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Analysis for quasi-stationary electromagnetic field with ferromagnetic objects present within

***Abstract**. A method for calculating a distribution for a quasi-stationary electromagnetic field in the systems containing ferromagnetic objects of non-linear properties is proposed. An iterative version of fundamental solutions method was applied in the presented research. Numerical tests performed on a simple, model system successfully proved the procedure to be correct and convergent. The report continues our research aimed at developing a method competitive to widely applied domain methods such as FEM, or FDM.*

Streszczenie**.. W prezentowanej pracy zaproponowano metodę obliczania rozkładu quasi-stacjonarnego pola elektromagnetycznego w układach zawierających ciała ferromagnetyczne o nieliniowych właściwościach, wykorzystującej iteracyjną wersję metody rozwiązań fundamentalnych. Na prostym układzie modelowym przeprowadzono testy numeryczne wykazujące poprawność I zbieżność opisanej procedury. Praca jest kontynuacją badań zmierzających do opracowania metody konkurencyjnej wobec powszechnie stosowanych metod obszarowych (MES, MRS). **Metoda obliczania rozkładu quasi-stacjonarnego pola elektromagnetycznego w układach zawierających ciała ferromagnetyczne o nieliniowych właściwościach.

Keywords. fundamental solution method, nonlinear problems, electromagnetic field, iteration
Słowa kluczowe. metoda rozwiązań fundamentalnych, zagadnienia nieliniowe, pole elektromagnetyczne, iteracja

Introduction

Computing the electromagnetic field distribution in systems containing ferromagnetic objects of non-linear properties provides one of the toughest challenges to face in technical electrodynamics. For the dynamic problems it proves practically impossible to take into account all the physical effects influencing the real magnetic induction dependence on the magnetic field intensity, thus it is obligatory for any computation model dealing with the concerned problem to assume a set of simplifications. Within this aspect the farthest simplification is to make the problem linear, i.e. to assume magnetic permeability to be constant overall the system under analysis. As reported in [1-3], such an assumption on linear properties of materials, works well for high power transformer models, with real shape details for their fundamental structural elements, such as coils wirings, cores, tank and screens. To successfully reproduce complex, three dimensional geometry of the concerned models boundary methods to solving partial problems – such as integral equation methods [1, 2] and fundamental solution methods (FSM) [3] shall be applied. Despite linearization of the problem the results proved satisfactory, well corresponding to findings of the experimental research performed on the physical model of the transformer. Advances made recently in computational field and progress in developing FSM, as well as their likely implementation to non-linear problems [4-6], have made the authors to re-invoke their interests, in this subject. It seems reasonable to expect that the computation method developed in the quoted reports can be generalised in such a manner that it would, to some degree, allow to include non-linear effects due to magnetisation curve.

The presented paper provides the first step in this undertaking. With a relatively simple model it clearly aims at testing the correctness and iterative efficiency of the FDM version applied to analysing computations for a quasi-static electromagnetic field in three dimensional systems containing objects of non-linear magnetic properties.

System and assumptions

The system under consideration consists of domains of currents inducing the primary field Ω^0,dielectric domains Ω^I of constant material parameters, conducting domains Ω^{II} of constant conductivity γ and magnetic permeability μ depending on the magnetic field intensity. It is assumed that all the domains disjoint areas. With no detriment to the generality of the problem it is further assumed that the considered system contains only one specific domain of each type, as illustrated in Fig.1.

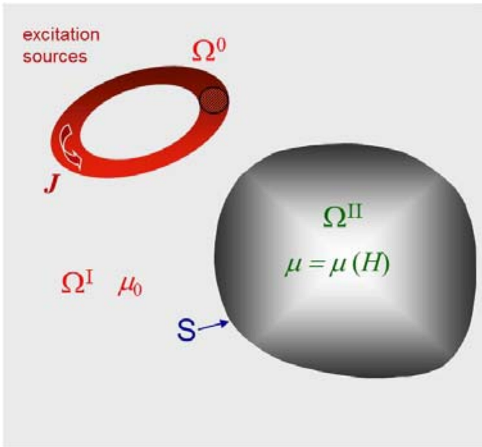

Fig 1. The system under consideration

We have assumed the following simplifications
1. the considered problem is quasi-static,
2. the conductive materials dimensions and the curvature radius of their surfaces are much bigger than the equivalent depth of electromagnetic field penetration,
3. exciting currents and field components are sinusoidal in time,
4. the distribution for the exciting currents is known,
5. the magnetic induction is uniquely dependant on magnetic field strength and this relation is known.

