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A mathematical model of a synchronous drive with protrude poles, an analysis using variational methods

***Abstract**. In the paper a mathematical model of a synchronous drive with protrude poles in physical cooeridantes of magnetic couplings. The system is considered as having concentrated parameters. For formulation of differential state equations a novel interdisciplinary method based on a modification of the well-known Hamilton-Ostrogradsky principle. On the basis of the model the transient states of the drive system with synchronous motor were analyzed. The results of computer simulations were presented in the graphical form.*

***Streszczenie.** W pracy przedstawiono model matematyczny napędu synchronicznego o biegunach jawnych w fizycznych współrzędnych sprzężeń magnetycznych. System rozpatrywany jako układ o parametrach skupionych. Dla sformułowania różniczkowych równań stanu wykorzystano nawą interdyscyplinarną metodę, która bazuje na modyfikacji znanej zasady Hamiltona-Ostrogradskiego. Na podstawie modelu poddano analizie stany nieustalone pracy układu napędowego z silnikiem synchronicznym. Wyniki symulacji komputerowej przedstawiono w postaci graficznej. (**Model matematyczny napędu synchronicznego z biegunami jawnymi, analiza z zastosowaniem metod wariacyjnych**).*

Słowa kluczowe: zasada Hamiltona-Ostrogradskiego, Euler-Lagrange'a systemy, napęd synchroniczny, nieliniowe równania różniczkowe.
Keywords: Hamilton-Ostrogradsky's rule, Euler-Lagrange's system, synchronous drive, nonlinear differential equations.

Introduction

Synchronous drives play a significant role in heavy industry. One of their advantages is their constant rotational velocity in a wide range of changes of load moment, what is particularly important in heavy work regimes, e.g. drives of drilling machines, mine drives, elevators, rolling mill machines, fans, etc. [2]. But the main applications of these machines are energy sources in different power engineering stations. There exist two types of synchronous machines: the first one – machines with high rotational velocity – are turbogenerators and turbomotors with hidden poles. For those machines the stators are produced for one or two pole pairs. Another type are machines with low rotational velocity (with protrude poles), which have more than two pole pairs.

In the present paper a mathematical model of a synchronous model in physical coordinates of magnetic couplings, the so-called Ψ-model is presented. One important aspect is that the model is based on a modification of the well-known Hamilton-Ostrogradsky principle [1, 4], what makes it possible to avoid decomposition of the uniform electromechanical system. Such approach is useful in multi-machine electromechanical systems, when load moments of the motors are rather complicated.

The aim of the paper is to develop a mathematical model of synchronous drive with protrude poles, starting from an interdisciplinary method, as well as to analyze electromechanical states on the basis of the developed model.

Mathematical model of the system.

In the mathematical model, the coordinate system related to rotor, i.e. (d,q) system is used [3]. In the generalized coordinates it was assumed: electric charges in motor windings $q_{1-3} = \bar{Q}_{1-3}$; $q_4 = \bar{Q}_4 \equiv \bar{Q}_D$ – charge of attenuation circuit along *d* axis of the rotor, $q_5 = \bar{Q}_5 \equiv \bar{Q}_Q$ – charge of attenuation circuit along *q* axis of the rotor, $q_6 = \bar{Q}_6 \equiv \bar{Q}_f$ – charge of excitation circuit along *d* axis of the rotor; rotation angle of the rotor and the driven inertial component – $q_7 = \gamma$. By analogy, the currents in those widings have been introduced $\dot{q}_{1-6} = i_{1-6}$ and the rotational velocity of rotor and the driven mechanism $\dot{q}_7 = \omega$, where k – the number of generalized coordinates ie. three currents in the stator winding i_{SA}, i_{SB}, i_{SC}, three currents in the rotor winding i_D, i_Q, i_f and a single one rotational velocity, ie. together $k = 7$.

It should be remembered that the rotor of the synchronous motor includes a cage winding for asynchronous start-up [3] or for decreasing motor vibrations, if the machine operates as generator.

