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Estimating Parameters in Production Function based on Quadratic Synchronous Artificial Fish- Particle Swarm Optimization

***Abstract**. Investment analysis is an important element of the process of economic activities, and the rational analysis and selection of production function as well as the parameter estimation, are important part of the investment analysis. Common production functions are characterized by too strong non-linearity for the use of traditional method to estimate parameters; therefore a fast, simple and robust algorithm becomes a hot research interest to optimize the production function. To this end this paper presents QSAFPSO for solving this problem. The algorithm enhances the gene-level exchange of information between individuals, creates a genetic template, employs genetic template evolution, mutation and other operations to improve the convergence speed, solution accuracy, and better helps algorithm out of local optimum. The typical function tests show that QSAFPSO, compared with like algorithms, features fast convergence and higher solution precision. A simulation based on the annual output value from 1820 to 1926, capital investment and labor input in Massachusetts shows that the algorithm is characterized by fast optimization of production function parameter estimation and by small residual sum of squares.*

***Streszczenie.** W artykule opisano opracowany algorytm decyzyjny QSA-FPSO służący do analizy inwestycji. Jego działanie opiera się na budowie szablonu genetycznego i między genowej wymianie informacji. Wykonane testy funkcjonalności algorytmu pokazują jego większą, w porównaniu z innymi algorytmami, precyzję i szybkość osiągnięcia rozwiązania. Przedstawiono także wyniki badań symulacyjnych, dokonanych na rzeczywistych danych, potwierdzające wysoką skuteczność działania. (**Estymacja parametrów funkcji produkcji algorytmem optymalizacji rojem cząstek QSA-FPSO**).*

Keywords: production function; parameter estimation; particle swarm algorithm; non-differentiable point; residual sum of squares
Słowa kluczowe: funkcja produkcji, estymacja parametrów, optymalizacja rojem cząstek, punkt nierozróżnialny, szczątkowa suma kwadratów.

Introduction

A production function, the concept first proposed by American mathematician Cobb and the economist Douglas in 1928, is obtained by modeling production process to reflect the relationship between the investment elements and output; it can be used to analyze the role of capital, labor and other production factors in economic development, or to analyze the important role of scientific and technological progress and other information in economic development. In addition, the modeling and solution of production function can not only provide information useful for the acceleration of economic construction and reasonable allocation of limited resources, maximize the utilization of resources through the best combination, but also provide data and theoretical basis for the decision-making bodies at all levels. Therefore, the research of production function is practically and theoretically significant.

In establishing a production function, one needs historical data to estimate its parameters. The common estimation methods are: (1) the least linear squares method [1], which features simple calculation of a large set of equations while its disadvantage is: residual error of large regression equation and thus low prediction accuracy, resulting in a model less reflective of the objective reality. (2) quasi-Newton method for unconstrained optimization [2], however, this method, during the parameter optimization process, will be trapped by local minimum, resulting a model deviating from the actuality,in addition, the method requires partial derivatives for the optimized function, given the hardly satisfiable conditions, the application of the method in the economic field is seriously limited. It is an important research interest to find a fast, simple and effective algorithm to estimate the parameters of production function. Li Zhe, Wang Dongdong et al [3] propose to optimize the production function by using the artificial fish swarm algorithm. Compared with the previous two methods, this method is improved to some extent, and this paper will use TCAFSAPSO which is easy for programming and understanding.

Artificial Fish-Swarm And Particle Swarm Optimization [4-5]

(1) Foraging behavior: assume the current state of artificial fish is X_i , randomly select a state X_j in its perception, in seeking the minimum, if $X_i > X_j$, we move one step towards X_j; again randomly select the state X_j to determine the satisfaction of forward condition or not; if the forward condition is not satisfied despite repeated attempts, we move a random step.

(2) Aggregation behavior: Assume the current state of the artificial fish is X_i, search after the number of companion in the current exploration area (i.e., $d_{ij} <$ Visual) n_f , and the central location. $Y_C / n_f > \delta Y_i$, indicates there is much food in the central location which is not crowded, then we can move one step forward towards the central location, otherwise we will see foraging behaviors implemented.

(3) Rear-end behavior: assume the current state of artificial fish is X_i, among the companions in the exploration area (i.e., $d_{ij} <$ Visual) Y_j has a maximum fitness companion X_j . If $Y_j / n_f > \delta Y_i$, indicate companion X_j enjoys high concentration of food and uncrowded space, then we may move one step forward towards the companion X_j; otherwise we will see the foraging behavior implemented. If $n_f = 0$, we will also see the foraging behavior.

