

# Stiffness analysis of electromechanical transducer for nozzle flapper piezoelectric servo valve

**Abstract.** An electromechanical transducer is designed to replace the torque motor in the traditional nozzle flapper servo valve. The electromechanical transducer is constructed by two identical piezoelectric bimorphs and a beryllium bronze board. The stiffness values are obtained respectively by theory, simulation and experiment. Comparison and analysis results of the stiffness values prove that the proposed electromechanical transducer can provide a bigger stiffness and meet the requirements of the nozzle flapper piezoelectric servo valve.

**Streszczenie.** Przedstawiono elektromechaniczny przetwornik zaprojektowany z myślą o zastąpieniu silnika w układach serwo mechanizmu. Przetwornik składa się z dwóch identycznych bimorfów piezoelektrycznych na podłożu z brązu berylowego. (Analiza sztywności przetwornika elektromechanicznego do piezoelektrycznych zaworów dyszowych)

**Keywords:** piezoelectric bimorphs, electromechanical transducer, stiffness, servo valve.

**Słowa kluczowe:** przetwornik piezoelektryczny, zawór

## Introduction

Curie brothers (Jacques and Pierre Curie) are the first who found the piezoelectric effect in  $\alpha$ -quartz in 1880 [1], piezoelectric effect means that some anisotropic crystals produce charge proportionally under mechanical stress and some anisotropic crystals applied to the external electric field will produce geometric deformation proportionally, this phenomenon is called the inverse piezoelectric effect.

Piezoelectric actuators are a kind of functional device having the ability of micro-displacement driving, which are designed by the inverse piezoelectric effect. As a kind of piezoelectric actuator, piezoelectric bimorph has many merits, such as short reaction time, no magnetic field, low cost and simple structure. It can be used in ultrasonic wave transducers [2-4], valves [5-8], pumps [9-11], micro-displacement stages [12, 13] and other fields. In this paper, the piezoelectric bender made of piezoelectric bimorph is used as electromechanical transducer instead of the torque motor in the traditional nozzle flapper servo valve.

When the electromechanical transducer is being designed, the flow force effect to the electromechanical transducer must be considered, that need the stiffness of electromechanical transducer must be stronger than the stiffness of the flow force. Therefore the stiffness of the electromechanical transducer is researched in this paper. First, the structure and working principle of the electromechanical transducer are introduced, and then the theoretical calculation equations of electromechanical transducer's stiffness are analyzed, finally, simulations and experiments of electromechanical transducer's stiffness are researched.

## Structure and working principle of the electromechanical transducer

In this paper, the mechanical mounting form of the electromechanical transducer is cantilever. Piezoelectric bimorph with high electromechanical coupling coefficient is chosen, because the structure of electromechanical transducer has symmetry, and the symmetry is also necessary when it is working; so it needs to choose two identical piezoelectric bimorphs. Two piezoelectric bimorphs have opposite polarization directions, the electrical connection form is parallel and the middle layer is metal layer, the three layers are bonded together with the special technique. The structure is shown in Fig. 1. In this paper, the type of piezoelectric bimorph chosen in the experiment is PZT-5H. Substrate (metal layer) in this paper is beryllium bronze material. Compared with other metallic materials, beryllium bronze has good elasticity, in the same conditions

it can produce a larger deformation, piezoelectric bimorph and beryllium bronze board are bonded together; the upper surface and the lower surface of the electromechanical transducer are linked together with a thin wire, this is an electrode, another electrode is extracted from the substrate, and then connected to the control power.

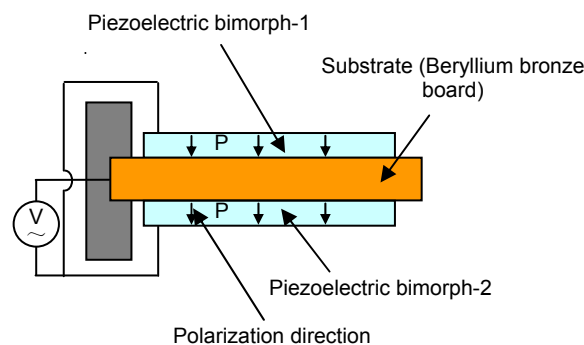


Fig. 1. Structure of electromechanical transducer

When voltage is applied to the piezoelectric bimorph, because the polarization directions of the two piezoelectric bimorphs are opposite, one piezoelectric bimorph has a shrinkage deformation in the length direction (see in Fig. 2 a)), and another piezoelectric bimorph has an elongation deformation in the length direction (see in Fig. 2 b)), shrinkage deformation and elongation deformation of the two piezoelectric bimorphs can produce a big bending tension which can overcome the tension of the substrate, and cause deformation displacement  $x$  on the free end of the electromechanical transducer (see in Fig. 2 c)).

