{I}}_{n+1} = -j\sin(u\mathbf{\Psi})\mathbf{\Psi}^{-1}\widetilde{\mathbf{V}}_n + \cos(u\mathbf{\Psi})\widetilde{\mathbf{I}}_n\end{cases}$$

being $u = L\omega/c$.
The transformation in (3) decouples the chain matrix equations and simplifies the following calculations. It is worth noting that in the modal domain the characteristic impedance matrix of the non-grounded MTL becomes a diagonal matrix too, and in particular

$$\text{(4)}\qquad \widetilde{\mathbf{Z}}_0 = -j\frac{c}{\omega}\mathbf{T}^{-1}\mathbf{Y}\dot{\mathbf{Z}}_0\mathbf{T} = \mathbf{\Psi}.$$

The chain matrix at the grounding point is, in the modal domain

$$\text{(5)}\qquad \begin{cases}\widetilde{\mathbf{V}}_{n+1} = \widetilde{\mathbf{V}}_n \\ \widetilde{\mathbf{I}}_{n+1} = -\widetilde{\mathbf{G}}\widetilde{\mathbf{V}}_n + \widetilde{\mathbf{I}}_n\end{cases}$$

being $\widetilde{\mathbf{G}}$ = $-j\omega/c\mathbf{T}^{-1}\mathbf{G}\mathbf{Y}^{-1}\mathbf{T}$, and $\mathbf{G}$ an $m\times m$ matrix where all the elements are zero but the last one, $\mathbf{G}_{m,m} = 1/R_g$, being R_g the grounding resistance. While $\mathbf{G}$ is almost an empty matrix, $\widetilde{\mathbf{G}}$ is a singular real positive full matrix. So the chain matrix of a single cell of a grounded MTL is given by the product of the chain matrices (3) and (5), obtaining

$$\text{(6)}\qquad \begin{cases}\widetilde{\mathbf{V}}_{n+1} = \cos(u\mathbf{\Psi})\widetilde{\mathbf{V}}_n - j\mathbf{\Psi}\sin(u\mathbf{\Psi})\widetilde{\mathbf{I}}_n \\ \widetilde{\mathbf{I}}_{n+1} = -\left[\widetilde{\mathbf{G}}\cos(u\mathbf{\Psi}) + j\sin(u\mathbf{\Psi})\mathbf{\Psi}^{-1}\right]\widetilde{\mathbf{V}}_n + \\ \qquad\qquad \left[j\widetilde{\mathbf{G}}\mathbf{\Psi}\sin(u\mathbf{\Psi}) + \cos(u\mathbf{\Psi})\right]\widetilde{\mathbf{I}}_n\end{cases}$$

It is possible to define $\widetilde{\mathbf{Z}}_c$ as the characteristic impedance matrix of the periodically grounded MTL in the modal domain, related to the actual characteristic impedance matrix by the relation $\dot{\mathbf{Z}}_c$ = $-j\omega/c\mathbf{Y}^{-1}\mathbf{T}\,\widetilde{\mathbf{Z}}_c\,\mathbf{T}^{-1}$according to (4). Being the structure semi-infinite, it is clear that voltages and currents are linked by the relation $\widetilde{\mathbf{V}}_i = \widetilde{\mathbf{Z}}_c\widetilde{\mathbf{I}}_i$, for every i = 0, 1, ... Enforcing this relation in (6), and considering that it has to be valid for every set of voltages and currents, by some manipulations it is possible to obtain a second order equation where the only unknown is $\widetilde{\mathbf{Z}}_c$,namely:

$$\text{(7)}\qquad \widetilde{\mathbf{Z}}_c\mathbf{C}\widetilde{\mathbf{Z}}_c - \widetilde{\mathbf{Z}}_c\mathbf{D} - \mathbf{A}\widetilde{\mathbf{Z}}_c + \mathbf{B} = 0$$

