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# A Simple Modeling Method to Identify the Kinematic Errors of Machine Tool Rotary Table 


#### Abstract

A Simple modelling method to identify the kinematic errors of machine tool table is proposed. According to the structure of the machine tool, based on rigid body kinematics, the motion coordinates of major components of the machine tool table was reasonable set up, and the error propagation of the machine tool rotary table was described. The mathematical expression of this model is simple, and the physical meaning is understandable. It is a theoretical foundationon for further error recognition and error compensation.

Streszczenie. W artykule przedstawiono metodę rozpoznania błędów kinematycznych stołu do obrabiarki. Wyznaczono funkcje ruchu najważniejszych elementów oraz opisano sposób dalszej propagacji powstałego uchybu. Na tej podstawie możliwe było rozpoznanie błędu i jego kompensacja. (Prosta metoda modelowanie błędów kinematycznych w obrotowym stole do obrabiarki).


Keywords: machine tool; trunnion axis; kinematic error; Modeling
Słowa kluczowe: obrabiarka, biegun, oś czopa, błąd kinematyczny, modelowanie.

## Introduction

Five-axis machine tools have two rotary axes, and the motion errors of the rotary axes are accumulated in the positioning error of a tool relative to the workpiece. With the application of five-axis machine tools, there have been more cases where five-axis machine tools are used in high-precision machining application. The improvement of their kinematic accuracy is a crucial demand in the market. The structural configurations can be divided into the following three types: the first one is a universal spindle head with two controlled axes (RRTTT), the second one is a swivel head with a controlled axis and a rotary table (RTTTR) and the third one is a tilting rotary table with two con-trolled axes (TTTRR) [1,2]. The third type of machines tools are also called crad-type machine tools. They are the most commonly used small and medium-sized machine tools. International Organization for Standardization developed the relevant evaluation criteria. ISO 10791-1~ 3 developed a method to measure and evaluate the kinematic errors for universal spindle head type five-axis machine tools[3]. There is currently no ISO standard for the assessment of other forms of five-axis machine tools. However, some researches have been proposed to identify the kinematic errors of the machines tool with a tilting rotary table, such as the assessment method by telescoping double ball bar (DBB)[4] and the "R-Test", and so on. The inclusion of the measurements for other types of five-axis machine tools into the revised ISO standards has been currently discussed in TC39/SC2/WG3.

## A Mathematical Model Of Kinematic Errors

A cradle-type five-axis machine tool is studied in this paper. The structural configuration can be seen in Fig.1.
(1) Kinematic errors to be identified

There are three linear displacement errors ${ }^{C} \vec{\delta}_{x}(\gamma),{ }^{C} \vec{\delta}_{y}(\gamma),{ }^{C} \delta_{z}(\gamma)$ and three angular errors ${ }^{C} \vec{\xi}_{x}(\gamma)$, ${ }^{c} \vec{\xi}_{y}(\gamma)$ and ${ }^{c} \xi_{z}(\gamma)$ in C-axis. Where, ${ }^{c} \vec{\delta}_{x}(\gamma),{ }^{c} \vec{\delta}_{y}(\gamma),{ }^{c} \delta_{z}(\gamma)$ are the linear shifts in X-direction, Y-direction and Zdirection with respect to A-axis , respectively; ${ }^{c} \vec{\xi}_{x}(\gamma),{ }^{c} \vec{\xi}_{y}(\gamma)$ and ${ }^{c} \xi_{2}(\gamma)$ are the angular errors of the centre line of $C$ axis about X -direction, Y -direction and Z-direction with respect to $A$-axis, respectively. There are three linear displacement errors ${ }^{A} \delta_{x}(\alpha),{ }^{A} \delta_{y}(\alpha),{ }^{A} \delta_{z}(\alpha)$ and three angular errors ${ }^{A} \xi_{x}(\alpha),{ }^{A} \xi_{y}(\alpha),{ }^{A} \xi_{z}(\alpha)$ in A-axis. Where,
${ }^{A} \delta_{x}(\alpha),{ }^{A} \delta_{y}(\alpha),{ }^{A} \delta_{z}(\alpha)$ are the linear shifts in X-direction, Ydirection and Z-direction with respect to A-axis,


Fig. 1 Structural configurations of cradle-type five-axis machine tool
respectively; ${ }^{A} \xi_{x}(\alpha),{ }^{A} \xi_{y}(\alpha)$ and ${ }^{A} \xi_{z}(\alpha)$ are the angular errors of the centre line of A -axis about X -direction, Y direction and $Z$-direction with respect to $Y$-axis, respectively.