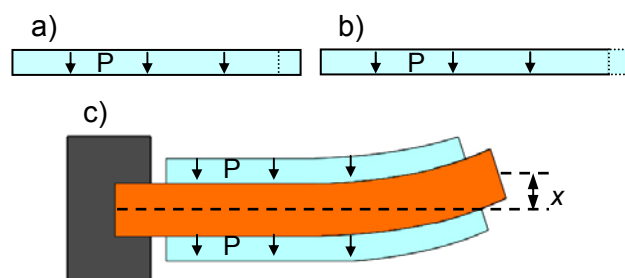


Fig. 2. a) Shrinkage deformation of piezoelectric bimorph in the length direction, b) Elongation deformation of piezoelectric bimorph in the length direction, c) Working principle of electromechanical transducer

The change of the displacement can change the pressure in the nozzle oil chambers at both sides of the electromechanical transducer, so there will be a pressure difference between the two working oil ports, and this pressure difference can drive the load.

### Analysis of flow force stiffness

When designing the electromechanical transducer, the acting force of the flow force must be considered. This increases the requirements of the electromechanical transducer's performance. Ignoring the frictional force of the electromechanical transducer supporting part and inertia force of the electromechanical transducer, the main resistance of the electromechanical transducer is flow force. When the electromechanical transducer is located in the middle of two nozzles, the forces acting on the electromechanical transducer are balance each other. When the electromechanical transducer moves off the middle position, the force acting on the electromechanical transducer is the difference value between the two nozzle's flow forces. Fig. 3 is the schematic diagram of the nozzle flapper piezoelectric servo valve.

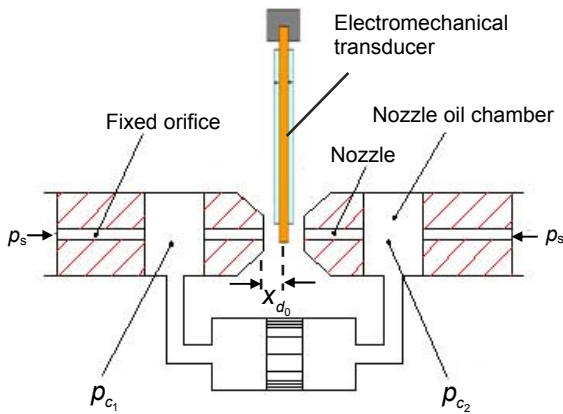


Fig.3. Schematic diagram of the electromechanical transducer

The flow force acting on electromechanical transducer is approximate [14]:

$$(1) \quad R = R_1 - R_2 = \frac{\pi d_p^2}{4} p_p - 8\pi C_p^2 x_{d0} x_d p_s$$

where,  $R_1$  and  $R_2$  are respective flow force of two nozzles,  $d_p$  is diameter of nozzle orifice,  $p_p = p_{c1} - p_{c2}$  is pressure difference of two nozzle oil chambers,  $C_p$  is discharge coefficient,  $x_{d0}$  is zero clearance,  $p_s$  is supply pressure.

According to Eq. (1), the stiffness of flow force can be obtained:

$$(2) \quad K_y = dR / dx_d = -8\pi C_p^2 x_{d0} p_s$$

where,  $C_p = 0.6$ ,  $p_s = 10 \text{ MPa}$ ,  $d_p = 0.5 \times 10^{-3} \text{ m}$ , and

$$(3) \quad x_{d0} = d_p / 16 = 3.125 \times 10^{-5} \text{ (m)}$$

The stiffness of flow force is:

$$(4) \quad K_y = 2827.4 \text{ (N / m)}$$

To ensure the stability of the electro-hydraulic servo valve, it is necessary to ensure the stiffness of the electromechanical transducer is bigger than the stiffness  $K_y$  of the flow force.

### Theoretical analysis of stiffness of the electromechanical transducer

The electromechanical transducer must have enough stiffness. Moreover, in order to ensure its dynamic characteristics, it should also meet certain stiffness requirements. Fig. 4 is the simplified model of the electromechanical transducer (in Fig. 4,  $L$  is length of electromechanical transducer,  $x$  is deflection on the free end of electromechanical transducer).

