

# Influence of supplying voltage on deformation of current in transformer

**Abstract.** The paper presents a new method for modeling transformer with a hysteresis loop of Jules-Atherton. The Micro-Cap program allows the modeling of magnetically coupled coils by using description of the parameters of the magnetic circuit connecting the coil and the shape of ferromagnetic hysteresis. The way of determination of parameters describing the shape of the magnetization curve, taking into account the phenomenon of ferromagnetic hysteresis are presented. With the assumed parameters of the magnetic circuit, the calculations illustrating the operation of the transformer for the selected voltages were performed.

**Streszczenie.** W artykule przedstawiono nową metodę modelowania transformatora z uwzględnieniem pętli histerezy Jules'a-Athertona. Program Micro-Cap pozwala na modelowanie cewek sprężonych magnetycznie poprzez opis parametrów obwodu magnetycznego łączącego cewki oraz kształt histerezy ferromagnetyka. Przedstawiono sposób wyznaczania parametrów opisujących kształt krzywej magnesowania użytego ferromagnetyka z uwzględnieniem zjawiska histerezy. Przy założonych parametrach obwodu magnetycznego wykonano obliczenia symulacyjne ilustrujące pracę transformatora dla wybranych napięć zasilających. (Wpływ napięcia zasilającego na odkształcenie prądu w transformatorze).

**Keywords:** transformer model, hysteresis loop, higher harmonics of current

**Słowa kluczowe:** model transformatora, pętla histerezy, wyższe harmoniczne prądu

## 1. Introduction

A prime example of ambiguous circuit current-voltage characteristics of an element is the with ferromagnetic core described dependence  $B=f(H)$  with the hysteresis [2]. Examples of such devices are inductors and transformers.

Appropriate modeling of hysteresis is important to determine the magnetizing current and hence current-voltage dependence. In the past, authors tried many methods of approximation and modeling of the magnetization curve. Model proposed in 1986 by David Jiles and David Atherton is one of the most popular and is widely used as a tool for modeling circuits, both magnetic and electromagnetic [1]. This model can be used to describe the hysteresis effect of any ferromagnetic material which describes the magnetization  $M$  as a function of magnetic field  $H$  through the nonlinear differential equation.

## 2. Core material model

The Jiles-Atherton model assumes that total magnetization  $M$  of ferromagnetic material is decomposed into an irreversible  $M_{irr}$  and a reversible  $M_{rev}$  magnetization component [1,4].

The form of total differential magnetization susceptibility  $dM/dH$  is:

$$(1) \quad \frac{dM}{dH} = (1 - C) \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} + C \frac{dM_{an}}{dH}$$

where  $M_{an}$  is anhysteretic magnetization curve represented by Langevin function:

$$(2) \quad M_{an} = M_s \left[ \operatorname{cth} \left( \frac{H+\alpha M}{a} \right) - \frac{a}{H+\alpha M} \right]$$

- $H$  magnetic field strength
- $M_{an}$  magnetization in the absence of hysteresis
- $M_s$  magnetization saturation
- $\alpha$  average parameter fields
- $a$  the shape parameter /in the model: A parameter/
- $K$  domain wall pinning constant, proportional to hysteresis losses
- $\delta$  is equal +1 where  $dH/dt > 0$  or -1 where  $dH/dt < 0$
- $C$  deformation parameter of non flexible domain walls

The induction  $B$  can be calculated using:

$$(3) \quad B = \mu_0(H + M)$$

Figure 1 presents the hysteresis loop with characteristic points for the transformer from figure 2.

Authors used a modified model of a nonlinear magnetic material proposed by the D. Jiles and D. Atherton in Orcad program [3].

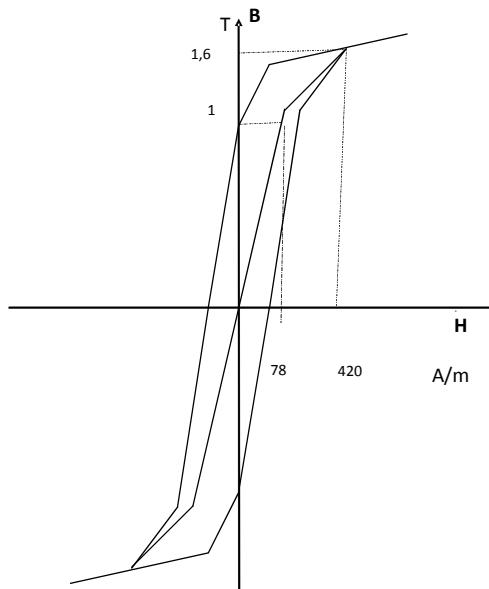


Fig. 1. The hysteresis loop with characteristic points

We have assumed the following core parameters of the magnetization curve characteristic: points as follows:

