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A hybrid optimization algorithm based on population migration algorithm and chaos theory

***Abstract**. A new hybrid optimization algorithm based on population migration algorithm (PMA) and chaos theory is proposed by introducing the logistic mapping of chaos theory into PMA. The proposed algorithm aims to improve solution accuracy and convergence, and avoid the prematurity of PMA. Experimental results show that the solution accuracy and convergence of the new algorithm can be effectively improved, and prematurity can be avoided by introducing ergodicity, randomicity, and regularity of the chaos theory into PMA.*

***Streszczenie.** W artykule przedstawiono hybrydowy algorytm optymalizacji, bazujący na algorytmie migracyjnym (PMA) i teorii chaosu. Proponowane rozwiązanie ma na celu zwiększenie dokładności, zbieżności oraz unikanie „wcześniactwa" PMA. Wyniki badań eksperymentalnych potwierdzają skuteczność proponowanego algorytmu. (**Hybrydowy algorytm optymalizacji – wykorzystanie algorytmu migracyjnego oraz teorii chaosu**).*

Keywords: Population Migration Algorithm, Chaos, Ergodicity.
Słowa kluczowe: algorytm migracyjny PMA, chaos, ergodyczność.

Introduction

Population migration algorithm (PMA) [1] is a new swarm intelligence optimization algorithm [2] provided by Zhou and Mao in 2003. PMA simulates a mechanism [3] in the social field wherein a population migrates with economic centre change and spreads with population pressure. When the relative surplus population or population pressure increases in a certain region, people will migrate to find better regions that suit their lifestyle. The diffusion mechanism causes the PMA algorithm to choose a better search field, whereas the emigration mechanism prevents the algorithm from getting into a local optimal solution to a certain degree. Many numerical experiments [4-6] reveal that PMA is an effective algorithm. However, several numerical analyses should be expanded because PMA is a new global optimization algorithm [7]. To improve the performance of PMA, the present paper combines chaos theory [8] with PMA. Ergodicity, randomicity, and regularity of the chaos theory are used to attain an improved algorithm by introducing the logistic mapping of chaos theory into PMA. Results show that the solution accuracy and convergence property of PMA can be effectively improved, and the prematurity phenomenon can be avoided by introducing ergodicity, randomicity, and regularity of chaos theory into PMA.

The remainder of this paper is organized as follows. Section II describes the chaos optimization algorithm. Section III provides the methodology. Section IV presents the experimental results. Section V concludes.

Chaos Optimization Algorithm

Chaotic motion is a widespread phenomenon that has properties of ergodicity, randomicity, and regularity existing in nonlinear systems. This motion has an ergodic effect on all states when it follows its own regulation without repetition in certain regions. The logistic mapping of chaos theory is chosen as follows:

(1) $$x_{n+1} = rx_n(1-x_n)$$

where: $r = 4$. A method similar to the carrier method is used to introduce chaos variables produced by logistic mapping into optimization variables. Meanwhile, the ergodic field of chaos is turned into a domain of definition of optimization variables. The chaos optimization algorithm is described below.

Step 1. Algorithm initialization: set $k=1, k'=1$; let x_n in Eq. (1) be endowed with i initial values with small difference, then i chaos variables $x_{i,n+1}$ are attained, which have different loci.

Step 2. The determined i chaos variables $x_{i,n+1}$ are changed into chaos variables $x'_{i,n+1}$ for optimization by Eq.

(2) $$x'_{i,n+1} = c_i + d_i x_{i,n+1}$$

where: c_i and d_i are the constants used for creating a scale variation of variables.

Step 3. Iterative search.

Let $x_i(k) = x'_{i,n+1}$ and count $f_i(k)$, then set $x_i^* = x_i(1)$, $f^* = f(1)$,

If $f_i(k) \le f^*$, set $f^* = f_i(k)$, $x_i^* = x_i(k)$; otherwise, give up $x_i(k)$. $k := k+1$.

Step 4. If f^* does not change after the search step in Step 3, it is carried by Eq. (3).

(3) $$x''_{i,n+1} = x_i^* + \alpha_i x_{i,n+1}$$

where: $\alpha_i x_{i,n+1}$ are small chaos variables in an ergodicity interval, and α_i are adjust constants. x_i^* is the best solution at present; otherwise, return to Step 3.