Aquadratic Synchronous Artificial Fish-Particle Swarm Optimization (Qsarpso)

Algorithm Thinking

To make full use of the information obtained during the algorithm evolution process, improve search efficiency and convergence speed, the paper, based on population co-opetition and individual information-sharing, presents TCAFSAPSO. The algorithm treats the set of variables contained in the individuals of a population as a genetic template, and different variables as genes. According to the algorithm, AFSA and PSO are two independent populations (each population includes a chaotic mapping agency), their global parallel search of the algorithm and the serial local search of simulation annealing algorithm are employed to search the entire solution space using the ergodicity without repetition according to their own characteristics of chaotic mapping within a certain range. The first collaboration, initialization of genetic template, the second collaboration, creation of a complete genetic template, genetic template evolution, genetic template mutation operations allow the two populations, AFSA and PSO, to keep track of each other's global optimal solution, and through real-time information migration and knowledge sharing, synchronize their evolution, improve the searching efficiency, solution accuracy and global search performance. The instructions for first collaboration, initialization of genetic template, the second collaboration, creation of a complete genetic template, genetic template evolution, genetic template mutation operations are as follows:

(1) The first collaboration and genetic template initialization. In the Algorithm, the collaborative competition among the population is seen as the first collaboration, and the variable sequence of the optimal individual generated by the collaboration is seen as the initial genetic template.

(2) The second collaboration and the creation of complete genetic template. By the evaluation criteria of the fitness function value of the initial genetic template, every individual substitutes the genes of its dimensions for the genes of corresponding dimension in the genetic template, and by comparing the fitness function value, and goes on the second competition, if superior to the genes in the same dimension in the template, its genes will change them. In this way, the optimal individual genes of the current dimensions of every individual in all populations will be selected out by dimension to form the genetic template not only with the optimal fitness value but also the all-round superior genetic template of the optimal gene sequence.

(3) The evolution of genetic template. With the current all-round superior genetic template as basic standard, a new population is generated after gene mutation operation, and evolved for certain generation with PSO algorithm to select the optimal individuals for competition with the all-superior excellent genetic templates. If superior to the genetic template, the individuals will replace the genetic template, at the same time replace the optimal individuals of AFSA and PSO to guide the search of these two populations, otherwise, the original genetic template will remain.

(4) The mutation of genetic template. If the current global optimum in the specified generation does not meet the evolution requirements, the genetic template can be regarded as too degraded to guide the evolution of populations, the jump mechanism of serial local search of simulation annealing algorithm will be employed to conduct mutation operation on the genetic template, if superior to the current genetic template, the genetic template generated from mutation will replace it as well as the current optimal individuals in AFSA and PSO populations.

The Steps of Algorithm

Step 1 Parameter initialization. Set the number of individual, the maximum number of iteration, the VisualL, Step, Try_number and δ of AFSA, and parameters c_1, c_2, w of PSO; and the parameters of simulation annealing.

Step 2 Population initialization. N individuals randomly generated in the feasible solution space as the initial AFSA population $X(0)$ and PSO population $X_1(0)$, and let $X(0) = X_1(0)$ (for verifying global convergence of the algorithm; the effect should be better with region division on paper);

Step 3 Calculate the initial fitness value for each individual, compare the individual initial fitness values, record their optimal values and population optimum label;

Step 4 Put population AFSA through aggregation behavior and rear-end behaviros;

Step 5 The chaos mechanism $CHAOS_1$ in the AFSA population puts the AFSA population through chaotic mapping according to the formulas (4) - (6) to calculate the optimal values;

Step 6 AFSA population competes with $CHAOS_1$ to generate the current optimal individuals and update the optimal individuals in the AFSA population;

Step 7 Initialize the particle velocity and displacement of the PSO population;

Step 8 Calculate the initial fitness value for each individual, compare the values, record their optimal values and population optimum label;

Step 9 Calculate w by $w = 0.7 - 0.3(t/M)$, update the particle position and velocity by (2) and (3);

Step 10 Calculate the fitness value of every particle, record the optimal value and the optimal solution;

Step 11 The chaotic mechanism $CHAOS_2$ in PSO population puts the PSO population through chaotic mapping by the formulas (4) - (6) and calculate the optimal value;

Step 12 PSO population competes with $CHAOS_2$ to generate the current optimal individuals and update the optimal individuals of the PSO population;

Step 13 Populations PSO and AFSA go through the first collaboration to produce the global optimal individuals (initial genetic template), and update the optimal individuals of the two populations.

Step 14 All the individuals in the populations PSO and AFSA are put through the second collaboration to produce the optimal individuals (the complete genetic template).

Step 15 Retain a very small number of optimal individuals (complete genetic template), put the template through mutation by the formulas (8) --- (9) to generate a new population and perform the evolution operation on the genetic template.

Step 16 Perform the operation of mutation on the genetic template. If the global optimum stops evolution in the specified generation, the simulation annealing will be carried out to genetic mutation operation on the template.

Step 17 Increase the number of iteration by 1 and check if it satisfies the algorithm termination condition, if not, move to Step 3; and if yes, find the optimal solution.