$$\begin{aligned} B_r &= 1 \text{ T} & H_c &= 200 \text{ A/m} \\ B_x &= 1.3 \text{ T} & H_x &= 789 \text{ A/m} \\ B_n &= 1.6 \text{ T} & H_n &= 4200 \text{ A/m} \end{aligned}$$

magnetic permeability of the demagnetization

$$(4) \quad \mu' = \frac{B_r}{\mu_0 * H_c} = \frac{1}{4 * \pi * 10^{-7} * 200} = 3968$$

average magnetic permeability

$$(5) \quad \mu = \frac{B_x}{\mu_0 * H_x} = \frac{1.3}{4 * \pi * 10^{-7} * 789} = 1330$$

deformation parameter of flexible domain walls

$$(6) \quad C = \frac{\mu - 1}{\mu' - \mu} = \frac{\mu_0 * H_c * (\mu - 1)}{B_r - \mu * \mu_0 * H_c} = 0,498$$

deformation parameter of non flexible domain walls

$$(7) \quad K = H_c \frac{\mu'}{\mu' - 1} = \frac{H_c * B_r}{B_r - \mu_0 * H_c} = 200$$

magnetization saturation

$$(8) \quad M_s = \frac{B_n}{\mu_0} - H_n = \frac{1,6}{4 * \pi * 10^{-7}} - 4200 = 1,27 * 10^6 \text{ A/m}$$

$$(9) \quad M_x = \frac{B_x}{\mu_0} - H_x = 1,03 * 10^6 \text{ A/m}$$

the shape parameter

$$(10) \quad A = \frac{\frac{H_x - C}{1+C} \frac{M_x}{\mu - 1}}{\frac{1}{1 - \frac{M_x}{M_s}} - 3 * \frac{M_x}{M_s}} = 264 \text{ A/m}$$

Figure 2 presents circuit model of the transformer described by equations 1-3 with parameters: input voltage  $U_1 = 1000V$ , numbers of primary winding turns  $N_1=230$  and secondary windings  $N_2=23$ , cross-sectional area of the core  $S=144 \text{ cm}^2$ , the average length of the magnetic flux path  $l_{av}=100 \text{ cm}$ .

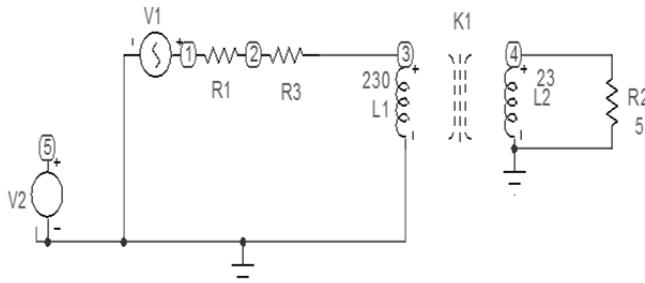


Fig. 2. Circuit model of the analyzed transformer with hysteresis loop

In numerical simulation we assumed the additional resistance ( $R_3$ ) of the value controlled we have by voltage source  $V_2$ , limiting the current at the time of switching of the circuit. Resistance value was changing from  $200 \Omega$  to  $0 \Omega$  in the first 80 ms of the period. Load resistance was equal  $R_2=5 \Omega$ .

The characteristic related to the magnetization curve of the coil  $L_1$  is dependent on the supply voltage level.

