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Design and Analysis of Odd-Harmonic Repetitive Control for Three-Phase Grid Connected Voltage Source Inverter

***Abstract**. This paper presents a novel design and analysis of an odd-harmonic repetitive control (ORC) for a two-level three-phase grid connected voltage source inverter. An LCL filter between the inverter and the grid is used to attenuate high frequency PWM switching harmonics. The controller lowers the memory requirement, compared to a conventional repetitive controller. The control scheme contains a traditional conventional tracking controller with a dual loop feedback system, and a zero-phase noncasual filter with add-on ORC. Our analysis and simulation results suggest that the proposed control scheme is able to provide high quality output current (THD=1.8 %) even in worst case scenario.*

Streszczenie.** W artykule przedstawiono nową strategię sterowania ORC (ang. Odd-harmonic Repetitive Control) dla trójfazowego, dwupoziomowego sieciowego przekształtnika napięcia. Dodatkowo wykorzystano filtr wyjściowy LCL. Algorytm redukuje ilość wymaganej pamięci. Jego struktura opiera się na sterowniku nadążnym z podwójną pętlą sprzężenia zwrotnego, filtrze o zerowym przesunięciu fazy oraz ORC. Analiza i symulacje wykazały, że proponowane sterowanie może zapewnić wysokiej jakości prąd (THD=1,8%). **(Opracowanie i analiza algorytmu ORC dla trójfazowego przekształtnika sieciowego).

Keywords: current control, pulse width modulation converter, system analysis and design, total harmonic distortion.
Słowa kluczowe: sterowanie prądem, przekształtnik PWM, projekt i analiza systemu, THD.

Introduction

Many renewable and conventional energy sources generate DC/AC power at inconvenient frequencies. A pulse width modulated (PWM) voltage source inverter as shown in Fig.1 is typically used to integrate the distributed generation plant into the grid. The quality of the output current of inverter introduced into the grid must conform to standards and regulations that define the total harmonic distortion (THD) limits [1].This is attained by a mixture of output filter and adequate designed PWM inverter with dynamic feedback control of the current introduced into the grid. Different other control arrangements and strategies [2], [3] have been used for grid-connected inverters but the key point is that most of the controllers provide comparatively low loop gain at fundamental frequency and its harmonics so tend to have less ability of disturbance rejection that consequences in less output current total harmonic rejection (THD) if the grid voltage total harmonic rejection THD is comparatively high. Repetitive feedback based control techniques can improve THD quality of grid-connected invertors by increasing loop gain at the basic frequency and related harmonics. The RC concept and technique has been widely used for different applications.

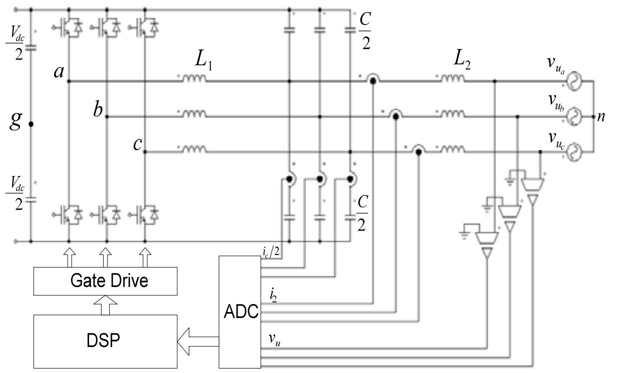

Fig. 1: Two-level grid connected inverter with LCL filter

The idea of repetitive control (RC) comes from the internal model principle (IMP) [4]. According to IMP, a periodic reference inputs with known period can be traced without steady state error if the stable closed loop system includes the model that generates these references. Based on the IMP, a periodic signal generator $(e^{-T_p s}/1-e^{-T_p s})$ could be included in the RC feedback loop as the internal model. Here, T_p is time period of periodic signal whose sampling time will be denoted by T_S later on. When the RC is applied in the discrete time domain with a sampling period, the transfer function of the periodic signal generator that needs to be included in the loop is:

$$G_R(z)=\frac{z^{-N}}{1-z^{-N}}=\frac{1}{z^N-1}\ ;\quad N=T_p/T_s \tag{1}$$

Normally N is a large number and hence requires a large memory buffers which is drawback of typical RC. Moreover, a typical RC with above periodic signal generator has a high gain at both even and odd-harmonics. Usually in power electronic systems we normally have odd voltage and current harmonics only [5]. By using a conventional RC, we increase the computational burden on the processor and which requires extra computational effort to get good asymptotic tracking and accuracy. A special type of RC, known as odd-harmonic repetitive control (ORC) offers high gain merely at odd-harmonic frequencies of interest [6], [7]. The ORC has the advantage of halving the memory and computational effort requirements. An ORC updates all memory cells after every sample intervals whereas traditional RC updates its memory cells after sample intervals. So ideally there should be two times faster error convergence for the ORC system as compared to traditional RC system [8]. This paper investigates the design and performance of a digital ORC three-phase grid connected inverter. The controller incorporates a phase lead compensator and a low pass filter. A method for analysing the ORC system from the frequency point of view is presented. A systematic way of optimizing the system performance is proposed. Table 1 illustrates the electrical constraints of the system. The overall block diagram of proposed control scheme is shown in Fig. 2.

Table 1. System parameters

Parameters	Symbol	Value
Grid phase voltage	V_u	230 V (rms)
DC link voltage	V_{dc}	750 V dc
1st inductor of filter	L_1	350 μH
2nd inductor of filter	L_2	50 μH
Capacitance	C	22.5 μF
Switching frequency	f_s	10 KHz
Grid frequency	f	50 Hz
Inverter rated current	I_2	100 A

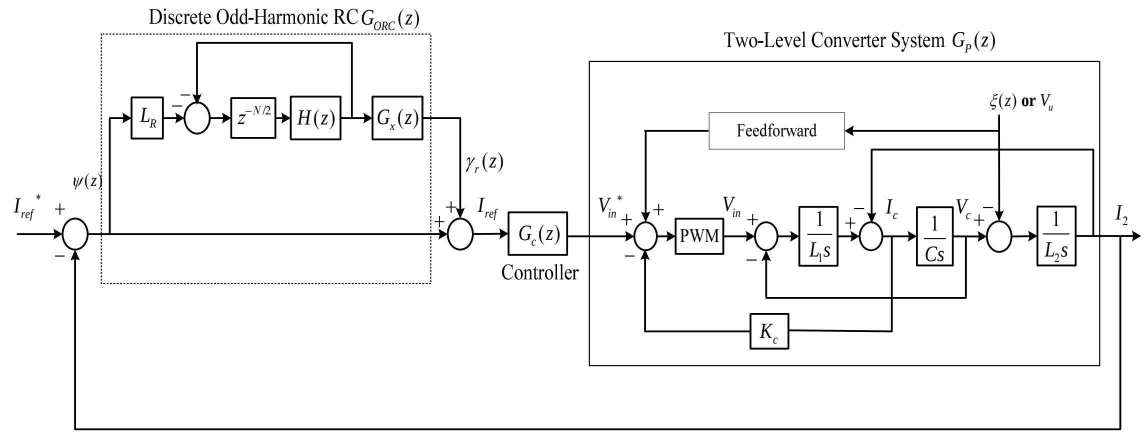

Fig. 2. Overall block diagram of the ORC system of the two-level converter incorporated in the basic classical controller

1) Odd-Harmonic repetitive control

a. Transfer Function

Structure of ORC: A periodic discrete time signal $w(n)$ and its Fourier series are given by equation (4):

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} w(n)e^{-j2\pi kn/N} \tag{4}$$

If the signal has odd harmonics only, then

$$w(n+N/2) = -w(n) \quad ;\forall\, n \in \mathbb{Z} \tag{5}$$

The corresponding periodic discrete transfer function of a periodic signal generator can be represented by the following function:

$$G_{OR}(z) = -\frac{z^{-N/2}}{1+z^{-N/2}} = -\frac{1}{z^{N/2}+1} \tag{6}$$

The above equation has poles at $z = e^{j(2k+1)2\pi/N}$ and $k = 0,1,2,........,(N/2-1)$ .It offers infinite gain at all odd harmonic frequencies $W_k = e^{j(2k+1)2\pi/N}$. In Fig. 3 , the general structure of an ORC based control scheme is shown. Here, $X(z)$ is the reference signal, $V(z)$ is the output, $\psi(z) = X(z) - V(z)$ is the error signal, $\xi(z)$ is the disturbance and $\gamma_r(z)$ is the output of ORC. Moreover $G_c(z)$ represents a conventional feedback controller of the plant $G_p(z)$. The low pass filter $H(z)$ is there to enhance the robustness of system. The compensator $G_x(z)$ is added to have a stable closed loop system. The transfer function of ORC can be written as:

$$G_{ORC}(z) = -\frac{L_R H(z) z^{-N/2}}{1+H(z)z^{-N/2}} G_x(z) \tag{7}$$

b. Stability Analysis

From Fig. 4, the transfer function $G_o(z)$ of the closed loop system without add-on RC is given by equation (8).