In the theory of electric machines a multi-phase system of rotor currents is replaced with an equivalent three-phase system. Such approach results in a substantial simplification of mathematical modelling at the same time it does not influence the results of simulation calculations.

For the assumed electromechanical system the Lagrangian components [1] are given with the relationships:

$$\tilde{T}^* = \sum_{j=1}^{3} \int_0^{i_{Sj}} \Psi_{Sj}(i_{Sj}) di_{Sj} + \sum_{n=1}^{3} \int_0^{i_n} \Psi_n(i_n) di_n + \frac{J_\Sigma \omega^2}{2}; \tag{1}$$

$$\Phi^* = \frac{1}{2}\int_0^t \sum_{j=1}^{3} r_{Sj} i_{Sj}^2 d\tau + \frac{1}{2}\int_0^t \sum_{n=1}^{3} r_n i_n^2 d\tau, \qquad P^* \equiv 0; \tag{2}$$

$$D^* = \int_0^t \sum_{j=1}^{3} u_{Sj} i_{Sj} d\tau + \int_0^t u_f i_f d\tau + \int_0^t \int_0^\omega M_{EM} d\omega d\tau -$$

$$\int_0^t \int_0^\omega M(\omega) d\omega d\tau, \qquad j = A, B, C, \qquad n = D, Q, f, \tag{3}$$

where: $\tilde{T}^*$ – kinetical coenergy, P^* – potential energy, Φ^* – dissipation energy, D^* – energy of external forces, M_{EM} – elektromagnetic moment of the motor, $M(\omega)$ – load moment of the drive, J_Σ – total inertial moment of the electric drive, $\bar{Q}$ – circuit charge, $i = \dot{\bar{Q}}$ – circuit current, Ψ – total magnetic coupling, R_S– resistance of stator winding, Ψ – total magnetic coupling; R_n– resistances of rotor windings, u_S – phase voltage of motor supply, u_f – constant supply voltage of the rotor, τ – additional integration variable $\mathbf{q} \equiv (\bar{Q}_{SA}, \bar{Q}_{SB}, \bar{Q}_{SC}, \quad \bar{Q}_D, \bar{Q}_Q, \bar{Q}_f, \gamma)^{\mathrm{T}}$;

$\dot{\mathbf{q}} \equiv (i_{SA}, i_{SB}, i_{SC},\ i_{RA}, i_{RB}, i_{RC}, \omega)^{\mathrm{T}}$ – columnar vectors of generalized coordinates and velocities.

The Hamilton variation of action functional [1, 4] was equated to zero:

(4) $$\delta S = \delta \int_0^{t_1} L^* dt = \int_0^{t_1} \delta L^* dt = 0 .$$

The Hamilton variation of action function shall be equal to zero only in the case, when the dynamical system is subject to Euler-Lagrange equations [2]

(5) $$\frac{d}{dt}\frac{\partial L^*}{\partial \dot{q}_s} - \frac{\partial L^*}{\partial q_s} = 0, \qquad L^* = T^* - P^* + \Phi^* - D^* ,$$

where L^* – modified Lagrange function [1].

On the basis (1) – (3) the Lagrangian was obtained, which was substituted next to Euler-Lagrange equation, obtaining finally:

(6) $$\Psi_{SA} + \Psi_{SB} + \Psi_{SC} = 0, \quad i_{SA} + i_{SB} + i_{SC} = 0 ;$$

(7) $$\mathbf{u}_S \equiv \begin{bmatrix} u_A \\ u_B \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2u_{SA} - u_{SB} - u_{SC} \\ 2u_{SB} - u_{SA} - u_{SC} \end{bmatrix}$$

The state equations have the following form:

(8) $$\frac{d\Psi_{SA}}{dt} = u_A - R_S i_{SA}, \quad \frac{d\Psi_{SB}}{dt} = u_B - R_S i_{SB} \ ;$$

(9) $$\frac{d\Psi_D}{dt} = -R_D i_D , \quad \frac{d\Psi_Q}{dt} = -R_Q i_Q , \quad \frac{d\Psi_f}{dt} = u_f - R_f i_f \ ;$$