## (2) Machine tool coordinate system settings

According to the structure of the machine tool table, the coordinate systems were reasonably set up with a simple method in this paper, as can be seen in Fig.2. Firstly, establish the reference coordinate system $F$ in the centerline intersection point of A -axis and C -axis (point O ) when A-axis and C-axis remain stationary. Secondly, establish the coordinate system A (nominal coordinate system of A-axis) when the nominal displacement of A-axis is $\alpha$. Thirdly, when the actual displacement of $A$-axis is $\alpha$, the position of $A$-axis changes from $A_{1}$ to $A_{2}$. The center line of C -axis changes from $\mathrm{C}_{1}$ to $\mathrm{C}_{2}$. The intersection point of $A_{2}$ and $C_{2}$ is Point $O_{2}$. Establish the actual coordinate system of A-axis ( $A^{\prime}$ ) and the nominal coordinate system of $A$-axis $(C)$ in Point $O_{2}$. The nominal displacement of Coordinate system C to Coordinate system $A^{\prime}$ is $\gamma$. When the actual displacement of A-axis is $\alpha$ and the actual displacement of C -axis to Coordinate system $A^{\prime}$ is $\gamma$, the position of the centerline of C -axis changes from $C_{2}$ to $C_{3}$ because of the kinematic error. The center line of $A$-axis remains at the position of $A_{2}$. The intersection point of $C_{3}$ and $A_{2}$ is Point $O_{3}$. At last, establish the actual coordinate system of A-axis ( $C^{\prime}$ ) in Point $\mathrm{O}_{3}$.

The position of the workpiece is described in Coordinate system $C^{\prime}$.


Fig. 2 Kinematic error modeling of A-axis

## (3) The modeling process of the kinematic errors

The modeling process of the kinematic errors is described as follows:
(a) The differential transformation

There is a differential transformation from coordinate system $\mathrm{C}^{\prime}$ to coordinate system C . The differential transformation is completed by the differential movement transformation and the differential rotation transformation.

$$
\begin{equation*}
{ }^{c} \vec{W}={ }^{C} \vec{\delta}(\gamma)+{ }_{C^{\prime}}^{C} R^{C^{\prime}} \vec{W}_{N} \tag{1}
\end{equation*}
$$

Where, ${ }^{c} \vec{\delta}(\gamma)$ is the differential motion vector of coordinate system $\mathrm{C}^{\prime}$ to coordinate system C when A -axis turned the angle of $\gamma$ around Z-axis. ${ }^{c} \vec{\delta}(\gamma)$

$$
{ }^{c} \vec{\delta}(\gamma)=\left[\begin{array}{l}
{ }^{c} \vec{\delta}_{x}(\gamma)  \tag{2}\\
{ }^{c} \vec{\delta}_{y}(\gamma) \\
0
\end{array}\right]
$$

Based on the assumptions of small angle, the operator of the differential rotation transformation from coordinate system $\mathrm{C}^{\prime}$ to coordinate system C can be represented as
(3)

$$
{ }_{C^{c}}^{C} R=\left[\begin{array}{ccc}
1 & 0 & { }^{c} \xi_{y}(\gamma) \\
0 & 1 & { }^{-} \xi_{x}(\gamma) \\
-{ }^{c} \xi_{y}(\gamma) & { }^{c} \xi_{x}(\gamma) & 1
\end{array}\right]
$$

Similarly, there is a differential transformation from coordinate system $A^{\prime}$ to coordinate system A, too. The differential transformation is completed by the differential movement transformation and the differential rotation transformation.

$$
\begin{equation*}
{ }^{A} \vec{W}={ }^{A} \vec{\delta}(\gamma)+{ }_{A^{\prime}}^{A} R^{A^{\prime}} \vec{W}_{N} \tag{4}
\end{equation*}
$$

Based on the assumptions of small angle [10], the operator of the differential rotation transformation from Coordinate System $A^{\prime}$ to Coordinate System A can be represented as

$$
{ }_{A^{A}}^{A} R=\left[\begin{array}{ccc}
1 & -{ }^{A} \xi_{z}(\alpha) & { }^{A} \xi_{y}(\alpha)  \tag{5}\\
{ }^{A} \xi_{z}(\alpha) & 1 & -{ }^{A} \xi_{x}(\alpha) \\
-{ }^{A} \xi_{y}(\alpha) & { }^{A} \xi_{x}(\alpha) & 1
\end{array}\right]
$$

(c) Rotation transformation

There are two rotation transformation operators ${ }_{A}^{Y^{\prime}} R$
and ${ }_{C}^{A^{\prime}} R .{ }_{C}^{A^{\prime}} R$ is represented as
(6)

$$
{ }_{C}^{A^{\prime}} R=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \gamma & -\sin \gamma \\
0 & \sin \gamma & \cos \gamma
\end{array}\right]
$$

${ }_{A}^{Y^{\prime}} R$ is represented as

$$
{ }_{A}^{Y^{\prime}} R=\left[\begin{array}{ccc}
\cos \alpha & -\sin \alpha & 0  \tag{7}\\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{array}\right]
$$

From the above analysis, the workpiece position vector in coordinate system F can be obtained.

## Conclusions

A simple mathematic model of the kinematic errors of the machine tool rotary table is established by optimizing the machine tool coordinate system settings, and the mathematical expressions of the tool position in the workpiece coordinate system is deduced. It provides a mathematical basis for developing the machining tests to identify the kinematic errors.

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