where

$$\text{(8)}\qquad \mathbf{A} = -\cos(u\mathbf{\Psi}),$$

$$\text{(9)}\qquad \mathbf{B} = -j\mathbf{\Psi}\sin(u\mathbf{\Psi}),$$

$$\text{(10)}\qquad \mathbf{C} = \widetilde{\mathbf{G}}\cos(u\mathbf{\Psi}) + j\sin(u\mathbf{\Psi})\mathbf{\Psi}^{-1},$$

$$\text{(11)}\qquad \mathbf{D} = \cos(u\mathbf{\Psi}) + j\widetilde{\mathbf{G}}\mathbf{\Psi}\sin(u\mathbf{\Psi}).$$

This equation is a non-symmetric algebraic Riccati equation (NARE) with complex coefficients, probably one of the worst cases among the Riccati equations. Such a problem has been widely studied in literature and some methods have been proposed to solve it numerically [19], [20]. Before presenting solution methods for this equation, a consideration on the MTL per unit length parameters frequency dependence has to be done. If we suppose that the three parameters are frequency independent, the diagonalisation matrix $\mathbf{T}$ is frequency independent too. The diagonalisation of $\mathbf{YZ}$ has to be performed just once and the method can be applied easily. A frequency dependence of the line parameters can be introduced anyway with minimum effort, for instance to take into account the skin effect into $\mathbf{r}$. In this case the proposed method can be applied as well, but the diagonalisation matrix $\mathbf{T}$ is frequency dependent and so the diagonalisation of $\mathbf{YZ}$ has to be repeated at each frequency step.

Solution of the Riccati equation

Now we will present two ways of solving the Riccati equation, which will then be implemented; the first one is an iterative algorithm based on a Newton-Raphson method [21], [22], the second method leads to a straightforward computation of the solution by means of the decomposition of the Hamiltonian matrix of the equation [23]. In order to implement the iterative algorithm based on a Newton-Raphson method, it is preferable to operate with real equations. By introducing the unknown resistance and reactance matrixes from $\widetilde{\mathbf{Z}}_c = \widetilde{\mathbf{R}}_c + j\widetilde{\mathbf{X}}_c$, the NARE (7) can be split into two second order matrix equations. If the line has m conductors, $2m^2$ real equations are obtained by this process. Anyway, due to the reciprocity of the characteristic impedance matrix and so to its symmetry, only $m(m + 1)$ equations are needed. If we consider a general equation

$$\text{(12)}\qquad f_i\left(x_1,\dots,x_{m(m+1)}\right) = 0, \qquad i = 1,\dots,m(m+1),$$

an iterative procedure can be set-up as follow

$$\text{(13)}\qquad f_i\left(x_1^{(k)},\dots,x_{m(m+1)}^{(k)}\right) + \sum_{j=1}^{m(m+1)} \left.\frac{\partial f_i}{\partial x_j}\right|_{x^{(k)}} \left(x_j^{(k+1)} - x_j^{(k)}\right) = 0$$

Since the equations includes quadratic terms at most, the computation of the derivatives is simple. In a frequency sweep, the best starting point for the iteration at a given frequency is the solution found at the previous frequency step, adopting a step frequency not so large compared to the variations of the characteristic impedance. In case of single frequency, the starting point has to be considered carefully since the problem admits multiple solutions. The second method refers to non-iterative methods. Most of them are based on the decomposition of the Hamiltonian matrix of the problem. The Hamiltonian matrix of (7) is defined as

$$\text{(14)}\qquad \mathbf{H} = \begin{pmatrix}\mathbf{D} & \mathbf{C} \\ -\mathbf{B} & -\mathbf{A}\end{pmatrix}.$$

If it possible to decompose the Hamiltonian matrix as

$$\text{(15)}\qquad \mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{U}^{-1},$$

if the matrix $\mathbf{U}$ is partitioned as $\mathbf{U} = \begin{pmatrix}\mathbf{U}_{11} & \mathbf{U}_{12} \\ \mathbf{U}_{21} & \mathbf{U}_{22}\end{pmatrix}$ and $\mathbf{U}_{11}$ is nonsingular, then the solution of the (7) is obtained as

$$\text{(17)}\qquad \widetilde{\mathbf{Z}}_c = -\mathbf{U}_{21}\mathbf{U}_{11}^{-1}.$$

