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Some aspects of numerical modeling of cable lines

Abstract. *In the article some results of analysis of basic factors that determinates the physical models of cable systems were shown and their influence on calculations accuracy were discussed. A special attention was put on the typically accepted simplifications of electrical and thermal models. The differences between simplified and accurate models were calculated and discussed.*

Streszczenie*. W niniejszej pracy podano wybrane zasady modelowania układów linii kablowych. Podstawowym celem prowadzonych analiz była determinacja czynników gwarantujących możliwość prowadzenia obliczeń możliwie wiernie odzwierciedlających rzeczywiste warunki pracy kabli wysokiego napięcia. Analizie poddano modele matematyczne i numeryczne ze szczególnym uwzględnieniem wartości błędów wprowadzanych przez typowe uproszczenia rzeczywistych zjawisk fizycznych zachodzących w układach tej klasy. (****Dokładność modeli numerycznych linii kablowych****).*

Słowa kluczowe: linie kablowe, dokładność obliczeń, modelowanie.
Keywords: cable lines, modeling, calculations accuracy.

Introduction

Well known and usually used calculating procedures for cable systems [3] and the information from cables manufacturer's catalogues [4], enable to analyze precisely only the most typical solutions and geometrical configurations . For the real cable systems, such conditions occur relatively rarely. Analysis of a long-term load of high-voltage cables placed, for example, in cable trays or in enclosure of other cables, shows some defects of classical calculating procedures. In such conditions, cross sections of conductors are selected with a large safety stock, which is a non- optimal. Based on these reasons, designing processes are often supported with a specialized calculating techniques that enables to account the untypical operating conditions of cable systems. To meet the basic requirements for long-term load of cables, modeling of heat transfer phenomenon in cable systems seems to be a very helpful technique. The numerical FEM calculating systems, widely used in many domains, enable to carrying out calculations of any circuits. But the necessity of obtaining a high accuracy results, requires to meet a number of criteria, defined in any stage of calculations. The significant errors of modeling of cable systems results from not accurate consideration of the real operating conditions. The errors of numerical methods can be omitted in cases of correct construction of models.

In the article some results of analysis of basic factors that determinate the physical models of cable systems were shown and their influence on calculations accuracy were discussed. A special attention was put on the typically accepted simplifications of electrical and thermal models. The differences between simplified and accurate models were calculated and discussed.

Analytical description of heat generation in conductors of cylindrical geometry results from Maxwell equations. Taking into consideration the field effects of current conduction within the conductor, there is a possibility to describe the magnetic field intensity vector sum (proportional to the current density) as the function of radius (1).

(1) $$\frac{d^2H}{dr^2}+\frac{1}{r}\frac{dH}{dr}-(j\omega\mu\gamma+\frac{1}{r^2})H=0$$

where: H - magnetic field intensity; r - radius of conductor; ω - pulsation; μ- magnetic permittivity.

Taking into consideration the boundary conditions (2), there is a possibility to calculate the values of magnetic field intensity. For the cylindrical geometries, usage of the Bessel functions seems to be a very attractive possibility (3).

(2) $$H_0=\frac{I}{2\pi r_0}\Leftrightarrow r=R\,;\ H=0\Leftrightarrow r=0$$

(3) $$H(r)=H_0\frac{J_1\left(\frac{\sqrt{2}r}{\delta}\cdot j^{\frac{3}{2}}\right)}{J_1\left(\frac{\sqrt{2_0}r}{\delta}\cdot j^{\frac{3}{2}}\right)}$$

Taking into consideration the magnetic field distribution, the current densities and volumetric Joule heat generation rate within the cylindrical conductors, are given by (4) and (5).

(4) $$\frac{dH}{dr}+\frac{1}{r}H=J$$

(5) $$p_V=\rho\cdot|J|^2$$

Presented approach (1) ÷ (5) is a accurate, simplified solution for every AC conductors. Basic problem during determination of construction conditions and solving the numerical models is determination of error values results from the simplifications of assumption of homogenous current density within the cross section. To optimal determination of mathematical model, two tasks were solved and results were compared.