Assumption 1 means that the considered problem varies slowly in time, which allows to neglect the expression related to displacement currents in Maxwell's equations.

Assumption 2 allows to replace classic boundary conditions for the electrodynamics on the conducting materials surfaces with the impedance type condition.

Assumption 3 means that the sinusoidal distortion effect for the field-time dependencies related to non-linear material magnetic properties systems, can be neglected. Hence, all time dependent physical quantities can be represented with their complex amplitudes and complex Maxwell equations are applicable.

Assumption 5 means that the magnetic hysteresis effect is negligible.

Mathematical formulation of the problem

With the assumptions made within the dielectric domains of the system the complex amplitude of the magnetic field strength $\boldsymbol{H}$ can be expressed with the scalar magnetic potential φ defined by the relation

$$\boldsymbol{H} = -\mathbf{grad}\varphi \tag{1}$$

satisfying the Laplace equation

$$\Delta\varphi = 0 \tag{2}$$

and the impedance boundary condition on the surface of the conducting domains [1, 2, 3]:

$$\widetilde{\Delta}\varphi = \beta \frac{\partial \varphi}{\partial s_3} \tag{3}$$

where:

$$\beta = \sqrt{j\frac{\omega\gamma\mu_0}{\mu_r(H)}} \tag{4}$$

$$\widetilde{\Delta} = \frac{1}{h_1 h_2}\left[\frac{\partial}{\partial s_1}\left(\frac{h_2}{h_1}\frac{\partial}{\partial s_1}\right) + \frac{\partial}{\partial s_2}\left(\frac{h_1}{h_2}\frac{\partial}{\partial s_2}\right)\right] \tag{5}$$

$$h_i = \sqrt{\sum_{k=1}^{3}\left(\frac{\partial x_k}{\partial s_i}\right)^2} \quad i = 1, 2, 3 \tag{6}$$

s_1, s_2, s_3 – orthogonal system coordinates, where s_1, s_2, vary along the tangents to the boundary surface S, while s_3, in the direction perpendicular to it, i.e. towards the inside of the conducting domain.

To simplify the forms of the further formulas the following operator is introduced

$$\mathcal{L} = \widetilde{\Delta} - \beta\frac{\partial}{\partial s_3} \tag{7}$$

which transforms (3) into the form

$$\mathcal{L}(\varphi) = 0 \tag{8}$$

The function φ can be written in a for

$$\varphi = \varphi^{exc} + \varphi^{ind} \tag{9}$$

where φ^{exc} stands for the primary field induced by exciting currents flowing in the domain Ω^0, whereas φ^{ind} is the field induced by domain Ω^{II}. Both φ^{exc} and φ^{ind} satisfy the equation (2).

With the assumption 4 it is a separate problem to find field distribution φ^{exc}, independent of the problem of computing the field φ^{ind}. For sources with closed current lines it can be determined from the general relation [3]:

$$\varphi^{exc}(\boldsymbol{r}) = \frac{\mu}{4\pi}\iint_{S_0}\Phi(\boldsymbol{r},\boldsymbol{r}')\boldsymbol{J}(\boldsymbol{r}')\cdot \mathrm{d}s_0, \quad \boldsymbol{r}' \in \Omega^0 \tag{10}$$

where:

$$\Phi(\boldsymbol{r},\boldsymbol{r}') = \iint_{A(Q)}\frac{\boldsymbol{R}\cdot \boldsymbol{da}}{R^3}, \quad \boldsymbol{R} = \boldsymbol{r}' - \boldsymbol{r} \tag{11}$$

Φ is a spherical angle at which A surface is spanned on the infinitely narrow current stream coming through point $\boldsymbol{r}$ of domain Ω^0. The remaining symbols are explained in Fig. 2. Function φ^{exc} is considered known for further considerations.

