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Research and Simulation of a Full-Scale Attitude Algorithm for Strapdown Inertial Navigation Based on a Rotation Vector

Abstract. *The compensation for non-swappable error of traditional quaternion algorithm is not enough in attitude matrix solution. It can only apply to the attitude solution of low dynamic carriers. An optimized algorithm based on a 3-subsample rotation vector is presented. The full-scale attitude of a strapdown inertial navigation system can be calculated through resolving a attitude change quaternion. The algorithm is validated through simulation. The results show that the algorithm can satisfy real-time and precision requirements. The influence of direction drift on the attitude solution of the strapdown inertial navigation system is also reduced.*

Streszczenie. *W artykule przedstawiono zoptymalizowany algorytm kompensacji błędów w podejściu kwaternionowym w metodzie macierzy orientacji, na potrzeby systemu nawigacji bezwładnościowej typu strapdown. Określanie orientacji pełnowymiarowej odbywa się poprzez obliczeniowe rozwiązanie kwaterniona zmiany orientacji. Opracowany algorytm poddano badaniom symulacyjnym, których wyniki potwierdzają skuteczność proponowanego rozwiązania. (**Badania i symulacje algorytmu pełnowymiarowego określania orientacji w nawigacji bezwładnościowej typu strapdown – wykorzystanie wektora rotacji**).*

Keywords: strapdown inertial navigation system, quaternion algorithm, rotation vector, attitude solution.
Słowa kluczowe: nawigacja bezwładnościowa strapdown, algorytm kwaternionowy, wektor rotacji, metoda orientacji.

Introduction

A strapdown inertial navigation system connects the inertial measurement unit composed of gyroscopes and accelerometers with a carrier. The measured angular motion and linear motion parameters of the carrier are the parameters in the carrier coordinate system. The navigation computer can decompose acceleration data into the navigation coordinate system by calculating an attitude matrix. Then, speed and position can also be calculated [1]. This is called mathematics platform. Attitude solving is the key technology of the strapdown inertial navigation system. The carrier posture and navigation data can be gotten with the attitude matrix. It is an important work of strapdown inertial navigation algorithm. Euler Angle algorithm direct calculates yaw angle, pitch angle and roll angle by solving Euler Angle differential equations. Euler Angle differential equations are simple and clear. They are intuitive and easy to understand. Besides, their solution doesn’t need orthogonalization. However, because the equations include trigonometric calculation, it may bring difficulties for the real-time computation. What is more, when the pitch angle is close to 90°, equation degeneration phenomenon will appear. This case can be considered as the lock of inertial platform. So this method is only applied to the situation where horizontal attitude changes not too greatly. It is not suitable for the determination of full-scale attitude [2]. Compared with other sub-sample methods, it can also minimize the algorithm drift by optimizing coefficients. So it is particularly applicable for attitude update of the carrier with fierce angle frequency and serious angle vibration [3, 4].

Both quaternion algorithm and the rotation vector algorithm can realize attitude update by calculating the attitude quaternion. But the former direct solves attitude quaternion differential equations, and the latter solves the attitude quaternion by solving attitude change quaternion. In the paper, an optimized algorithm based on 3-subsample rotation vector is presented to realize full-scale attitude calculation. The algorithm is validated to be viable through simulation.

Quaternion algorithm

Quaternion attitude expression includes four parameters. Its basic principle is that the transformation from a coordinate system to another one can be realized by the rotation of vector μ defined in the reference coordinate system. Defined q as a quaternion, which is a vector includes four elements that are the functions of the direction and rotation of q.

$$q=\begin{bmatrix}a\\b\\c\\d\end{bmatrix}=\begin{bmatrix}\cos(\mu/2)\\(\mu_x/\mu)\sin(\mu/2)\\(\mu_y/\mu)\sin(\mu/2)\\(\mu_z/\mu)\sin(\mu/2)\end{bmatrix}\tag{1}$$

Where, μ_x, μ_y and μ_z represent the components of the angle vector, μ represents its size.

According complex operation rules, the product of quaternion $q=a+ib+jc+kd$ and $p=e+if+jg+kh$ can be expressed with matrix as follows.

$$q.p=\begin{bmatrix}a&-b&-c&-d\\b&a&-d&c\\c&d&a&-b\\d&-c&b&a\end{bmatrix}\begin{bmatrix}e\\f\\g\\h\end{bmatrix}\tag{2}$$

Define vector r^b in the carrier coordinate system. We can direct use the quaternion to express it as r^n in the reference coordinate system. First, define a quaternion $r^{b'}$ .Its imaginary part is equal to the corresponding of components of r^b and its scalar is zero.

$$r^b=ix+jy+kz$$

$$r^{b'}=0+ix+jy+kz$$

In the reference coordinate system, $r^{n'}$ is expressed as follows.

$$r^{n'}=qr^bq^*\tag{3}$$

Where, $q^*=a-ib-jc-kd$ is the complex conjugate of q. We express it as a matrix.

$$r^n=C_b^nr^b\tag{4}$$

(5) $$C_b^n = \begin{bmatrix} (a^2+b^2-c^2-d^2) & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & (a^2-b^2+c^2-d^2) & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & (a^2-b^2-c^2+d^2) \end{bmatrix}$$

Direction cosine matrix can be expressed with quaternion or Euler angle. If use the quaternion attitude expression, it is necessary to solve the following equation.