This result can be proved since $\begin{bmatrix}1 \\ \widetilde{\mathbf{Z}}_c\end{bmatrix}$ is an invariant subspace of $\mathbf{H}$. The decomposition (15) can be performed in different ways, a diagonalisation of $\mathbf{H}$ is the preferable solution since it doesn't require a significant computational effort. More generally a Jordan decomposition can be adopted; this method includes also the cases when $\mathbf{H}$ cannot be diagonilized. A Schur decomposition can be used as well, but it requires higher computational effort, however, the algorithms are more stable than the ones implementing the Jordan decomposition. In order to analyze and evaluate these methods, some numerical simulations are now performed. The line geometry considered for the numerical simulations is a two-conductor line made by a non-grounded wire at an height of 14.8 m, located beneath a periodically grounded wire at an height of 16 m (these are typical heights of an Italian power transmission line). The cross section and the per-unit length resistivity are 148.5 mm^2 and 0.2282 Ω/km respectively, for the grounded cable, and 227.8 mm^2 and 0.1576 Ω/km for the lower cable. A reference length L = 100 m is assumed between two following grounding points. A grounding resistance of 1 Ω, 10 Ω and 100 Ω has been considered. The characteristic impedance has been computed by both methods, and relative difference between the two is always lower than $1 \cdot 10^{-13}$, that proofs that the two methods give practically the same solution. Further, by substituting the solutions into (7), the norm of the residue is lower than $4 \cdot 10^{-13}$ compared to the norm of the solution and that proves the correctness of the solution. In Figures 2-7 the real and imaginary parts of the different terms of the characteristic impedance are presented, as function of the frequency and for different values of the grounding resistance. By making the different simulations, we observe an important result: either including or omitting the per unit length resistance in the MTL parameters has no observable effects on the characteristic impedance. The significant result is that we can always simplify the problem by considering the line as lossless. From a strict mathematical point of view, we observe that for p.u.l. resistances higher than 10 Ω/km differences show up for the characteristic impedance, but these values are not realistic and so we can absolutely consider the MTL as lossless.

Conclusions

The problem of the characteristic impedance of a multiconductor transmission line with one conductor periodically grounded has been discussed and formulated in terms of a NARE equation. Two methods have been presented and applied in order to solve the equation. Further investigations are required to test and compare the robustness and efficiency of the two methods for more complex MTL configurations.

