MILAN ŠIMKO, MILAN CHUPÁČ

University of Žilina, Faculty of Electrical Engineering, Slovak Republic

Generalisation of transfer functions of inter-digital transducer and filter on basic of surface acoustic waves

Abstract: *In the report we deal with the generalisation of transfer function which is derived from the three-gate model of the inter-digital transducer. Stated generalisation of transfer function includes the influence of adapter circuits attached to the transducer and allows its application for the description of any model´s functioning.*

Streszczenie: *Analizo przetwornik używany do zasilania i detekcji elementu SAW – z powierzchniową falą akustyczną. W modelu uwzględniono także układ adaptera. (**Uogólnienie funkcji transferu przetwornika i filtru bazującego na elemencie z akustyczną fala powierzchniowa SAW**)*

Keywords: inter-digital transducer, surface acoustic waves, complex transfer function, diffusion coefficients, diffusion loss,.
Slowa kluzcowe*:* powierzchniowa fala akustyczna SAW.

Introduction

Analyzing the activity of inter-digital transducer (IDT) which is used for excitation and detection of surface acoustic waves (SAW) at the moment of realisation of acoustic-electronic components we stem most commonly from the three-gate circuit model (1). Depicted model allows us to state all three-gate transducer transfer functions with the presence of so called secondary features like acoustic reflections created at the edges of electrodes as a consequence of weighted electric load on the surface of the pad (2), (3), which is at the same time often applied to make calculations of delay circuits (DC) or filters with double IDT and various matching circuits. To calculate the transfer functions taking the secondary features into consideration it is convenient to use three-gate matrixes. Calculated transfer functions based on the models of "weak binding" does not include the influence of matching circuits and this method does not allow us to calculate reflection acoustic coefficient of transducer.

Reflections from the electrodes of the transducer are often little, eventually they can be strongly choked down by the use of doubled electrodes. In the depicted case it is possible to use three-gate model to derive the generalisation of transfer function in which there is included the influence of matching circuits and allows us to derive the nexus between the tree-gate activity model and various geometrical models of "weak binding".

Complex transfer function, diffusion coefficients and diffusion loss

In the Fig.1 there is a filter with SAW with linear phase characteristics which is most often composed of two IDT.

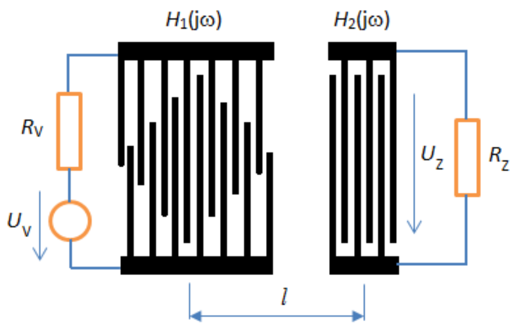


Fig.1. Most common filter arrangement with PAV

The left IDT (apodized) changes electric signal on the SAW and the second transducer, most commonly homogeneous integrates the energy of incoming wave and converts it to the electric signal. This process is linear, reciprocal and in the ideal case we can mathematically express it using a convolution, or equivalently in the following form (4):

(1) $$H(j\omega)=U_z/U_\upsilon = H_1(j\omega)H_2(j\omega)e^{-j\omega\tau},$$

where: $\tau = l/\upsilon$ is a SAW delay between IDT and υ is a SAW velocity.

The filter transfer function as it results from the equation (1) is fully determined by the features of IDT because the pod material does not have dispersal features. The transducers can find themselves in a different distance from each other; they can be "weighted" and can have a different geometry of electrodes. The right IDT (homogeneous) is most commonly broadband. Then we can approximately express the transfer function with the apozited transducer $H_1(j\omega)H(j\omega)=H_1(j\omega)$.

Based on the circuit theory it is possible to describe IDT in a complex way by using the matrix coefficients Y_{ij}, which are defined by the formula

(2) $$I_i=\sum_{j=1}^{3}Y_{ij}U_j,\ (i=1,2,3).$$

To simplify we will further assume IDT to be lossless (e.g. we do not consider the creation of volume waves, electrode resistance, diffraction and loss by the SAW diffusion), then coefficients X_{ij} are often imaginary.