$$G_o(z) = \frac{G_c(z)G_p(z)}{1+G_c(z)G_p(z)} \tag{8}$$

$$\begin{aligned}\frac{V(z)}{X(z)} &= \frac{\left(1+G_{ORC}(z)\right)G_c(z)G_P(z)}{1+\left(1+G_{ORC}(z)\right)G_c(z)G_P(z)} \\ &= \frac{\left(1+H(z)z^{-N/2}+L_R H(z)z^{-N/2}G_x(z)\right)G_o(z)}{1+H(z)z^{-N/2}\left(1-L_R G_x(z)G_o(z)\right)}\end{aligned} \tag{9}$$

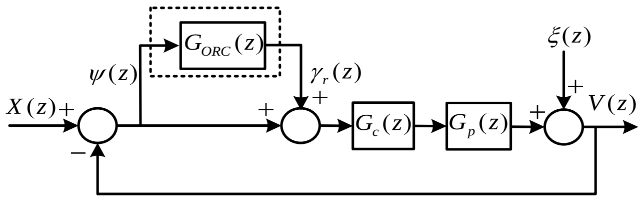

Fig. 3. General block diagram of control scheme

The transfer function of the overall system with respect to disturbance $\xi(z)$ can be derived as

$$\begin{aligned}\frac{V(z)}{\xi(z)} &= \frac{1}{1+(1+G_{ORC}(z))G_c(z)G_P(z)} \\ &= \frac{1+H(z)z^{-N/2}}{\left(1+G_c(z)G_p(s)\right)\left(1+H(z)z^{-N/2}(1-L_R G_x(z)G_o(z))\right)}\end{aligned} \tag{10}$$

The error transfer function $G_\psi(z)$ of the overall system is

$$\begin{aligned}G_\psi(z) &= \frac{\psi(z)}{X(z)-\xi(z)} \\ &= \frac{1+H(z)z^{-N/2}}{\left(1+G_c(z)G_p(s)\right)\left(1+H(z)z^{-N/2}(1-L_R G_x(z)G_o(z))\right)}\end{aligned} \tag{11}$$

Using (7)-(11), we can deduce the stability condition based on the following theorem:

Theorem: Assume two plants G_1 and G_2 are joined in a feedback loop, then the closed loop plant is input-output stable if $\|G_1\|$. $\|G_2\|$ < 1

As in above mentioned theorem the general stability conditions can be developed as:

Condition1: The roots of the characteristic equation, $1+G_c(z)G_p(z)=0$ of conventional two-loop plant (no RC) should be inside the unit circle.

Condition 2: From equation (11)

$$\left\|H(z)(1-L_R G_f(z)G_o(z)\right\| < 1 \quad ;\forall z = e^{j\omega} \quad 0 < \omega < \frac{\Pi}{T_p} \tag{12}$$

The stability conditions of the ORC are similar to the traditional RC [8].

2) Modeling of Grid Connected Inverter and Conventional Tracking Controller

The model of grid connected and conventioanl tracking controller are similiar to [8]. This can be seen in Fig.2 by ignoring OHR loop. The overall output transfer fucntion is,

(18) $$I_2 = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} I_{ref} - \frac{G_p(s)}{1+G_c(s)G_p(s)} V_u(s)$$

Where, $G_p(s)$ is the ratio of output to input of the two-loop plant given by,

(19) $$G_p(s) = \frac{I_2}{V_{in}*} = \frac{1}{(L_1L_2C)s^3 + (K_cL_2C)s^2 + (L_1+L_2)s}$$

The discrete form of $G_p(s)$ including a sample and hold circuit can be obtained by taking the Z-transform as:

(20) $$G_p(z) = Z\left(\frac{1-e^{-Ts}}{s} G_p(s)\right)$$

We choose a simple proportional controller, having $G_c(z) = K_P$ .The values of $K_p = 3.2$ and $K_c = 13.4$ were chosen to provide a trade-off between disturbance rejection, speed of response and stability. The plant has a phase margin of 71.8^o and a gain margin of 13.6 dB at 50 Hz as shown in Fig. 4. The loop gain is 24 dB and decreases further at higher frequencies. So the disturbance rejection at frequency 50 Hz and its harmonics will be decreasing with reduction in loop gain. However, we can't increase the loop gain further due to stability constraints.