The analysis was made for cables of conductor cross section of 1000 mm^2. All calculations were made with assumption of nominal current. For this case, the current value was at 1169 A [2]. The cross-bonding of return conductors was assumed. For this case, the zero - value of back current was used. The accurate model required the coupling electromagnetic and thermal calculations. At first, the distribution of heat generation rate was determined, based on electrodynamics analysis.

Heterogeneous distribution of magnetic field density within the cable cross - section, as in dielectric as in conducting materials shows unequal distribution of current density within the main conductor. The consequence of this phenomenon is a very large differentiation of the Joule heat generation in high voltage cables. The calculation results were shown in figure 1, where the conduction current densities (figure 1.a) and heat generation rates (figure 1.b) were discussed.

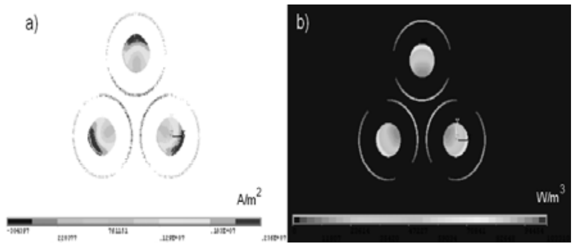

Fig. 1. Current density (a) and Joule heat (b) distribution within the cross section of high voltage cable

Using the calculated Joule heat generation enables to calculation of temperature distribution in all model. To compare the calculation results, the simplified model was used. As it was written, simplification was received the homogenous heat generation within the cross section of conductors. The resistivity, dimensions and other parameters of cables were assumed basing on manufacturer catalogue [4]. The calculation results have been shown in figure 2.

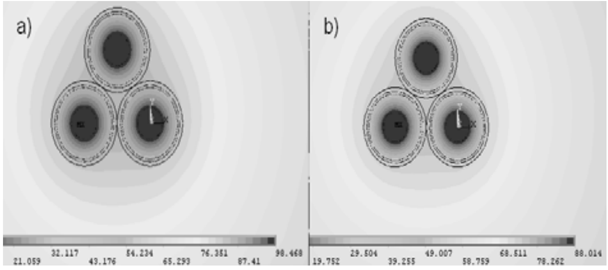

Fig. 2. The comparison of temperature fields within the cables for full electromagnetic and thermal analysis (a) and for the simplified model

The temperature values for two analyzed models were significantly different. Despite of equal total power values in both cases (36.11 W/m for full model and 35.25 W/m for simplified model), the maximal temperatures were different by 10%. The numerical model errors and discretization errors were omitted because of optimal model construction. Analysis of temperature field in main conductors (figure 3) allow for assess the sources of differences between two analyzed cases.

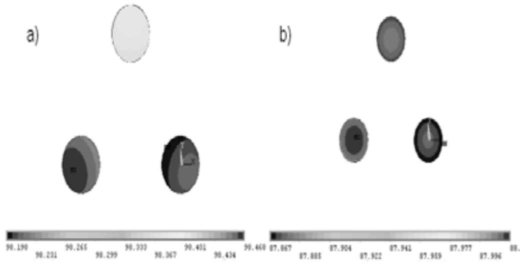

Fig. 3. Comparison of temperature fields within the main conductors for full (a) and simplified (b) models

For full model, the temperature distribution was equal to the Joule heat distribution. The phenomenon was observed despite of a high value of coppers thermal diffusivity, causing a strong homogeneity of temperatures within the conductors. The differences of temperature distribution in two models were observed especially for high temperature values. In the case of low load of cables, such effect was not visible. However, possibility of precise determination of heat generation in cable systems, under conditions of skin effect and the proximity effect as well as carrying calculations for maximal cable operating temperatures, clearly determines the necessity of using the full coupling electromagnetic and thermal models.