It is convenient besides the admittance coefficients to define the system of complex transfer functions

(3) $$T_{ij}(j\omega)=2\sqrt{\frac{G_i}{G_j}\frac{U_i}{U_j}},$$

where U_i is a voltage of transfer or reflection SAW on the load G_i, i-level gate if a voltage U_j is attached to the j-level gate with conductance G_j.

Then it is possible to identify the diffusion coefficients p_{ij} with the nexus after equation adjustment (3), applies the following:

$$U_i^2G_i=\frac{1}{2}T_{ij}^2U_j^2G_j \quad \text{and}$$

(4) $$P_i=\frac{1}{2}T_{ij}^2P_j,\quad p_{ij}=\frac{P_i}{\left(P_{Useful}\right)_j}=\left|T_{ij}\right|^2,$$

where P_i is transferred or reflected power of i-level gate and $P_{(Useful)}$ is a utilizable from adjusted generator on the j-level gate. Then P_{ij} is a part of power reflected by i-level gate and p_{ij} (i≠j) is a part of transferred power j-level gate to i-level gate (e.g. characterizes the conversion of electric energy to acoustic energy and vice-versa).

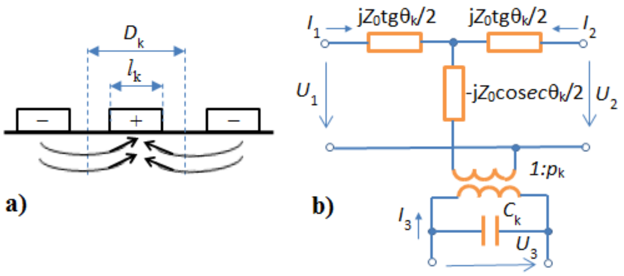
Based on the IDT reciprocity and symmetry results that $p_{11} = p_{22}, p_{13} = p_{23}$ and $p_{ij} = p_{ji}$, respectively $T_{ij} = T_{ji}$. Then we can define the diffusive (or transfer) loss as following:

(5) $$b_{ij} = -10\log p_{ij}, \quad \text{[dB]}$$

where b_{ij} is a loss caused by a reflection i-level gate and b_{ij} (i≠j) is transfer (insertion) loss between the i-level j-level gates.

Three-gate model of inter-digital transducer

We derive the IDT transfer function while coming from a single-electrode IDT which we will model by using one-dimensional cross field model, depicted in the fig. 2a and 2b. We can also use an alternative longitudinal field model which is the most convenient one for some pad materials with low coefficients of electro-mechanical binding K^2 [1]. Using the most common materials like $LiNbO_3$or $Bi_{12}GeO_{20}$ we stem from the cross field model which characterises better the IDT and at the same time is mathematically more accurate.



a) electric field division, b) equivalent electric scheme

Fig. 2. Single-electrode IDT

Like shown in the fig. 2b, single-electrode model has three gates (one being electric and other two) symmetrically electric). Using the equivalent currents and voltage we can express the particles velocity and mechanic power on acoustic gates. The dependence between these currents and voltages (i.e. $I_i = Y_{ij}U_i$)) can be expressed by the elements of admittance matrix. For the k-level IDT leg applies:

(6) $$\begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix}_k = \begin{vmatrix} -j\cot g\Theta_k, j\cos ec\Theta_k, -jp_k tg\left(\Theta_k/2\right) \\ j\cos ec\Theta_k, -j\cot g\Theta_k, -p_k tg\left(\Theta_k/2\right) \\ -jp_k tg\left(\Theta_k/2\right), -jp_k tg\left(\Theta_k/2\right), j\omega C_{pk} + 2p_k^2 tg\left(\Theta_k/2\right) \end{vmatrix} \begin{vmatrix} U_1 \\ U_2 \\ U_3 \end{vmatrix}_k$$

where $\Theta_k = \frac{2\pi f D_k}{\upsilon} = \frac{\pi f}{f_k}$ is a phase shift emerging on the k-level leg, $f_k = \upsilon/2D_k$ is a synchronised frequency k-level leg and D_k is k-level leg length.