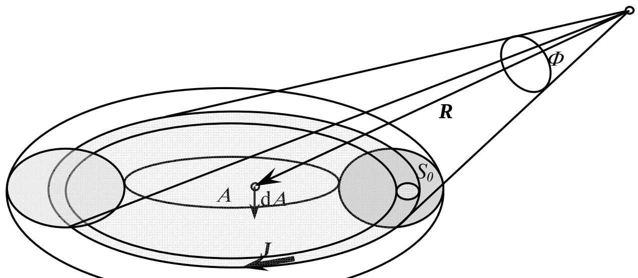

Fig. 2. Illustration to formulas (10), (11)

Assuming (9) condition (8) can be written as

$$\mathcal{L}(\varphi^{ind}) = F_0 \tag{12}$$

$$F_0 = \mathcal{L}(\varphi^{exc}) \tag{13}$$

To sum up, a mathematical formulation for finding magnetic field distribution in the dielectric domain of the system under consideration is reduced to finding a scalar complex function $\boldsymbol{\varphi}^{ind}$ satisfying Laplace equation and condition (12) for the core surface.

Having solved the problem defined above distribution for the electromagnetic field can be found in the conductive domain on the base of the following relations [1]:

$$H_i(s_1,s_2,s_3) = H_i(s_1,s_2,0^+)\,\mathrm{e}^{-\alpha s_3}, i = 1,2 \tag{14}$$

$$H_3(s_1,s_2,s_3) = \mu_r(H)H_3(s_1,s_2,0^+)\,\mathrm{e}^{-\alpha s_3} \tag{15}$$

$$E_1(s_1,s_2,s_3) = H_2(s_1,s_2,0^+)\,\mathrm{e}^{-\alpha s_3} \tag{16}$$

$$E_2(s_1,s_2,s_3) = -H_1(s_1,s_2,0^+)\,\mathrm{e}^{-\alpha s_3} \tag{17}$$

Solving method

The major problem while solving the above is to incorporate the relation, implicit in the boundary condition (12) on the magnetic field function (see (4), (7)). The solution can be found by an iterative self-consistent process for field $\boldsymbol{H}$ distribution on the surface S with function $\mu_r(H)$. In the following steps of this procedure linear problems are solved, under the assumption that the function local value on surface S is determined with the local value of H calculated in the preceding step, i.e. in such a manner as if the domain Ω^{II} was non-homogeneous with respect to magnetic properties. At the first step $\mu_r(H)$ on surface S is determined by the inducing field. So formulated linear problems can be solved with another iterative procedure based on FSM [4-6]. Both procedures, however, can be merged into one, which leads to a simpler final algorithm, which as demonstrated with numerical experiments performed, converges even quicker.

According to FSM the sought function $\boldsymbol{H}^{ind}$ at any point $\boldsymbol{r}$ of the dielectric domain can be approximated with a function

(18) $$\tilde{\varphi}^{ind}(\boldsymbol{r}) = \sum_{k=1}^{N} q_k G_k(\boldsymbol{r})$$

where

(19) $$G_k(\boldsymbol{r}) = \frac{1}{|\boldsymbol{r} - \boldsymbol{r}_k|}, \ \boldsymbol{r}_k \notin \overline{\Omega}^{\mathrm{I}}, k = 1,...,N$$

are fundamental solutions to Laplace equation. It should be noted that singularities $\boldsymbol{r}_k$. by definition are outside the domain $\overline{\Omega}^{\mathrm{I}}$, concerned with solution (18), that is inside the domain Ω^{II}. Hence, regardless of the values of q_k coefficients the function defined with formula (18) remains within domain $\overline{\Omega}^{\mathrm{I}}$ a restricted class C^{∞} function, and thus an exact solution to Laplace's equation.

Though it is arbitrary how we chose $\boldsymbol{r}_k$ points, it may still significantly influence the convergence of the procedure in question.