(10) $$\frac{d\omega}{dt} = \frac{1}{J_\Sigma}\big(M_{EM} - M(\omega)\big), \qquad \frac{d\gamma}{dt} = \omega ;$$

The equations of magnetic couplings were written in the form
[1, 3]

(11) $$\mathbf{\Psi}_S \equiv \begin{bmatrix} \Psi_{SA} \\ \Psi_{SB} \end{bmatrix} = \boldsymbol{\alpha}_S^{-1}\mathbf{i}_S + \boldsymbol{\psi}_S = \begin{bmatrix} \alpha_S^{-1} & \\ & \alpha_S^{-1} \end{bmatrix}\begin{bmatrix} i_{SA} \\ i_{SB} \end{bmatrix} + \begin{bmatrix} \psi_{Sd} \\ \psi_{Sq} \end{bmatrix};$$

(12) $$\mathbf{\Psi}_R \equiv \begin{bmatrix} \Psi_D \\ \Psi_Q \\ \Psi_f \end{bmatrix} = \boldsymbol{\alpha}_R^{-1}\mathbf{i}_R + \mathbf{B}\boldsymbol{\psi}_R = \begin{bmatrix} \alpha_D^{-1} & & \\ & \alpha_Q^{-1} & \\ & & \alpha_f^{-1} \end{bmatrix}\begin{bmatrix} i_D \\ i_Q \\ i_f \end{bmatrix} + \begin{bmatrix} 1 & \\ & 1 \\ 1 & \end{bmatrix}\begin{bmatrix} \psi_{Rd} \\ \psi_{Rq} \end{bmatrix}, \quad \mathbf{B} \equiv \begin{bmatrix} 1 & \\ & 1 \\ 1 & \end{bmatrix},$$

where: $\boldsymbol{\psi}_S, \boldsymbol{\psi}_R$ – columnar vectors of fundamental (working) magnetic couplings of stator and rotor, respectively, ($\boldsymbol{\alpha}_S = \mathbf{L}_S^{-1}, \boldsymbol{\alpha}_R = \mathbf{L}_R^{-1}$) – matrices of inverse inductances of winding dissipation of stator and rotor, respectively; $\mathbf{B}$ – topological matrix.

We shall write down the equation of stationary magnetic coupling between fundamental couplings of stator and rotor, respectively [1]:

(13) $$\boldsymbol{\psi}_R = \mathbf{\Pi}\boldsymbol{\psi}_S = \boldsymbol{\psi}, \quad \boldsymbol{\psi}_S = \mathbf{\Pi}^{-1}\boldsymbol{\psi}, \quad \boldsymbol{\psi} \equiv (\psi_d, \psi_q)^{\mathrm{T}} .$$

On the basis of the second Kirchhoff law for magnetic circuits:

(14) $$\boldsymbol{\psi} = \mathbf{L}(\mathbf{i})\mathbf{i}, \quad \boldsymbol{\psi}_S = \mathbf{\Pi}^{-1}\mathbf{L}(\mathbf{i})\mathbf{i}, \quad \mathbf{i} = \mathbf{\Pi}\mathbf{i}_S + \mathbf{B}^{\mathrm{T}}\mathbf{i}_R, \quad \mathbf{i} = \mathbf{i}(\boldsymbol{\psi}) ,$$

where $\mathbf{\Pi}$ - Park matrix.

Expression (14) including (11), (12) takes the form:

(15) $$\boldsymbol{\psi} = \mathbf{L}(\mathbf{i})\Big(\mathbf{\Pi}\boldsymbol{\alpha}_S(\mathbf{\Psi}_S - \mathbf{\Pi}^{-1}\boldsymbol{\psi}) + \mathbf{B}^{\mathrm{T}}\boldsymbol{\alpha}_R(\mathbf{\Psi}_R - \mathbf{B}\boldsymbol{\psi}\Big) .$$