Transformation rate is given by the nexus:

(7) $$p_k = (-1)^k \sqrt{2 f_k C_{pk} K^2 Z_0}, \left[K\left(1/\sqrt{2}\right)/K(q_k)\right],$$

where C_{pk} is k-level leg capacity, (as characteristic impedance we chose the unit one, Z_0=1). The factors $K\left(1/\sqrt{2}\right)$ and $K(q_k)$ are totally ecliptic first-order integrals with $q_k = \sin\left[\pi\left(l_k/2D_k\right)\right]$ module, where l_k is a width of k-level integral. These integrals can be found in the equation (7) as a consequence of capacity dependence on the rate of electrode and gap width. The k-level leg capacity is given by the formula:

(8) $$C_{pk} = \left(w_k/2\right)\sqrt{\varepsilon_1\varepsilon_2}, \left[K(q_k)/K\left(\sqrt{1-q_k^2}\right)\right],$$

where w_k is k-level leg aperture. The permittivities ε_1 and ε_2 express anisotropy features of pad material in direction of diffusion to the pad. The value C_{pk} like it stems from the nexus (8) increases with the growing rate l_k/D_k Next, to simplify, we will suppose that the width of the electrode and the gap is equal ($l_k = D_k/2$). Then the elliptic integrals in the equation (8) equal one.

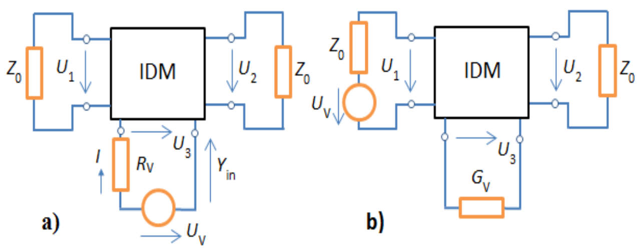
IDT Transfer function

IDT transfer function at generating the SAW at certain stated conditions can be expressed by the following formula:

(9) $$H_1(j\omega) \doteq T_{13}(j\omega),$$

and IDT transfer function at SAW detection by the nexus:

(10) $$H_2(j\omega) \doteq T_{23}(j\omega).$$



a) generation regime, b) detection regime

Fig.3. IDT equivalent model scheme as a triple gate:

But with the symmetrical IDT it is $|U_1| = |U_2|$ *and* $|T_{13}| = |T_{23}|$ that means it is enough to explore the transfer function $T_{13}(j\omega)$ given by the nexus:

(11) $$T_{13}(j\omega) = \frac{2}{\sqrt{Z_0 G_\upsilon}} \frac{U_1}{U_\upsilon}.$$

Solving the equation (11) we firstly calculate the U_1/U_2 rate then we will suppose that the ideal voltage source be connected to the electric gate ($G_\upsilon \to \infty$). Considering that every from *n* electrodes can with the circuit shown on the fig. 2., electric circuit short bond has a consequence that acoustic transfer conductances transfer reflection-free SAW and at the same time these are completely unbound and independent on the electric gate. Subsequently we can explore *n* IDT electrodes as a set of independent electric sources and we can set acoustic power (commensurable to U_1) for the IDT in the process of generalisation as a sum of mechanical voltages amplitudes (with convenient phase) that are generated by *n* electrodes.

The admittance coefficient given by the equation (6) will be used to calculate the amplitude of mechanic voltage generated by a single electrode. K-level IDT electrode generates the waves of mechanic voltage in both directions and acoustic power in the centre of electrode is given by the following formula:

(12) $$F_k = jU_3 p_k \sin\left(\Theta_k/2\right).$$

We reach the total acoustic power if we multiply the previous equation by the nexus $e^{-j2\pi f t_k}$ expressing SAW diffusion towards the end of transducer (e.g. to the gates n.1 or 2) and the sum of particular electrodes contributions, while the following applies:

(13) $$U_1 = jU_3 \sum_{k=1}^{n} p_k \sin\left(\frac{\pi f}{2 f_k}\right) e^{-j2\pi f t_k}.$$

In case of IDT connection to the real source ($G_\upsilon \neq \infty$) applies between the voltages U_υ and U_3 the following:

(14) $$U_\upsilon = U_3 \left(\frac{G_\upsilon + Y_{in}(j\omega)}{G_\upsilon} \right),$$

where $Y_{in}(j\omega)$ is IDT input admittance and Gv is an anode slope conductance of the source (fig.3a).