Values for coefficients q_k are found numerically by making function (18) to satisfy the boundary condition (12) and thus provide a good approximation of the solution. To do so we need to define at first boundary error function determined on surface S

(20) $$\delta(\boldsymbol{r}) = F_0(\boldsymbol{r}) - \mathcal{L}\left(\varphi^{ind}(\boldsymbol{r})\right), \quad \boldsymbol{r} \in \mathrm{S}$$

By substituting (18) we obtain

(21) $$\delta(\boldsymbol{r}) = F_0(\boldsymbol{r}) - \sum_{k=1}^{N} q_k F_k(\boldsymbol{r})$$

where:

(22) $$F_k(\boldsymbol{r}) = \mathcal{L}(G_k(\boldsymbol{r}))$$

As a measure for the total boundary error of the solution (18) a quadratic mean norm of the boundary error function is defined

(23) $$\mathrm{E} = \sqrt{\iint_S |\delta|^2 \,\mathrm{d}s}$$

With the best approximation method employed it is possible to determine the complete set of coefficients q_k in the sum (18) and keep the value of boundary error of the solution minimised. It turns out, however, that much quicker (in real time) and nevertheless much simpler procedure, is the one where this functional value is minimised in the subsequent iterations, and each iteration step brings just one of the coefficients q_k as dependent on

(24) $$q_k = \frac{\iint_S |F_k|^2 \,\mathrm{d}s}{\iint_S F_k^* \delta_{k-1} \,\mathrm{d}s}$$

(25) where F_k^* denotes a function conjugate to F_k, δ_{k-1} - the boundary error function calculated with formula (21) after k-1 iteration steps.

Substituting the resulting value (18) we arrive at subsequent approximation of function $\boldsymbol{\varphi}^{ind}$, which provides an input for the new values of boundary error function δ_k necessary in the next step of iteration.

To merge the above procedure with a self-consistent procedure between field $\boldsymbol{H}$ and function $\mu_r(H)$ it is enough at each iteration step to compute its local values from the magnetic field distribution on the boundary surface S determined in the preceding step. Therefore, the self-refining process is carried out in parallel to the process of diminishing the boundary error of the solution.

Nevertheless, it should be noted that unlike solving linear problems, there is no guarantee for the procedure to be convergent, i.e. for the sequence of boundary errors E computed in the subsequent iterations steps to decrease. For that reason, to assess computational practicability, it is necessary to perform a series of such problem-oriented tests. Below the first of such tests is described.

Test

An initial assessment for the proposed method to be correct and convergent was performed with a numerical procedure aimed at computing magnetic field distribution for a model system illustrated in Figure 3.

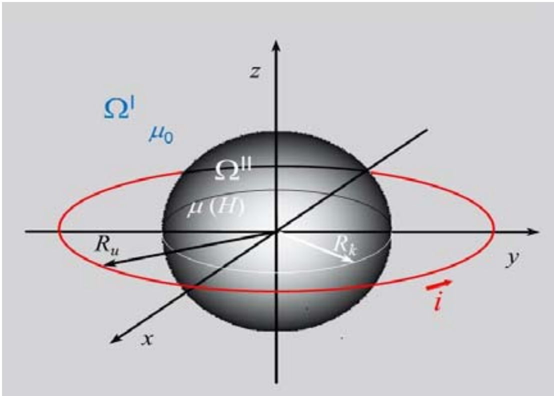

Fig. 3. The model system under consideration

The original source of the field is the sinusoidal alternating current $\boldsymbol{i}$ flowing in a circular coil, which surrounds a spherical solid of non-linear magnetic properties.

The assumed $\mu(H)$ dependence was that of ST35 steel (based on [7]) – see Figure 4.

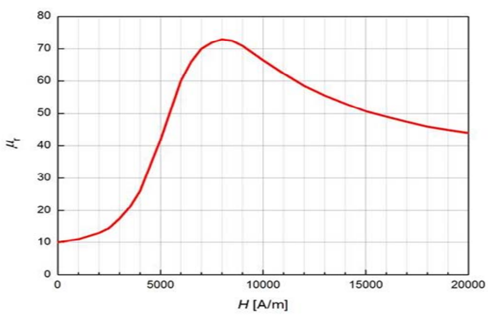

Fig. 4. $\mu(H)$ dependence (steel ST35)

The excitation field potential calculated from (10) and (11) is expressed as

(26) $$\varphi_1^0 = -\frac{i}{2\pi} z \int_0^{\pi} \frac{\rho R_u \cos\theta - \rho^2 - z^2}{(\rho^2 \sin^2\theta + z^2)\sqrt{R_u^2 - 2\rho R_u \cos\theta + \rho^2 + z^2}} d\theta$$

The plot shown in Figure 5 represents boundary error values(16) obtained at subsequent steps of the iteration procedure in question for various exciting current values.