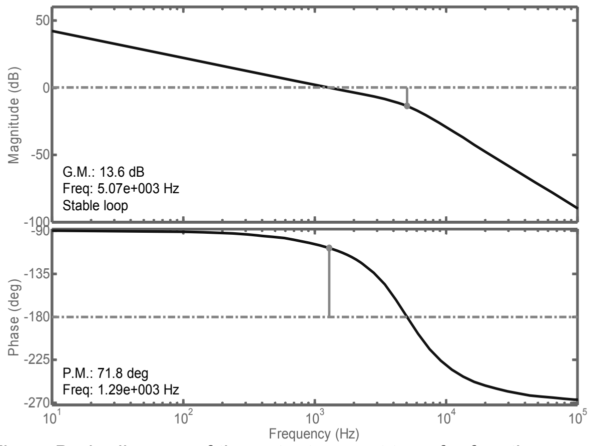

Fig. 4. Bode diagram of the output current transfer function

3) ORC Parameters:

a. Low-Pass filter

In practice normally a zero phase low-pass filter is used [8]:

(21) $$H(z) = \frac{\alpha_1 z + \alpha_o + \alpha_1 z^{-1}}{\alpha_o + 2\alpha_1} \quad ; \quad \alpha_o, \alpha_1 > 1$$

The normalized frequency response ($T_s = 1$) of equation (21) is $H(e^{j\omega}) = \alpha_o + 2\alpha_1 \cos(\omega)$ and $\omega \in (0,\pi)$. By considering $\alpha_o + 2\alpha_1 = 1$ for a unity gain response, we can write the magnitude of the normalized $H(j\omega)$ as

(22) $$|H(j\omega)| = \begin{cases} \alpha_o + 2\alpha_1 = 1 & \omega = 0 \\ \alpha_o + 2\alpha_1 \cos(\omega) & \omega(0,\pi) \\ \alpha_o - 2\alpha_1 & \omega = \pi \end{cases}$$

By selecting $\alpha_o = 0.5$ and $\alpha_1 = 0.25$, then

(23) $$|H(j\omega)| = \begin{cases} 1 & \omega = 0 \\ 0 < 0.5(1+\cos\omega) < 1 & \omega \in (0,\pi) \\ 0 & \omega = \pi \end{cases}$$

From (23) we can conclude that $|Q(j\omega)|$ is 1 at low frequencies and zero at high frequencies. We can achieve different low pass filtering by using different α_o and α_1. Typically a first order filter is sufficient and is given by,

(24) $$H(z) = 0.25z + 0.5 + 0.5z^{-1}$$

No causality problem exists because the filter is cascaded with the transfer function of the repetitive control having *N* period delay as shown in Fig. 2.

b. Learning Gain L_R and Compensator $G_x(z)$

The learning gain selection is linked to the transient response and stability of the ORC system. It must fulfill the stability condition given by equation (12). Increasing L_R improves the steady state error (SSE) for a given value of $H(z)$ and vice versa. We selected L_R value to be 0.3 as it gives fast enough transient (1 cycle to reach steady state) response while maintaining good stability. Since $G_0(z)$ in equation (8) is a minimum phase transfer function (no zeros outside unit circle), it is desirable to choose $G_x(z)$ to be the inverse of system model to achieve zero-phase error tracking. However in practice the grid impedance vary significantly and the transfer function is therefore not known. Instead a phase-lead compensation scheme is used such that,

(25) $$G_x(z) = z^m$$

This phase-lead compensator compensates the phase lag introduced by the transfer function of the inverter and hence improves the stability of the system. In addition, it can also compensate the unknown time delay, which is not modeled. A suitable value $m = 3$ is selected based on equation (12) to have just enough stability and robustness.