(6) $$q' = \frac{1}{2} q.p$$

where, $p = \begin{bmatrix} 0 & \omega^T \end{bmatrix}^T$.

The angle increment at sampling intervals is usually used for the output of the strapdown inertial gyroscope. In order to prevent the differential amplification of the noise, instead of converting the angle increment into angular velocity, we use the angle increment to determine the quaternion. Picard solution is a common algorithm to calculate the quaternion with the angle increment. If the direction of the rotation vector keeps invariable at computer refreshing intervals, the solution of the above equation can be written asfollows.

(7) $$q_{k+1} = \left[\exp \frac{1}{2} \int_{t_k}^{t_{k+1}} W dt \right] q_k$$

$$\int_{t_k}^{t_{k+1}} W dt = \sum = \begin{bmatrix} 0 & -\sigma_x & -\sigma_y & -\sigma_z \\ \sigma_x & 0 & \sigma_z & -\sigma_y \\ \sigma_y & -\sigma_z & 0 & \sigma_x \\ \sigma_z & \sigma_y & -\sigma_x & 0 \end{bmatrix}$$

$$q_{k+1} = \exp(\frac{\Sigma}{2}) q_k$$

With the expansion of exponential terms, similar to the solution of direction cosine, the exponential terms can be written with quaternion.

(8) $$q_{k+1} = q_k . r_k$$

(9) $$r_k = \begin{bmatrix} a_c \\ a_s \sigma_x \\ a_s \sigma_y \\ a_s \sigma_z \end{bmatrix}$$

$$a_c = \cos\left(\frac{\sigma}{2}\right) = 1 - \frac{(0.5\sigma)^2}{2!} + \frac{(0.5\sigma)^4}{4!} - ......$$

where, $$a_s = \frac{\sin(\sigma/2)}{\sigma} = 0.5(1 - \frac{(0.5\sigma)^2}{3!} + \frac{(0.5\sigma)^4}{5!} - ......)$$

$$(0.5\sigma)^2 = 0.25({\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2)$$

Bortz put forward the differential equation of the rotation vector.

(10) $$\phi' = \omega_{nb}^b + \frac{1}{2} \phi \times \omega_{nb}^b + \frac{1}{\phi^2} \left[1 - \frac{\phi \sin \phi}{2(1-\cos \phi)} \right] \phi \times (\phi \times \omega_{nb}^b)$$

where, ϕ is the corresponding equivalent rotation vector of carrier angular motion change, ω_{nb}^b is the angular velocity of the carrier. In practical calculation, we expand triangular functions with series development. Because the update cycle is short and ϕ is very small, high-order terms can be abandoned in practical calculation. An approximate equation can be presented as follows.

(11) $$\phi' = w_{nb}^b + \frac{1}{2} \phi \times w_{nb}^b + \frac{1}{12} \phi \times (\phi \times w_{nb}^b)$$

For the solution of rotation vector differential equation, parabola is used to fit angular velocity, namely, $w_{nb}^b = a + 2bt + 3ct^2$. The optimized 3-sub-sample algorithm of the rotation vector is represented as below.

(12) $$\phi = \theta_1 + \theta_2 + \theta_3 + \frac{9}{20}(\theta_1 \times \theta_3) + \frac{27}{40} \theta_2 \times (\theta_3 - \theta_1)$$

where, θ_1, θ_2 and θ_3 represent the angle increment at the interval $\left[t_{k,} t_k + \frac{T}{3} \right]$, $\left[t_k + \frac{T}{3}, t_k + \frac{2T}{3} \right]$ and $\left[t_k + \frac{2T}{3}, t_{k-1} \right]$ respectively, T is the sampling period.

We can extract angle increment according to the following equations.

(13) $$\theta_1 = \left[5w_{nb}^b(t_k) + 8w_{nb}^b(t_k + \frac{T}{3}) - w_{nb}^b(t_k + \frac{2T}{3}) \right] . \frac{T}{36}$$

$$\theta_2 = \left[-w_{nb}^b(t_k) + 8w_{nb}^b(t_k + \frac{T}{3}) + 5w_{nb}^b(t_k + \frac{2T}{3}) \right] . \frac{T}{36}$$

$$\theta_3 = \left[-w_{nb}^b(t_k) + 8w_{nb}^b(t_k + \frac{T}{3}) + 5w_{nb}^b(t_{k+1}) \right] . \frac{T}{36}$$

Use the equivalent vector to update the quaternion.