The scale used is a double logarithmic one to visualise that boundary error series, despite their stochastic character, display clearly power-like convergence only after few dozens of iterations. It indicates that the presented procedure is strongly convergent.

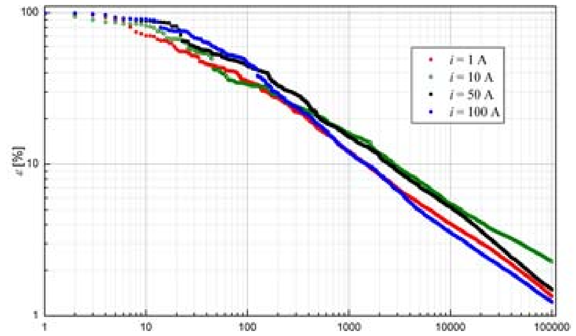

Fig. 5. The boundary error versus number of iterations for various exciting current values

In Figures 6 to 8 computed distributions for the magnetic potenteial and magnetic field intensity components within the system under analysis, within $y = 0$ plane are shown.

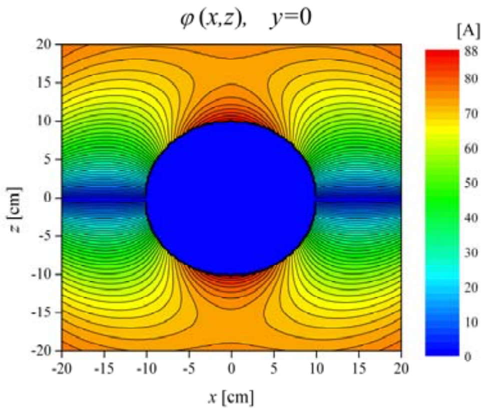

Fig. 6. Computed magnetic potential distribution

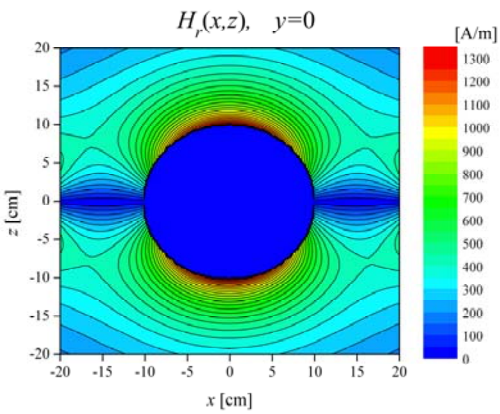

Fig. 7. Computed distribution for the radial component of the magnetic field intensity

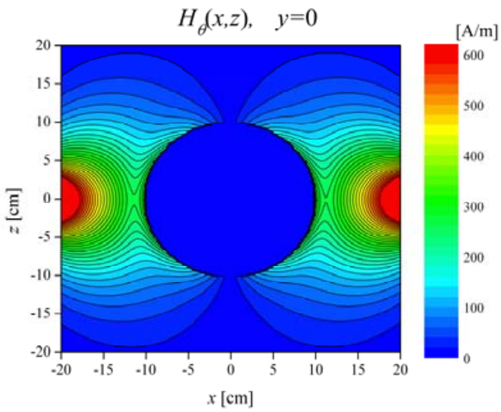

Fig. 8. Computed distribution for the zenith component of the magnetic field intensity

Summary

The presented paper continues the research into adapting fundamental solution method to solving nonlinear problems in electromagnetism. The method dealt with herein is concerned with a class of electrodynamics systems containing massive conducting objects of nonlinear magnetic properties exposed to low frequency sinusoidal electromagnetic field; a class of special importance to practical engineering. The interactive character of the procedure allows for its practicability, such as quick and easy assessment of solution errors, convergence rate monitoring, hence automatic interruption of computations once the required accuracy has been achieved. The performance test for convergence proved satisfactory. Despite assuming quite strong simplifications while defining the problem, the presented procedure may prove very useful for computations, though it definitely requires detailed analyses and tests on more complex systems reflecting real structural elements of technical facilities to be performed.

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