Finally the algebraic equation follows

(16) $$\boldsymbol{\psi} = \Big(\mathbf{1} + \mathbf{L}(\mathbf{i})(\boldsymbol{\alpha}_S + \mathbf{B}^{\mathrm{T}}\boldsymbol{\alpha}_R\mathbf{B})\Big)^{-1} \times \times\mathbf{L}(\mathbf{i})\Big(\mathbf{\Pi}\boldsymbol{\alpha}_S\mathbf{\Psi}_S + \mathbf{B}^{\mathrm{T}}\boldsymbol{\alpha}_R\mathbf{\Psi}_R\Big) .$$

In the case of linear dependence between magnetization currents and magnetic couplings the algebraic equation (16) takes the form:

(17) $$\boldsymbol{\psi} = \mathbf{L}^*\Big(\mathbf{\Pi}\boldsymbol{\alpha}_S\mathbf{\Psi}_S + \mathbf{B}^{\mathrm{T}}\boldsymbol{\alpha}_R\mathbf{\Psi}_R\Big) ,$$

where

(18) $$\mathbf{L}^* \equiv \mathrm{diag}\left(\frac{L_d}{1 + L_d(\alpha_S + \alpha_D + \alpha_f)}, \quad \frac{L_q}{1 + L_q(\alpha_S + \alpha_Q)}\right).$$

Currents in the stator and rotor windings on the basis of relationships (11) and (12) are given with

(19). $$\mathbf{i}_S = \boldsymbol{\alpha}_S(\mathbf{\Psi}_S - \mathbf{\Pi}^{-1}\boldsymbol{\psi}), \quad \mathbf{i}_R = \boldsymbol{\alpha}_R(\mathbf{\Psi}_R - \mathbf{B}\boldsymbol{\psi}) .$$

The start-up moment of the synchronous motor is calculated from [1]:

(20). $$M_E = \sqrt{3}p_0(\psi_{SA}i_{SB} - \psi_{SB}i_{SA}) ,$$

The full model of the drive system is described with differential equations: (8) – (10), (17) including (14), (18) – (20).

Results of computer simulations

For numerical analysis a synchronous fan drive with parameters: $P_N = 630$ kW, $n_N = 750$ ob/min; $U_N = 6$ kV; $u_f = 42$ V was assumed. The load moment of the motor $M_O = 5{,}007\omega + 14{,}8 \cdot 10^5\omega^3 + 2{,}21 \cdot 10^{11}\omega^5$.

In Fig. 1 the rotational velocity of the synchronous motor as function of time is presented. From the analysis of the Figure it follows, that for the fan synchronous drive the asynchronous state is unstable, what leads to oscillations with relatively high amplitude. This vibration amplitude is caused by complicated electromagnetic processes in the synchronous machine, what is depicted in Fig. 2. The amplitude of moment oscillations reaches 4 kNm and its own frequency is about 2 Hz.

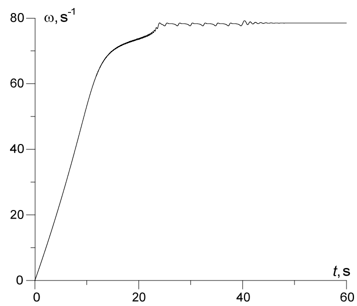

Fig. 1. Transient dependence of rotational velocity versus time for the synchronous drive

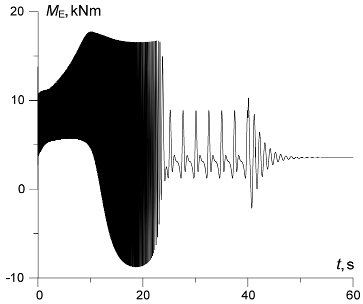

Fig. 2. Transient start-up moment for the synchronous motor

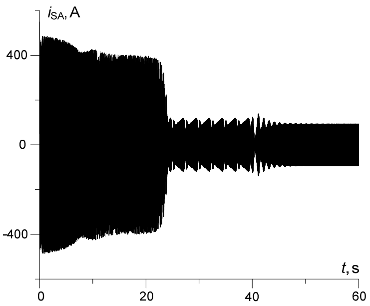

Fig. 3. Transient current in the phase A of the stator

Fig. 3 depicts the dependence of current in the phase A of the stator on time, whereas Fig. 4 the dependence of excitation current of the motor on time. From the analysis of these dependencies it can be stated, that high oscillations of current in the asynchronous state of the machine are visible, what is the result of current oscillations in the stator windigns. Such dependence of current is directly related to the reaction of machine armature.