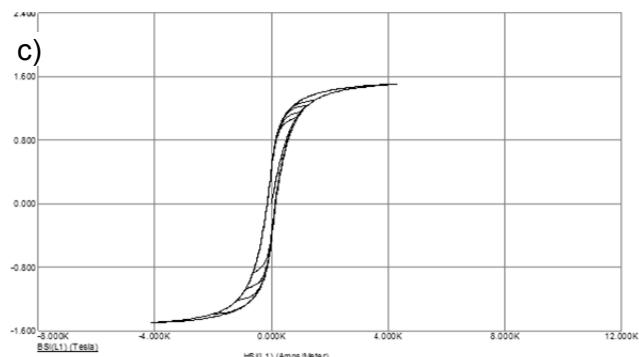
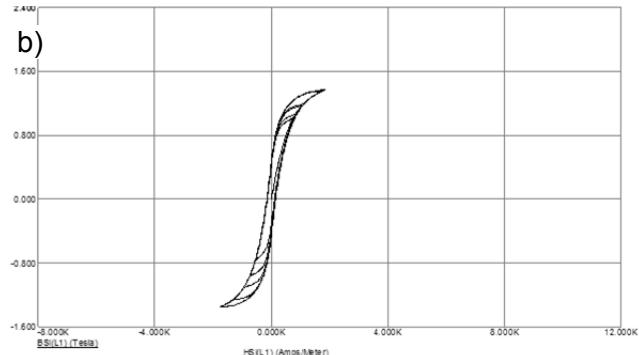
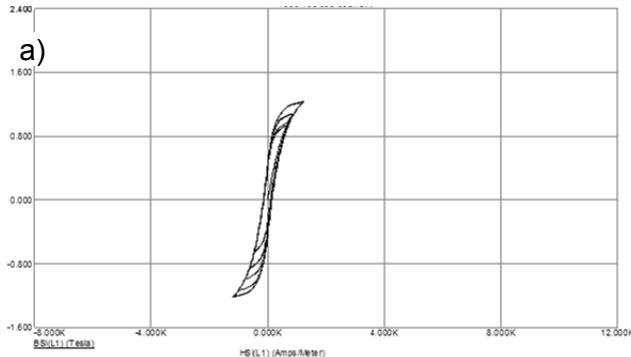


Fig. 3. The characteristics  $B=f(H)$  in the same coordinates scale for different input voltages: a) 900V, b) 1000V, c) 1100V

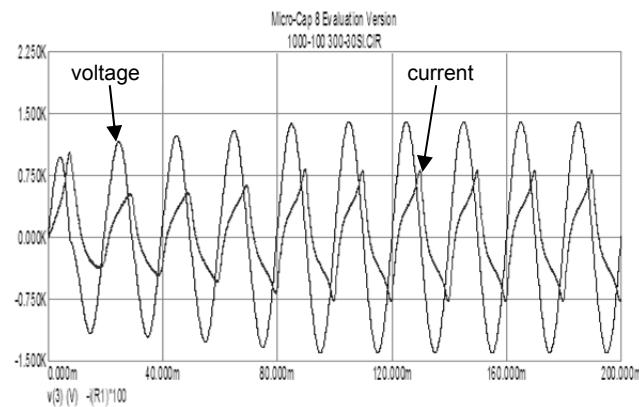


Fig. 4. Transient waveforms of current and voltage on the primary side of transformer (input voltage 1000V)

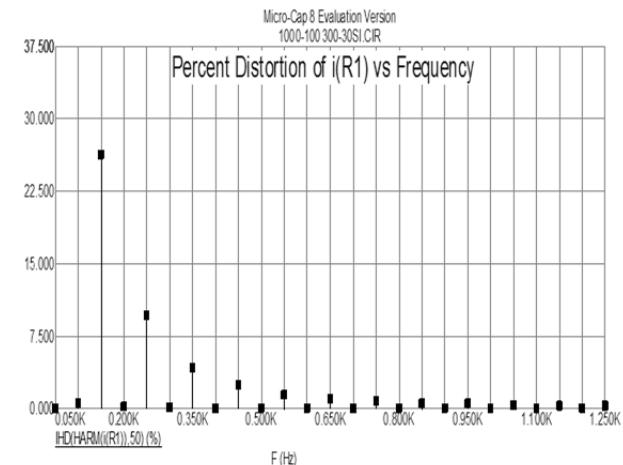


Fig. 5. Power spectrum of the current on the primary side of transformer (input voltage 1000V)

### 3. Numerical results

Exemplary computations were performed for following voltages:  $U_1=900$  V,  $U_2=1000$  V and  $U_3=1100$  V. The characteristics  $B = f (H)$  in the same scale coordinates are shown in Figure 3.

Transient waveforms of the current and voltage on the primary side of the modeled transformer with the power  $S_{Lem}$  voltage  $U_2=1000$  V are presented in figure 4. Figure 5 presents power spectrum of the current on the primary side of transformer.