Step 5. Continue iterative search. Set $x_i(k') = x''_{i,n+1}$, counting $f_i(k')$: If $f_i(k') \le f^*$, then $f^* = f_i(k')$, $x_i^* = x_i(k')$; otherwise, give up $x_i(k')$. $k' := k'+1$.

Step 6. Finish the search until the content terminate criterion is satisfied, then output the best solution x_i^*, f^*; otherwise, return to Step 5.

Chaos Theory Based PMA (CPMA)

This section proposes an improved PMA, which is a combination of PMA and chaos theory.

A. Basic rationales of CPMA

Considering the following single-aim optimal problem:

(4) $$\min f(x) \quad s.t.\ x \in S$$

where: $f: S \to R$ is an actual value, $x \in R^n$, $S = \prod_{i=1}^{n}[a_i, b_i]$ is a search space, $a_i < b_i$. Eq. (4) is assumed to contain optimum solutions.

In CPMA, optimization variable x represents habitual residence, and $x^i = (x_1^i, x_2^i, \cdots, x_n^i)$ represents the i th point. $x^i \in R^n$, x_j^i represents the j th component of the i th point. $\delta^i \in R^n$, δ_j^i means the region radius of the j th component of δ^i, $\delta_j^i > 0$, $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots n$; N represents the population scale. The objective function corresponds to the regions where the population migrates. The optimal solution is represented by the most fascinating region. The ascendence of algorithm corresponds to the immigrants in a preferential area. The algorithm fled in the local optimal solution is equal to the immigrant population in a preferential area. Population mobility corresponds to local random research [1].

B. CPMA steps

Step 1. Initialization: input population scale N, initial area radius $\delta(0)$, and floating population scale l. Randomly produce N unities $X^{(i)}(0)$ $(i = 1, 2, \cdots, N)$ to form the initial population living area:

$$\Omega(0) = \bigcup_{i=1}^{N} B(X^{(i)}(0), \delta(0))$$

where $B(x, r)$ represents a ball with the center x, radius is r. $X_{best}(0) = \max_{1 \le i \le N} J(X^{(i)}(0))$, $t := 0$.

Step 2. Evolution steps.

(2.1) Preparation: set $\eta(t) = \delta(t)$, $\delta(t) = C_t \delta(0)$

(2.2) Population mobility: Randomly produce l unities in each $B(X^{(i)}(t), \eta(t))$. Therefore, a population $Y(t)$ is attained, which contains Nl unities.

(2.3) Population migration

(2.3.1) Choose N best unities from $Y(t)$ to compose the middle population

$$Y_{best}(t) = (Y_{best}^{(1)}(t), Y_{best}^{(2)}(t), \cdots, Y_{best}^{(N)}(t))$$

(2.3.2) Form N preferential area $\left\{B(Y_{best}^{(i)}(t), \eta(t)), i = 1, 2, \cdots, N\right\}$.

(2.3.3) Produce l_i small difference unities in $B(Y_{best}^{(i)}(t), \eta(t))$, l_i prorata with $Y_{best}^{(i)}$. New unities $Y_j^{(i)'}(j = 1, 2, \cdots, l_i)$ are attained by substituting these unities in Eqs. (1), (2), and (3), respectively; use iteration search change $Y_j^{(i)'}(j = 1, 2, \cdots, l_i)$ to $Y_j^{(i)''}(j = 1, 2, \cdots, l_i)$, when $\left|Y_{j+1}^{(i)''} - Y_j^{(i)''}\right| < \delta_1$ ends. δ_1 is a specified parameter. Choose the best unity from $Y_j^{(i)''}(j = 1, 2, \cdots, l_i)$.

(2.3.4) Shirk preferential area: $\eta(t) := (1 - \Delta)\eta(t)$, $0 < \Delta < 1$.

(2.3.5) If $\eta(t) > \alpha(t)$, then $B(Y_{best}^{(i)}(t), \eta(y)) := B(Z_{best}^{(i)}(y), \eta(t))$, turn to step (2.3.3); otherwise, go to Step (2.3.6)

(2.3.6) Report the N best unities $Z_{best}(t) = (Z_{best}^{(1)}(t), Z_{best}^{(2)}(t), \cdots, Z_{best}^{(N)}(t))$.