The IDT transfer function can be calculated by instituting the equations (13) and (14) to the equation (11) in the following form:

(15) $$T_{13}(j\omega) = \frac{2G_\upsilon}{\sqrt{Z_0 G_\upsilon}} \frac{jU_3 \sum_{k=1}^{n} p_k \sin\left(\frac{\pi f}{2 f_k}\right) e^{-j2\pi f t_k}}{G_\upsilon + Y_{in}(j\omega)}.$$

Generalised IDT transfer function

Using the cross field model It is convenient to divide the electric input admittance $T_{13}(j\omega)$ which is a very important constant to calculate Y_{in} into capacity susceptance which is parallely connected to emitting admittance $Y_r(j\omega)$:

(16) $$Y_{in}(j\omega) = Y_r(j\omega) + j\omega C_T,$$

(where C_T is IDT static capacity). Reference frequency f_0 can be defined like non-dimensional emitting admittance given by the nexus

(17) $$y_r(j\omega) = \frac{Y_r(j\omega)}{\mathrm{Re}\{Y_r(j\omega)\}} = Y_r \frac{Q_r}{\omega_0 C_T},$$

where

(18) $$Q_r = \frac{\omega_0 C_T}{\mathrm{Re}\{Y_r(j\omega)\}},$$

is an emission quality coefficient. Similarly we can define "endurance" quality coefficient Q_υ by the following formula

(19) $$Q_\upsilon = \frac{\omega_0 C_T}{G_\upsilon}$$

and introduce non-dimensional transformation rate defined by the formula

(20) $$p'_k = p_k \sqrt{\frac{2Q_r}{\omega_0 C_T}} = (-1)^k \sqrt{\frac{2K^2 Q_r f_k C_{pk}}{\pi f_0 C^T}},$$

after adjustment we can define the transfer function $T_{13}(j\omega)$ as following (Z_0=1):

(21) $$T_{13}(j\omega) = \sqrt{\frac{2Q_\upsilon}{Q_r}} \frac{\sum_{k=1}^{n} p'_k \sin\left(\frac{\pi f}{2 f_k}\right) e^{j2\pi f t_k}}{1 + jQ_\upsilon \frac{f}{f_0} + \frac{Q_\upsilon y_r(j\omega)}{Q_r}}.$$

In the previous nexus Y_r and the sum in the dividend of the fraction approximately equal 1, function $T_{13}(j\omega)$ is normed in a way that $|T_{13}|^2 = p_{13}$ The coefficient Q_r depends on transducer´s geometry, is indirectly commensurable to K^2 and can be in a simple case expressed by the nexus

(22) $$Q_r \doteq \frac{\pi}{4K^2} \frac{\Delta f}{f_0}.$$

It is efficient to generalise the equation (21) at the further analysis. We suppose that a matching circuit is connected to the IDT electric gate according to the fig.4. In such a case the transfer function T_{13} and the reflection coefficient T_{11} can be expressed by the following generalisation:

(23) $$T_{13}(j\omega) = \sqrt{\frac{2Q_\upsilon}{Q_r}} \frac{A(j\omega)}{C(j\omega)},$$

(24) $$T_{11}(j\omega) = \frac{Q_\upsilon}{Q_r} \frac{[A(j\omega)]^2}{B(j\omega)}.$$

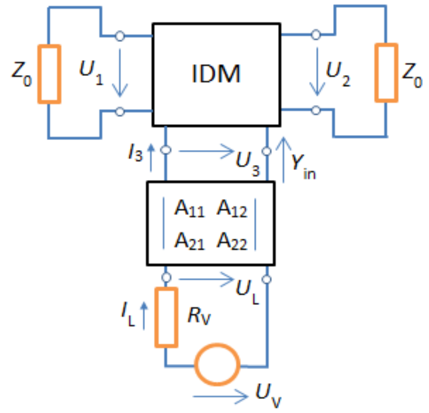


Fig.4. IDT equivalent scheme model on triple gate basis with optional double gate matching circuit