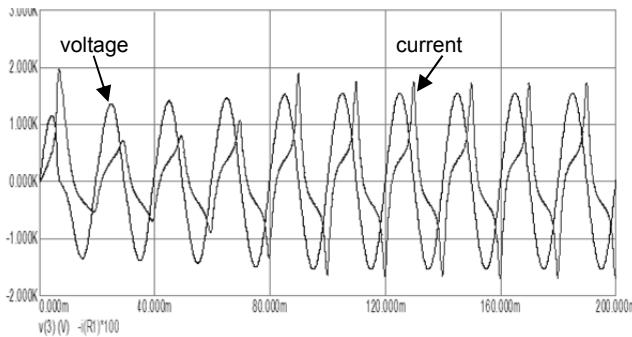


Fig. 6. Transient waveforms of current and voltage on the primary side of transformer (input voltage 1100V)

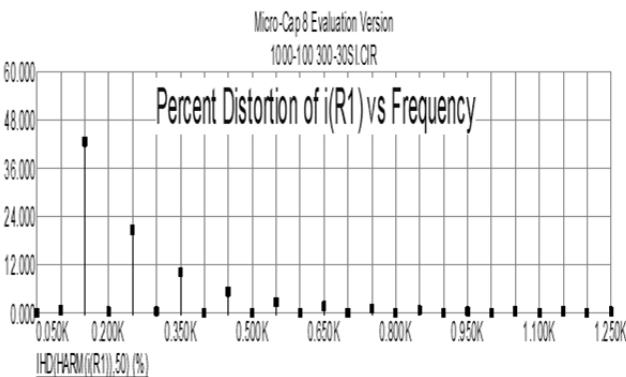


Fig. 7. Power spectrum of the current on the primary side of transformer (input voltage 1100V)

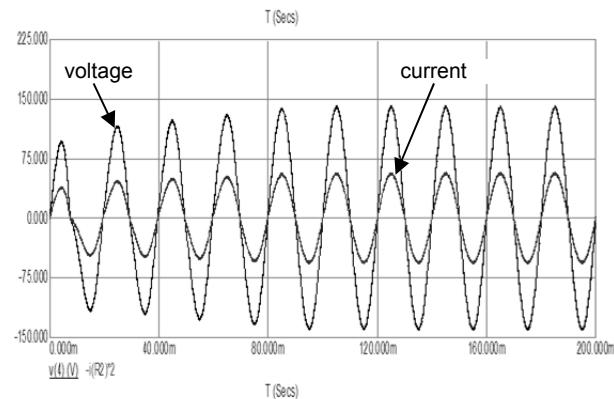


Fig. 8. Transient waveforms of current and voltage on the secondary side of transformer (input voltage 1100V)

Transient waveforms of the current and voltage on the secondary side of the modeled transformer with the power system voltage  $U_3=1100$  V are presented in figure 6. Figure 7 presents power spectrum of the current on the primary side of transformer. Transient waveforms of current and

voltage on the secondary side of transformer are shown in figure 8.

Voltages and currents on the secondary side have sinusoidal character (non deformed). Deformation of the current flowing through the resistance  $R_1$  shows the proportion of the magnetizing current, depending on the level of the supply voltage. The combined results of the analysis are shown in Table I.

Table I. Participation of higher harmonics of the supply current at different voltage levels on the primary side

voltage	Participation of higher harmonics							
	$I_3\%$	$I_5\%$	$I_7\%$	$I_9\%$	$I_{11}\%$	$I_{13}\%$	$I_{15}\%$	$I_{17}\%$
$U_1=0,9kV$	20,3	6,9	3,1	1,7	1,1	0,7	0,5	0,3
$U_2=1 kV$	26,3	9,61	4,1	2,3	1,3	1,0	0,7	0,5
$U_3=1,1kV$	42,7	20,9	10,1	5,2	2,7	1,6	0,9	0,7

### 4. Conclusions

On the basis of performed simulations we can draw the following conclusions:

- the higher odd (3,5,7,...) harmonics arise when transformer with hysteresis loop is applied,
- the voltage distortion is reduced at increasing the short-circuit power of the system.

The performed experiments allow to determine the propagation of the higher harmonics of the voltage & current created by the transformer with hysteresis loop applied in the power system. The model of transformer can be easily extended to the other nonlinear function including the hysteresis.

The numerical analysis conducted by the authors allows to estimate influence of the shape of magnetic hysteresis loop on the converter characteristic. Results of analysis can also be used to assess the propagation of higher harmonic for a current and voltage converters. Such converters are widely used to measure a current and voltage distortion in medium and high voltage power systems.

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