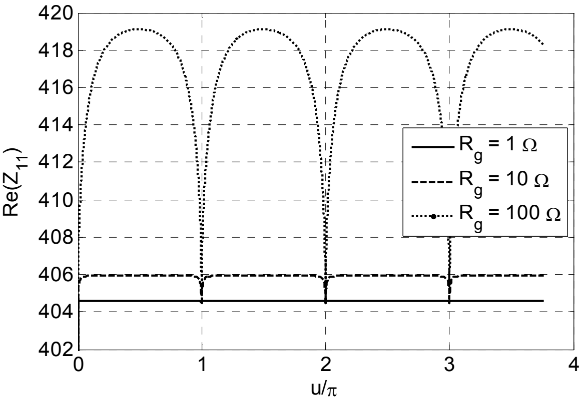

Fig.2. Real part of $\dot{\mathbf{Z}}_{11}$

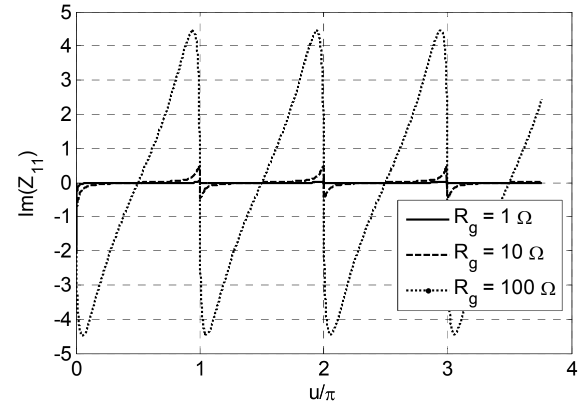

Fig.3. Imaginary part of $\dot{\mathbf{Z}}_{11}$

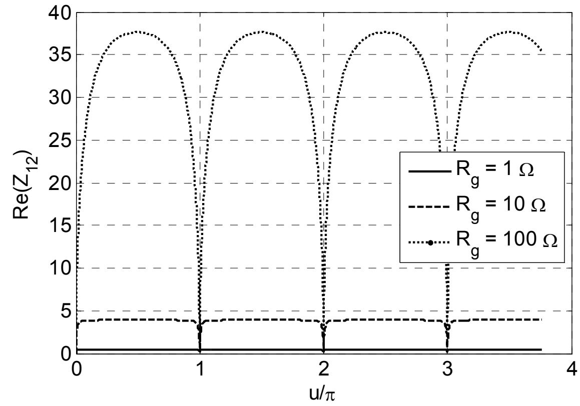

Fig.4. Real part of $\dot{\mathbf{Z}}_{12}$

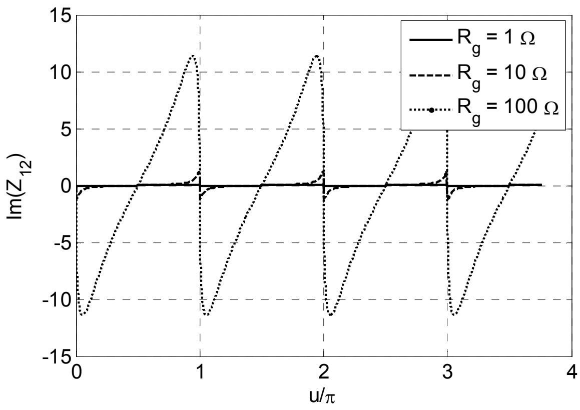

Fig.5. Imaginary part of $\dot{\mathbf{Z}}_{12}$

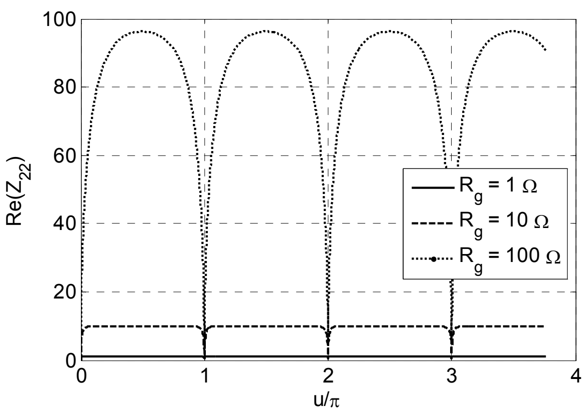

Fig.6. Real part of $\dot{\mathbf{Z}}_{22}$

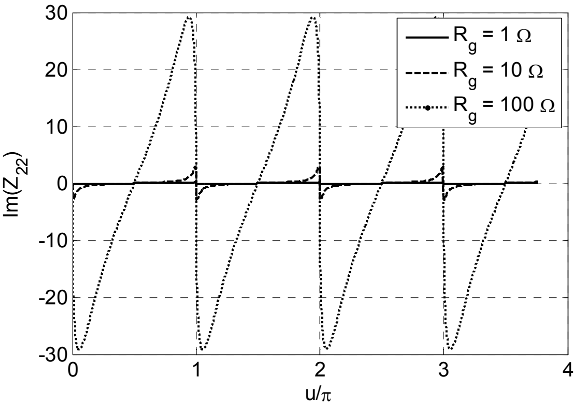

Fig.7. Imaginary part of $\dot{\mathbf{Z}}_{22}$

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