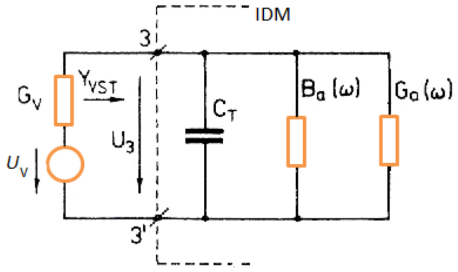


Fig.5. IDT equivalent scheme

The constant $A(j\omega)$ is the scheme coefficient and is dependant only on the IDT geometry. Circuit coefficients $C(j\omega)$ and $B(j\omega)$ are dependent on the input admittance $Y_{in}(j\omega)$ of the transducer, source impedance or load and on the features of matching circuit which are expressed by the elements of gradual cascade matrix A_{11}, A_{12}, A_{23}, A_{22}.

The scheme coefficient $A(j\omega)$ is in the case of cross-field model (as well as in the case of function δ model) a Fourier transformation of conveniently located SAW generating sources. Can be expressed by the nexus:

(25) $$A(j\omega) = \sqrt{\frac{2}{G_a(\omega_0)}} \sum_{k=1}^{n} e_k(j\omega) e^{-j2\pi f t_k},$$

where $e_k(j\omega)$ is called element coefficient of k-level electrode and exponential factor expresses the delay and space location of electrodes. Normalization coefficient $\sqrt{\frac{2}{G_a(\omega_0)}}$ is used for circuit models containing dimensional constants. As a consequence, $A(j\omega)$ equals one and insertion loss is initially given by the constants Q_υ and Q_r in the equation (23).

Element coefficients for two weak binding models and two different three-gate models are included in the Table n.1. Model of cross field and model of functions δ have similar element coefficients because the geometrical shape of exciter functions is almost identical. At the same time the element coefficient of circuit model generalisation resembles the element coefficient of weak binding model that stems from the solution of electric field because all the stated models use real IDT electric field as a exciter function.

Table 1. Element coefficients of some IDT activity models

Model	Circuit coefficients $e_k(j\omega)$
Of functions δ	$2l_k \cos\left(\pi f l_k / \upsilon\right)$, l_k is the distance between functions δ
Harmonic (direct solution of electric field)	F,T,(dE$_i$/dx), E_i is a part of intensity vector
Of cross field	$jK\sqrt{2 f_k C_k w_k / w_0}\,\sin\left(\pi f / 2 f_k\right)$ w_0 is the aperture of there apodized transducer
Generalised	$\frac{1}{2} j\chi F_k(\chi,\eta_k)$, $\chi = 2\pi f/\upsilon$ is wave number, η_k metallization coefficient

Table 2. Circuit coefficients for simple matching circuits

Matching circuit	Gradual cascade matrix	Circuit coefficients $C(j\omega)$, $B(j\omega)$
Without matching the circuit	$\begin{vmatrix}1,0\\0,1\end{vmatrix}$	$C(j\omega) = B(j\omega) = 1 + R_\upsilon Y_{in}$
Parallel admittance Y_p, e.g. inductor, resistor	$\begin{vmatrix}1,0\\Y_p,1\end{vmatrix}$	$C(j\omega) = B(j\omega) = 1 + R_\upsilon (Y_{in} + Y_p$
Serial impedance Z_s, e.g. inductor, resistor	$\begin{vmatrix}1,Z_s\\0,1\end{vmatrix}$	$C(j\omega) = 1 + Y_{in}(R_\upsilon + Z_s)$ $B(j\omega) = R_\upsilon\left[Y_{in} + \left(1/(Z_s + R_\upsilon)\right)\right]$
Other than parallel element	$\begin{vmatrix}A_{11},A_{12}\\A_{21},A_{22}\end{vmatrix}$	$\lvert C(j\omega)\rvert$ *and* $\lvert B(j\omega)\rvert$ can have various shapes and multiple dependence from R_v

Frequency dependences caused by matching circuit enable to include two circuit coefficients $C(j\omega)$ and $B(j\omega)$ while the circuit coefficient $C(j\omega)$ for transfer function T_{13} and circuit coefficient $B(j\omega)$ for the reflection coefficient T_{11} are formulated by the formule:

(26) $$C(j\omega) = A_{11} + A_{12}Y_{in} + R_\upsilon(A_{21} + A_{22}Y_{in}),$$

(27) $$B(j\omega) = R_\upsilon\left[Y_{in} + \frac{A_{11} + A_{21}R_\upsilon}{A_{12} + A_{22}R_\upsilon}\right].$$

The elements of gradual cascade matrix for random matching circuit are defined by the formule:

(28) $$\begin{vmatrix}U_L\\I_L\end{vmatrix} = \begin{vmatrix}A_{11}, A_{22}\\A_{21}, A_{22}\end{vmatrix} . \begin{vmatrix}U_3\\I_3\end{vmatrix}.$$

In the table n.2 we mention the values of both circuit coefficients for some frequently used matching circuits. In case of that the circuit coefficients equal $A_{12} = 0$, $A_{22} = 1$ (then $B(j\omega)=C(j\omega)$) then the circuit coefficient for coefficient of reflection is given by the following formula:

(29) $$T_{11}(j\omega) = \frac{1}{2}[T_{13}(j\omega)]^2 C(j\omega).$$

Based on the stated nexus which is valid only for the matching circuits composed of the parallel elements (or doubled conductor) we can assess the rate of insertion loss and three times transferred signal for the parallel tuning circuit.

Conclusion

In the report we derived generalisation of transfer function of transducer and reflection acoustic coefficients based on the defined complex transfer function, diffusion coefficients and diffusion loss with the generalisation being achieved by the introduction of system coefficient and circuit coefficient. The significant advantage of installed functions lies at the fact that they allow the application of random model on the process of modelling the IDT activity and at the same time they involve the influence of matching circuits.

REFERENCES

[1] Neveselý, M.: Akustoelektronika, Bratislava, ALFA, 1986.

[2] Dvornikov, A., Ogurcov, V., Utkin, G.:Stabiľnyje generatory s fiľtrami na poverchnostnych akustičeskich volnach, Moskva, Radio a svjaz, 1983.

[3] Šimko, M., Chupáč,M.: The theoretical synthesis and design of symmetrical delay line with surface acoustic wave for oscillators with single-mode regime of oscillation, Przeglad Elektrotechniczny, 12,2012, ISSN 0033-2097.

[4] Chicone, C., Feng, Z., C.: Synchronization phenomena for coupled delay-line oscillators, Physica D., 198, 2004, pp. 212–230, ISSN 0167-2789.

[5] Chicone, C., Feng, Z., C.: Synchronization phenomena for coupled delay-line oscillators, Physica D., 198, 2004, pp. 212–230,ISSN 0167-2789.

[6] L. Reindl, C. Ruppel, A. Kirmayr, N. Stockhausen, M. Hilhorst.: Passive radio requestable SAW water content sensor, in *Proc. Ultrason. Symp.*, vol. 1, 1999, pp. 461–466.

[7] Filipiak, J.; Solarz, L.; Steczko, G.: Surface acoustic wave vibration sensors for linear electronic warning systems. *Acta Phys. Pol. A* 2009.

[8] Cullen, C.; Montress, T.: Progress in the development of SAW resonator pressure transducers. In *Proceedings of the Ultrasonics Symposium*, Boston, MA, USA, 1980.

[9] Gutten, M., Bartlomiejczyk, M., Šebök, M.: Mathematical analysis of transformer insulation state by means of composite indicator, In: Przeglad Elektrotechniczny, Vol. 89, 2013, No. 3a, p. 132-135, ISSN 0033-2097

[10] Šebök, M., Gutten, M., Kučera, M.: Diagnostics of electric equipments by means of thermovision, In: Przeglad Elektrotechniczny, Vol. 87, 2011, ISSN 0033-2097, No. 10, p.313-317

Authors: Assoc. Prof. Milan Šimko, PhD., Assoc. Prof. Milan Chupáč, PhD.: Faculty of Electrical Engineering of the University of Žilina, Department of Measurement and Applied Electrical Engineering, Univerzitná 1, 010 26 Žilina, Slovak Republic, e-mail:simko@fel.uniza.sk, chupac@fel.uniza.sk.