(2.4) Population spread.

(2.4.1) Keep the best unity in $Z_{best}(t)$ (such as $Z_{best}^{(0)}(t)$).

(2.4.2) Substitute $N - 1$ unities in $\left\{Z_{best}(t) \setminus Z_{best}^{(0)}(t)\right\}$ with $N - 1$ unities in $\Omega(t)^c$, where $\Omega(t)^c$ is a complementary set of $\Omega(t)$.

(2.4.3) Define a new generation of population $X(t+1) = (X^{(1)}(t+1), \cdots, X^{(N-1)}(t+1), Z_{best}^{(0)}(t))$.

Step 3. Terminate test if $X(t+1)$ already contains the approximate solution, stop and output the best solution in $X(t+1)$; otherwise, set $t := t+1$ and return to Step 2.

Experimental Results

To check the performance of CPMA, it is tested with the typical functions as follows:

Function 1:

$$\max f(x) = 0.5 - \frac{\sin^2\sqrt{x_1^2 + x_2^2} - 0.5}{\left[1.0 + 0.001(x_1^2 + x_2^2)\right]^2}, \quad -100 \le x_i \le 100, \quad i = 1, 2.$$

This function has a global maximum value 1 at the point $(x_1, x_2) = (0, 0)$.

Function 2:

$$\min f(x) = \frac{1}{1/k + \sum_{j=1}^{25} f_j^{-1}(x_1, x_2)},$$

$$f_j(x_1, x_2) = c_j + \sum_{i=1}^{2}(x_i - a_{ij})^6, \quad K = 500, \quad c_j = j,$$

$$\left[a_{ij}\right] = \begin{bmatrix} -32 & -16 & 0 & 16 & 32 & -32 & -16 & \cdots & 0 & 16 & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & -16 & \cdots & 32 & 32 & 32 \end{bmatrix},$$

$-62.536 \le x_i \le 65.536$, $i = 1, 2$.

This function has a global minimum value 0.998 at the point $(x_1, x_2) = (-32, 32)$.

Function 3:

$$\min f(x) = -20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)) + 20 + e, \quad -30 \le x_i \le 30,$$

$i = 1, 2, \cdots, n$, $n = 20$.

This function has a global minimum value 0 at the point $(x_1, x_2, \cdots, x_{20}) = (1, 1, \cdots, 1)$.

Function 4:

$$\min f(x) = \sum_{i=1}^{n} i x_i^2, \quad -100 \le x_i \le 100, \quad i = 1, 2, \cdots, n, \quad n = 20.$$

This function has a global minimum value 0 at the point $(x_1, x_2, \cdots, x_{20}) = (0, 0, \cdots, 0)$ .Table 1. shows parameters in the proposed method. m is the maximum iterations, N is the population scale, l is the times of population mobility, Δ is the contraction coefficient, α is the vigilance parameters, and $\delta(0)$ is the radius of the prime area. Table 2. is the comparison of PMA and CPMA.

Table 1. Parameters of functions

Function	k	N	l	Δ	α	$\delta(0)$
1	10	3	10	0.1	10^{-4}	34
2	10	3	10	0.1	10^{-4}	20
3	5	3	10	0.01	10^{-6}	10
4	5	3	10	0.01	10^{-3}	20

Table 2. Experimental results and comparison

Function	Algorithm	The best value	The worst value	Convergence times	Average iteration times
1	PMA	1.000000	0.990284	14	6
	CPMA	1.000000	1.000000	20	3
2	PMA	0.998004	2.982105	16	4
	CPMA	0.998000	0.998002	20	3
3	PMA	1×10^{-5}	1×10^{-5}	20	3
	CPMA	0	0	20	2
4	PMA	1×10^{-3}	1×10^{-3}	20	4
	CPMA	0	0	20	2

Conclusions

The PMA has some faults, such as precociousness and lower search accuracy, among others. Considering the high convergence of the chaos algorithm, PMA was combined with the chaos theory to attain an improved algorithm with improved convergence and solution accuracy. The prematurity phenomenon was also avoided.

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