(14) $$Q(t_{k+1}) = Q(t_k) \otimes q(T)$$

Where, $\otimes$ represents quaternion multiplication, $Q(t_k)$ and $Q(t_{k+1})$ represent the attitude quaternion at moment t_k ad t_{k+1} respectively, $q(T) = \begin{bmatrix} a_c & a_s\phi_x & a_s\phi_y & a_s\phi_z \end{bmatrix}^T$, $a_c = \cos\frac{\phi_0}{2}$, $a_s = \frac{\sin \phi_0 / 2}{\phi_0}$, $\phi_0 = \sqrt{{\phi_x}^2 + {\phi_y}^2 + {\phi_z}^2}$ represents the size of the equivalent vector. According to the relationship between the quaternion and the direction cosine, the attitude transformation matrix is as follows.

(15) $$C_b^n = \begin{bmatrix} 1-2({q_3}^2+{q_4}^2) & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 + q_1q_3) \\ 2(q_2q_3 + q_1q_4) & 1-2({q_2}^2+{q_4}^2) & 2(q_3q_4 - q_1q_2) \\ 2(q_2q_4 - q_1q_3) & 2(q_3q_4 + q_1q_2) & 1-2({q_2}^2+{q_3}^2) \end{bmatrix}$$

Accordingly, the attitude angles can be solved as.

(16) $$\gamma = \arctan\left[\frac{2(q_3q_4 + q_1q_2)}{1-2({q_2}^2+{q_3}^2)} \right]$$

$$\theta = \arcsin\left[-2(q_2q_4 - q_1q_3) \right]$$

$$\psi = \arctan\left[\frac{2(q_2q_3 + q_1q_4)}{1-2({q_3}^2+{q_4}^2)} \right]$$

where, γ, θ and ψ represent roll angle, pitch angle and yaw angle respectively.

Calculation process

The attitude computation process of strapdown inertial navigation is shown in Fig. 1. First, the equivalent rotation vector of current cycle can be solved by the output of three-axis gyroscope. In the process, the 3-subsample optimization algorithm adopts parabolic fitting. Then, update the quaternion with the equivalent vector. And then, use the relationship between the quaternion and direction cosine to solve the attitude orientation cosine matrix. Finally, the attitude angles are computed..

Simulation results

In order to identify the effectiveness of the attitude angle calculation algorithm in engineering practice, Fig. 1 shows the relative error between theoretic value and calculated value of roll angle, pitch angle and yaw angle respectively. The calculated values are obtained by using the optimized 3-subsample algorithm of equivalent rotation vector to update the quaternion. The simulation results show that the algorithm can achieve the needed solution accuracy.

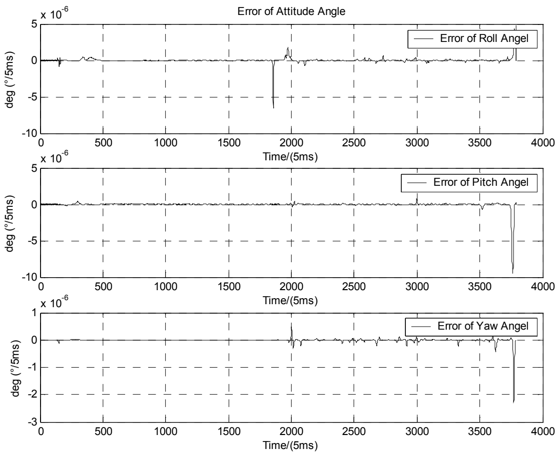

Fig. 1. The relative errors of the attitude angles

Conclusion

Based on the analysis of quaternion differential equation method and rotation vector method to solve the quaternion, the optimized 3-subsample rotation vector calculation algorithm is presented to solve the full-scale attitude angles. The simulation results confirm the correctness of the attitude angle calculation algorithm of tapdown inertial navigation system based on the rotation vector. Moreover, the errors are small. This research has some significance for the project practice of strapdown inertial navigation technology.

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REFERENCES

[1] Y. Y. Qin, Inertial Navigation, Beijing: Science Press, 2006
[2] David H. Titterton, John L. Weston, Strapdown Inertial Navigation Technology (2nd Edition), IEE, 2004
[3] Y. J. Yang, Study on the Crucial Technology of SINS, Harbin Engineering University, 2001
[4] H. Huang, Z. L. Deng, A New Expression for Rotation Vector Attitude Algorithm, Journal of Astronautics, 22(2001), No. 3, 92-97

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