Dielectric loss

Direct heat generation within the dielectric bodies is a result of different polarization effects, sometimes reinforced of electron, ion and hole conduction effects. The most important quantity defined during such analysis, the dielectric dissipation, (6) contains components of polarization and Joule - Lentz heat generation.

$$tg\delta = \frac{P}{\sqrt{S^2 - P^2}} \tag{6}$$

The formula above enable to determinate the effective conductivity of dielectric body. The conductivity account as polarization effects as heat loses (7).

$$\gamma_e = \omega\varepsilon' tg\delta \tag{7}$$

Active power of workpiece, relative to the unity volume is determined by formula, similar to the Joule law (8).

$$p_V = \frac{1}{\gamma_e} {J_R}^2 = \gamma_e E^2 = \omega\varepsilon' tg\delta E^2 \tag{8}$$

where: J_R - current density; E- effective electric field intensity

According to the equations above, the power value in dielectric bodies increase proportionally to the frequency and electric field intensity. Determination of heat power in the dielectric require the knowledge of electric field intensity distribution, the frequency of the field and two additional parameters characterizing the material: ε_r' i $tg\delta$. Due to the complex geometry of cable systems, determination of electric field distribution is possible only by using numerical methods. To designate of dielectric loss influence on total energetic balance of analyzed system, some calculations were done. The model of cable line system, described in previous section of the article was used. The analogous boundary conditions and the sources were used. To compare the calculation results, two series were done. At first, the dielectric insulation of cable was established as lossless. The second analysis was done with lossy dielectric of polyethylene parameters. The temperature fields for both cases has been shown in figure 4.

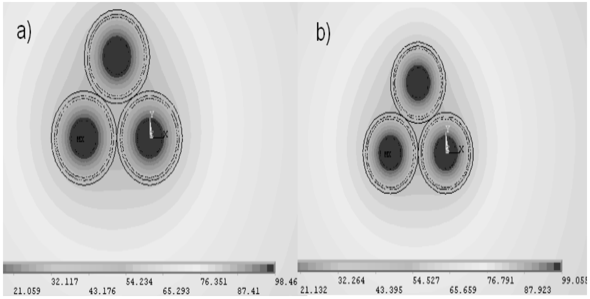

Fig. 4. Comparison of temperature fields in cable line environment without (a) and with (b) taking into consideration the dielectric loses

Comparative analysis shows that dielectric energy loses in cable lines is negligible in all energy balance of the system. There is no necessity to take into consideration such phenomenon during designing of cable line systems.

Heat loses in conduits

For cable lines placed in conduits fulfilled with fluids or in cable trays, a very important factor influencing on heat transfer phenomenon is thermal convection. In arrangement characterized for cable systems, the heat transfer occur between surfaces located in proximity. In general, determination of convection heat transfer requires the solution of complex system of differential equations of amount of substance balance, the momentum balance and the energy balance, with appropriate conditions of uniqueness of the solution. The momentum balance of unitary volume of fluid, named the Navier - Stokes equation (9) is equal for constant coefficient of viscosity. The equation determines the time change of momentum of fluid (of mass ρ) induced by the gravity (ρ·g_x), pressure gradient (-(∂p/∂x)) and the viscous force.

(9) $$\rho\frac{dw_x}{d\tau}=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 w_x}{\partial x^2}+\frac{\partial^2 w_y}{\partial y^2}+\frac{\partial^2 w_z}{\partial z^2}\right)+\frac{1}{3}\mu\frac{\partial}{\partial x}\left(\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{\partial w_z}{\partial z}\right)$$

The energy balance equation for single-component fluids (10), despite a number of simplifying assumptions, enables to simultaneous determination of kinetic energy and the heat transfer.

(10) $$\rho\frac{du}{d\tau}=\lambda\nabla^2 t-\rho\frac{pd\upsilon}{d\tau}+\mu\Theta_V+q_V$$

where: u - internal energy; λ - thermal conductivity; τ - time; ρ - density; υ- the dynamic coefficient of viscosity; q_V - heat generation rate; θ - Rayleigh dissipative function.

For the perfect gases, there is a possibility to define the Fourier - Kirchhoff equation as (11).

(11) $$\rho c_p\frac{dt}{d\tau}=\lambda\nabla^2 t+\frac{dp}{d\tau}+\mu\Theta_V+q_V$$

where: p- pressure; c_p- specific heat.

Despite simplifications of presented model, resulting inter alia from disregard of turbulences and variability of physical parameters of the fluids, direct solution of systems of presented equations require the utility of advanced techniques. The approach presented does not always guarantee a high precision of calculations.