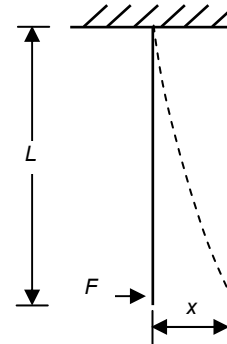


Fig.4. Simplified model of the electromechanical transducer

The deflection on the free end is:

$$(5) \quad w = \frac{FL^3}{3EI}$$

Make:

$$(6) \quad K_w = \frac{3EI}{L^3}$$

Deformation of (5) is:

$$(7) \quad F = K_w w$$

where,  $K_w$  is mechanical elastic stiffness.

Fig. 5 is the cross-sectional graphic of the electromechanical transducer (in Fig. 5,  $b$  is width of single piezoelectric bimorph,  $t_p$  is the thickness of single piezoelectric bimorph,  $t_m$  is the thickness of the substrate). Ignore the effect of the adhesive layer, namely that the piezoelectric bimorphs and the substrate are bonded together under ideal conditions. Using the parameters value of electromechanical transducer in Table 1, equivalent bending stiffness  $EI$  of the electromechanical transducer can be deduced.

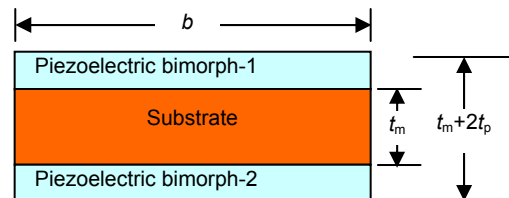


Fig.5. Cross-sectional graphic of the electromechanical transducer

Table 1. The parameters of electromechanical transducer

Symbol	Quantity	Value
$L$	Length of electromechanical transducer	$30 \times 10^{-3} \text{ m}$
$b$	Width of electromechanical transducer	$10 \times 10^{-3} \text{ m}$

$t_p$	Thickness of piezoelectric bimorph	$0.25 \times 10^{-3}$ m
$E_p$	Young's modulus of piezoelectric bimorph	110 GPa
$E_{max}$	Maximal electric field strength of piezoelectric bimorph	$1.5 \times 10^6$ V/m
$t_m$	Thickness of substrate	$0.5 \times 10^{-3}$ m
$E_m$	Young's modulus of substrate	125 GPa

\*Note: In order to simplifying the theoretical analysis and simulation, the length of piezoelectric bimorph and substrate are considered as the same value.

Derived process of the equivalent bending stiffness EI can be written as [15]:

$$(8) \quad I_{eff} = 2I_p + I_m$$

According to the parallel moving equation:

$$(9) \quad I_p = \frac{1}{12}bt_p^3 + bt_p \left( \frac{t_p + t_m}{2} \right)^2 = \frac{bt_p^3}{3} + \frac{bt_p^2 t_m}{2} + \frac{bt_p t_m^2}{4}$$

$$(10) \quad I_m = \frac{1}{12} \frac{E_m}{E_p} bt_m^3$$

Therefore, the equivalent bending stiffness is:

$$(11) \quad EI = E_p I_{eff} = E_p (2I_p + I_m)$$

Assume that the stiffness of the free end in the vertical direction in small deformation range is a constant value, namely, its displacement under the effect of external force is linear. Because the structure form of the electromechanical transducer is cantilever beam, according to deflection equation of the cantilever beam structure, and substitute Eq. (11), (12), (13) into Eq. (6), the stiffness on the free end of the electromechanical transducer in the vertical direction can be deduced as follow:

$$(12) \quad K_w = \frac{3b}{L^3} \left( \frac{2}{3} E_p t_p^3 + E_p t_p^2 t_m + E_p \frac{1}{2} t_p t_m^2 + \frac{1}{12} E_m t_m^3 \right) = E_p \left( \frac{2bt_p^3}{3} + bt_p^2 t_m + \frac{1}{2} bt_p t_m^2 + \frac{1}{12} \frac{E_m}{E_p} bt_m^3 \right) = \frac{2}{3} E_p bt_p^3 + E_p bt_p^2 t_m + \frac{1}{2} E_p bt_p t_m^2 + \frac{1}{12} E_m bt_m^3$$

Based on the parameter values in Table 1 and Eq. (12), the theoretic stiffness value of the electromechanical transducer can be calculated:

$$(13) \quad K_w = 4370(N/m)$$

### Simulation research

Finite element simulation analysis is researched for the electromechanical transducer in this paper; the stiffness of the free end in the vertical direction can be calculated. The specific process is: first, the geometric model of the electromechanical transducer is established; coupled with constraints, and then a vertical force is applied on the free end, as shown in Fig. 6.

Assume that the deformation on the free end is  $x$ , the stiffness can be obtained by the Eq. (7). The vertical force  $F$  is 5 N. The deformation result under the load force  $F$  is shown in Fig. 7.