Discussion and Simulation Results

Comprehensive simulation has been supported using the MATLAB Simpower Systems Toolbox. The controller constraints are shown in Tables 2. A high grid THD value has been considered to test controller in a worst case scenario. The reference current was 100 A (peak). Table 3 shows the grid voltage harmonics when THD is 10.4 %.

Table 2. Controller parameters

Parameters	Symbol	Value
Outerloop controller gain	K_P	3.2
Inner loop capacitor gain	K_C	13.4
ORC gain	L_R	0.3
No. of samples per period	N	200
Lead step for phase lead compensator	m	3

Table 3. Grid voltage harmonics when THD is 10.4%

Harmonic Number	3rd	5th	7th	9th	11th	13th
Fundamental Component	26	16	13	6.5	0.16	0.08

Fig. 5 demonstrates the output current without the ORC system. Whereas, Fig. 6 shows the output current with ORC system. The total harmonic distortion of the output current improves considerably when the ORC is used. The output current THD with the ORC improves from 10.4 % to 1.8 % without steady state error. Moreover it is noticeable in Fig. 6 that almost two times faster transient response is achieved as compared to traditional RC presented in [8]. The ORC scheme proved to be better in terms of disturbance rejection.

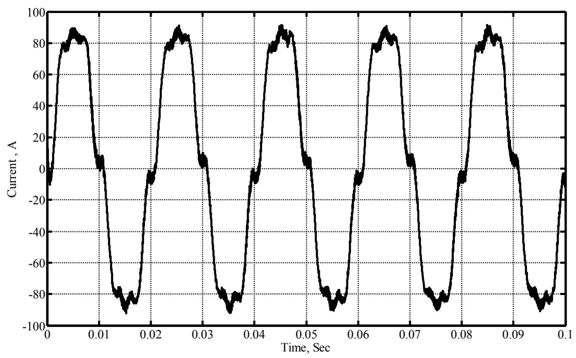

Fig.5. Output current without ORC

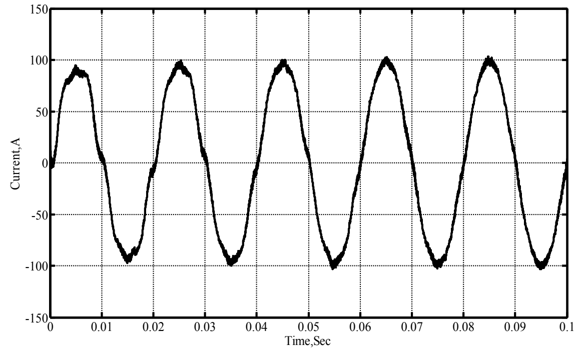

Fig.6. Output current with ORC

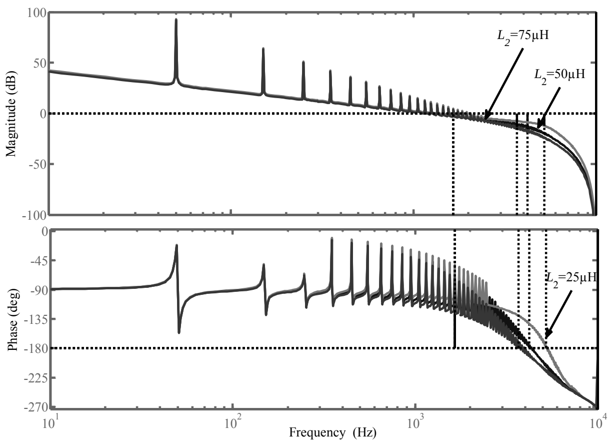

Fig.7. Effect of variations in L_2 on the ORC system

Robustness of ORC System

The value of the inductor L_2, which is determined by the grid impedance, can vary considerably depending upon the site where the inverter is fixed. This ambiguity should be considered to confirm that the plant can grab these uncertainties in the worst situation. To evaluate the robustness of the plant, the vagueness in the value of L_2 was varied by ±50% and it was observed that ORC system can handle these uncertainties very well. This can be clearly seen in Fig. 7, where the system is always stable.

Conclusion

A digital add-on ORC was designed for a three-phase PWM grid connected inverter to achieve high quality of sinusoidal current by rejecting grid harmonics. The proposed control scheme significantly improved the output current THD and steady state error as indicated by simulation results. A zero-phase noncasual filter was designed to improve the robustness. The data memory requirement by ORC is half of the conventional RC. It eases implementation and faster convergence of tracking error.

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