In practical issues, calculations of convection heat transfer are accomplished using the similarity theory and the dimensional analysis. Utility of such rules enables to composition the set of dimensionless similarities (dependent on physical quantities determine the phenomenon) and criteria equations (dependent on the similarities). Such approach enables to analyses of convection phenomenon in simple way, basing on test results of considerable generality. In geometrical closed systems (the example of closed system is a conduit fulfilled by fluid) there is a possibility to use the basic equations of heat conduction instead of complicated balance equations. Approximate analysis of heat transfer phenomenon in closed systems can be used to determinate the heat resistance of the fluid gap of a width δ and thermal conductivity λ (12).

(12) $$W=\frac{1}{\alpha_{ot1}\cdot F}+\frac{\delta}{\lambda\cdot F}+\frac{1}{\alpha_{dt2}\cdot F}$$

where: α_{ot1} - heat transfer coefficient in warmer surface; α_{dt2} - heat transfer coefficient in colder surface; F- size of the gap cross-section.

During determination, the thermal resistance of gas gaps, the equation (12) is not usually used, due to the difficulty to appoint α_{ot1} and α_{dt2} coefficients. In practice, the formula (13) is very popular.

(13) $$W=\frac{\delta}{\lambda_e\cdot F}$$

where: λ_e is so-called the equivalent thermal conductivity. The quantity can be determinate basing on equations presented in table 1.

Table 1. Criteria equations for determination of equivalent thermal conductivities

Nazwa	Zależność
Neumanna	$\lambda_e=\left[1+\left(\frac{c1\cdot Gr\cdot Pr^n}{Gr\cdot Pr+c2}\right)\right]\cdot\lambda(t_{ob})$
Jakoba	$\lambda_e=0{,}075(Gr\cdot Pr)^{0.3}\cdot\lambda(t_{ob})$
Michiejew	$\lambda_e=C\cdot(Gr\cdot Pr)^n\cdot\lambda(t_{ob})$
Kraussold	$\lambda_e=0{,}4(Gr\cdot Pr)^{0.2}\cdot\lambda(t_{ob})$

Heat transfer within the gaps, between external surfaces of cable conduits is calculated by using the same types of equations, like for the thermal conduction. The real value of thermal conductivity is not used for fluids, but for the equivalent conductivity. During the calculations the influence of physical model of convection heat transfer in the cable conduits on results quality was examined.

The 2D numerical model was created. The model consist the main conductor, one layer of the insulation of thermal conductivity of 1 W/(mK) and steel conduit cover. Between external surface of cable insulation and inner surface of conduit cover was the gas of air parameters. Two calculations variants were performed for different boundary conditions:

I) For constant temperature of main conductor (90°C) and constant temperature of conduits case (35°C);

II) For constant heat power in main conductor (the value corresponding to 1200 A) and constant temperature of conduits case (35°C).

The calculations were aimed at determination influence of simplifications of convection heat transfer by using equivalent thermal conductivities and full analysis (coupled flow and thermal fields) on results quality. In figure 5 the temperature distributions within the model were shown. The results of simplified (fig. 5.a) and full (fig. 5.b) model were done for the first variant for constant temperatures (I).

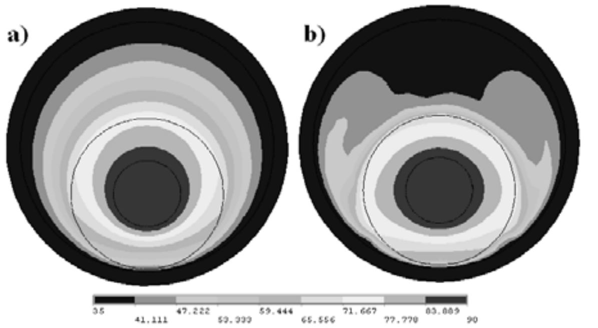

Fig.5. Temperature fields within the conduits models

Presented results show a different temperature distribution within the cross section of the model for analyzed cases. Full model (fig. 5.b) show a slightly worse intensity of heat transfer from the simplifying model.

Observed trend seems to be consistent with expectations. Relative small fluid velocity inside the conduits (maximal values was about 0,12 m/s (figure 6)) provides a conclusion of underdeveloped convection in analyzed cases.