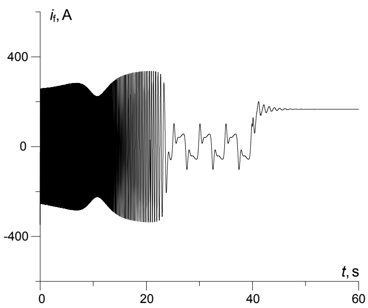

Fig. 4. Transient current of excitation rotor

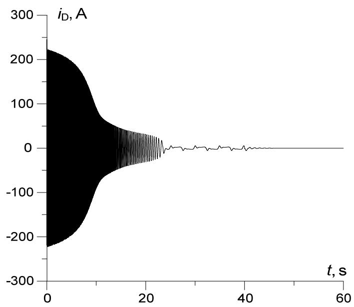

Fig. 5. Transient current in the attenuation winding of the rotor along *d* axis

In Fig. 5 the current in the attenuation winding in the *d* axis is shown. During start-up the oscillation frequency in the rotor cage drops from 50 Hz downto 1-2 Hz (in the asynchronous state). Oscillations of the current and its amplitude in the rotor cage in the range $t \in [22-40]$ are related to oscillations of current in the armature, i.e. to the armature reaction. Fig. 6 depicts transient current in the attenuation winding in the *q* axis. Here the transient dependences are similar to those in Fig. 5. Analyzing the aforementioned Figures it can be stated, that in the synchronous state all electromechanical processes fade out to constant values, what corresponds fully to the classical theory of electric machines.

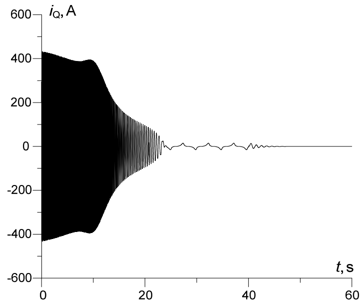

Fig. 5. Transient current in the attenuation winding of the rotor along *q* axis

Conclusions

1. Application of novel interdisciplinary method, developed in Ref. [1] made it possible to develop a mathematical model of the synchronous drive, avoiding at the same time the decomposition of the uniform electromechanical system, what is very efficient during modelling of multi-machine electromechanical systems, whose one component are synchronous machines.

2. On the basis of computer simulations realistic depictions of movements of functional dependencnies of the electromechanical system were obtained, what gives foundations for the analysis of those systems.

3. On the basis of computer simulations complicated physical processes in the synchronous drive were analyzed. Transient processes in the asynchronous drive were described, what of practical importance during analyses of similar systems, which include elastic mechanical components, e.g. a long elastic shaft, elastic clutches, lines, etc.

LITERATURA

[1] *Chaban A.* Modelowanie matematyczne procesów oscylacyjnych w systemach elektromechanicznych. (Wydanie drugie, zmienione i uzupełnione). – *Lwów: W-wo T. Soroki* 2008. – 328 s.

[2] *Kharchenko E.* Procesy dynamiczne w maszynach górniczych. – *Lwów: Sweet*, 1991. – 176 s.

[3] *Rusek A.* Model symulacyjno-komputerowy asynchronicznego silnika indukcyjnego z uwzględnieniem w modelu matematycznym nasycenia i zjawiska wypierania prądu, *Przegląd Elektrotechniczny R. 86* NR 12/2010, s.127-130.

[4] *Ortega R., Loria A., Nicklasson P.J., Sira-Ramirez H.* Passivity-Beast Control of Euler-Lagrange Systems: Mechanical, Electrical and Electromechanical Applications. *London: Springer Verlag*, 1998, 543 s

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