According to the finite element simulation analysis, when the applied force  $F$  is 5 N, the vertical deformation on the free end is  $x=1.128e-3$  m, by use of the Eq. (7), the mechanical elastic stiffness can be obtained

$$(14) \quad K_w = F/x = 5/(1.128e-3) = 4433(N/m)$$

Compared with Eq. (13) and (14), it can be known that the simulation result and the analysis result of the stiffness are very similar.

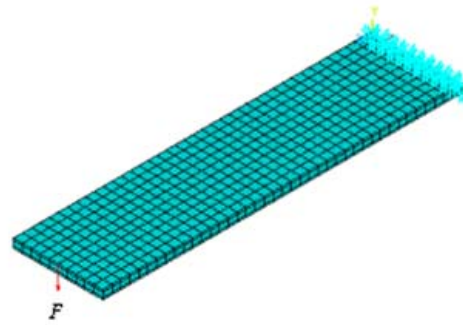


Fig.6. The stiffness analysis model

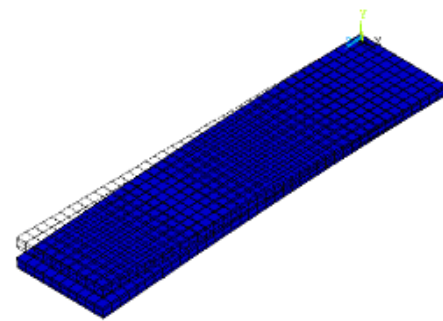


Fig.7. The deformation result

### Experimental research

In the stiffness experiment, the electromechanical transducer is fixed on the test bench, put a 100 g weight on the free end of electromechanical transducer, the displacement under the effect of the weight is  $2.09e-4$  m, then the mechanical elastic stiffness can be calculated by Eq. (15)

$$(15) \quad K_w = F/x = (0.1 \times 9.8)/(2.09e-4) = 4689(N/m)$$

According to Eq. (13), (14) and (15), the theoretical stiffness, simulation stiffness, and experimental stiffness can be obtained (see Table. 2), and the relationship between displacement and force is shown in Fig. 8.

Synthesize the theoretical, simulation and experimental results, when the load force is small, the theoretical, simulation and experimental results are almost identical.

By comparing Eq. (5) and Eq. (15), it can be known that the mechanical elastic stiffness  $K_w$  is much larger than the flow force stiffness  $K_y$ , which ensures the stability of the nozzle flapper piezoelectric servo valve.

Table 2. Stiffness comparison

Theoretical stiffness	Simulation stiffness	Experimental stiffness
4370 N/m	4433 N/m	4689 N/m

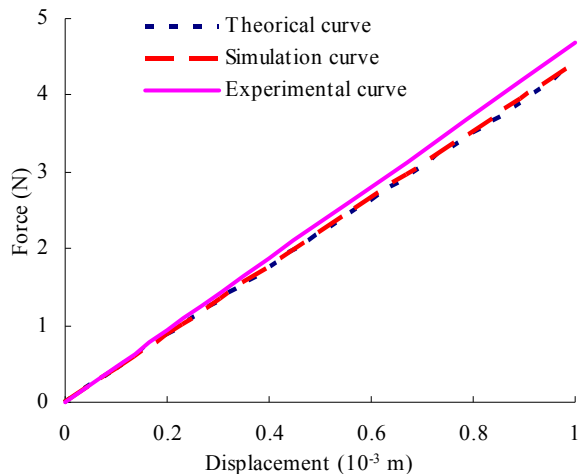


Fig.8. Comparison curves of stiffness

### Conclusions

In this paper, the electromechanical transducer of nozzle flapper piezoelectric servo valve is designed with the piezoelectric bimorph and beryllium bronze. Its structure and the working principle are introduced in this paper. The stiffness values are obtained respectively by theoretical calculation, simulation analysis and experiment research. The research results show that the three results are almost consistent. Finally it has been proved that the mechanical elastic stiffness is much larger than the flow force stiffness, this ensures the stability of the nozzle flapper piezoelectric servo valve from the stiffness.

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**Authors:** assoc. prof. Miaolei Zhou, Jilin University, 130022 Changchun, China, E-mail: [zml@jlu.edu.cn](mailto:zml@jlu.edu.cn); assoc. prof. Wei Gao, Jilin University, 130022 Changchun, China, E-mail: [gaow@jlu.edu.cn](mailto:gaow@jlu.edu.cn); prof. Zhigang Yang, Jilin University, 130022 Changchun, China, E-mail: [yzq@jlu.edu.cn](mailto:yzq@jlu.edu.cn)

The correspondence address is:  
e-mail: [zml@jlu.edu.cn](mailto:zml@jlu.edu.cn)