Numerical simulations

The authors use the Rosenbrock, Grienwank function in reference [8] and compare MPSC algorithm to verify its validity [8], assigning ten dimensions to the two functions. 20 independent experiments were conducted for each function with TCAFSAPSO. The following simulations are programmed in Matlab6.5. The test function, and parameters are set as follows:

Test Functions

Function from the range of the variable with the most advantages as follows:

(1) Rosenbrock function

$$f_1(x) = \sum_{i=1}^{n-1} 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2; x \in [-100,100],$$

where when $x=(1,\cdots,1)$, the global optimal value of function is 0. The search range is [-100， 100]

(2) Rastrigrin function

$$f_2(x)=\sum_{i=1}^{n}((x_i^2-10\cos(2\pi x_i))+10),\quad x_i\in[-10,10],$$

The optimal point is $x=(0,\cdots,0)$, the optimal value, 0, and the search range, [-10, 10].

The Algorithm Parameters

The maximum evolution generation of the function: 400 generations, while the number of iteration of the function from the reference [8] is 10, 000; the function runs 20 times, which is the same with reference [8]; other parameters are set as follows:

(1) The parameter settings of artificial fish swarm

Visual = 2.5, Step = 0.3, Try_number = 50, Friend_number = 50, δ = the optimal value of the current generation, the population number *N* = 20.

(2) Parameter settings for particle swarm: set the parameters in accordance with the requirements of reference [8] as follows:

Population number *N* = 20, learning factors c = 2.05, c_2 = 2.05; inertia weight factor w decreases from 0.7 to 0.3 from generation to generation;

(3) Parameter Settings for Simulation Annealing:

Set the decay parameter to 0.95, the step size to 0.02, the initial temperature to 100, and tolerance to 1e-8;

Simulation Results and Analysis

The Table 1 suggests, the TCAFSA provides better average fitness than the algorithm does for the former features high accuracy and that the global optimum values of the two functions each can be found in 400 generations, of which the best average fitness values, in 20 times, of Rastrigrin function are the global optimums, with the standard deviation of zero. As far as the convergence speed is concerned, the functions Rosenbrock and Rastrigrin deliver the results in Table 1 at the 341th and 214th generations. TCAFSAPSO can meet the test requirements in 400 generations. In terms of population size, TCAFSAPSO takes 3 populations, each consisting of 20 individuals, which is more than two times the population size of TCAFSAPSO algorithm. Thus, TCAFSAPSO, through twice collaborations of population and individual, makes full use of the effective gene during the populations evolution at the genetic level, so that the genetic template delivers the current optimum fitness values and genetic makeup; besides, the algorithm, through the evolution and mutation on the template, can more efficiently guide population evolution and obtain more satisfactory results with smaller population size and less evolution generations.

Table 1 The comparison of average best fitness values over 20 runs of TCAFSAPSO (mean + standard deviation)

Function	PSO	FPSO	HPSO	PSCO	QSARPSO
f_1	317.78+ 55.16	190.70+ 57.33	176.83+ 40.66	100.60+ 54.47	1.9746e-26+ 8.514e-26
f_2	24.92+ 4.90	22.33+ 2.71	0.047+ 0.023	11.99+ 1.58	0+ 0

The Application Of The Algorithm In The Estimation Of Production Function Parameters

A production function refers to the relationship, in a period of time without any change in technology, between various factors used in the production and the maximum possible yield. It is of great significance to construct a production function model and reasonably select the method for estimating the production function parameters to obtain a rational economic function in the actual production.

Given a variety of input factors, the common production function model is

$$y=\alpha x_1^{\alpha_1}x_2^{\alpha_2}\cdots x_n^{\alpha_n}e^{\mu}\ .$$

Where y is output, x_i is the input of the i th production element, α is efficiency coefficient，α_i is the output elasticity of the i th input element，$\alpha,\alpha_i(i=1,2,\cdots n)$ all are parameters needing estimating，μ is the random disturbance item ， and $\mu\sim N(0,\sigma^2)$ .Given $\mu\sim N(0,\sigma^2)$，$\mu=0$ in the actual calculation. Based on the modeling with the maximum residual sum of squares, the optimization model is:

$$\arg\min_{\alpha,\alpha_1,\alpha_2,\cdots\alpha_n}\sum_{k=1}^{l}[y_k-x_{k1}^{\alpha_1}x_{k2}^{\alpha_2}\cdots x_{kn}^{\alpha_n}]^2$$

Based on the statistics of [10], the parameters obtained with the algorithm presented by the paper is $\alpha=1.0317,\alpha_1=0.3595,\alpha_2=0.4431$ and the residual error, 0.8241. The model of production function of Massa obtained is as follows:

$$y=1.0317x_1^{0.3595}x_2^{0.4431}e^{\mu}$$

Conclusion

The paper addressing the non-linearity estimation of production function parameters, based on the particle swarm optimization, artificial fish swarm algorithm, simulation annealing mechanism, the collaboration strategy and chaotic strategy, presents the TCAFSAPSO to estimate the production function parameters and the results are more desirable.

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