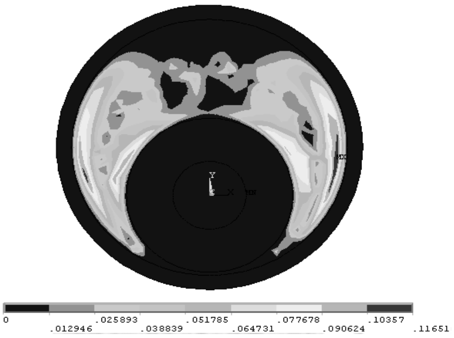

Fig. 6. Fluid velocity in exemplary cable conduit

To show the influence of used thermal convection heat transfer model on temperature values inside the system of cable conduits , some further analysis were done. The loads and boundary conditions were defined above as (II). Temperature distribution in steady state for two models were shown in figure 7.

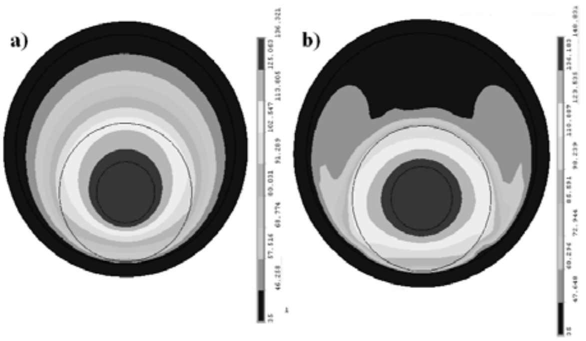

Fig. 7. Temperature fields for simplified (a) and full (b) model of convection heat transfer for the case of constant heat power

Maximal differences between calculation results for both cases not exceed the values at 12 K. In addition to reached temperatures, maximal relative error was about 8%. However, reached results seems to be positive. Absence of turbulence convection inside the conduits (figure 6) enable to claim of possibility to use the simplified model with equivalent thermal conductivity instead of the full model. A fact that maximal errors were at 8% seems to be a very interesting phenomenon. But the average error values were a significant lower. In the figure 8 temperature distributions in vertical axis of the model (in the air region) were shown for both models.

The result shows that the differences between analysed models were relatively small. Especially in areas, important from the cable systems designing point of view, temperatures were almost the same. Different nature of temperature distributions inside the air gap is not significant and results from location of transition resistance between the solid state and gas for the full model. However, temperatures in external surfaces of the air gap were almost identical. The fact enables to using alternative resistances method to precise calculations of cable conduits.

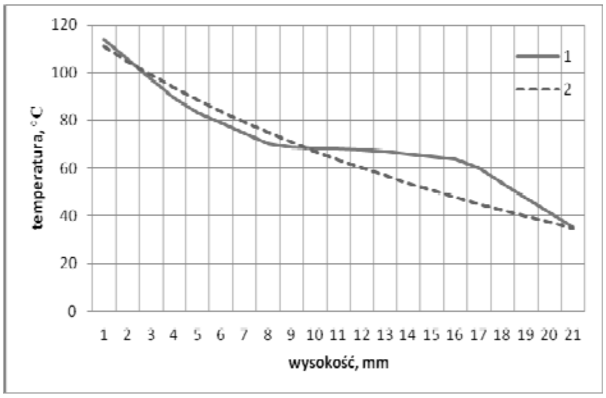

Fig. 8. Temperature distributions in air regions, in vertical axis of models. 1- full model of convection; 2- simplified model

Summary

In the article some results of analysis on cable systems modeling rules were described. The basic errors results from commonly accepted simplifications were analyzed. The influence of such errors on quality and accuracy of simulations have been shown.

It was demonstrated that acceptance of simplifications as omission of skin effects is unacceptable for high voltage cables of conductors cross section larger than 630 mm^2. The precise analysis of long time thermal load in such systems requires to usage of full coupled electromagnetic and thermal models.

In cases of cable systems, both insulating material parameters and operating current and frequency range, enables to omitting of dielectric loses. Total relative error, resulting from such simplifications, has not reach the value of 2%.

In the article some aspects of using of inexact thermal models were discussed. It was shown that in cases of cable conduits or cable trays systems, where cables are surrounded by fluids, there is a possibility to use the simplified convection heat transfer model. In most cases it is possible to use the equivalent thermal conductivity theory and simulation the convection phenomenon in the same way as in case of thermal conduction. Despite from different temperature distributions in the fluid areas, total differences of the heat fluxes in both cases has not reached the value of 5%.

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