

# Rotating Speed Identification and Simulation Research of Synchronous Motor based on MRAS

**Abstract.** Rotational speed difference is an important factor that effects synchronism and stability of vibratory machines. This paper establishes the motor dynamical differential equation based on MRAS (model reference adaptive system). Speed regulator is designed based on PID regulator and adaptive controller. Then using MRAS to control, identify and simulate the speed of two rotors. Simulation results show that adopting MRAS is better than common PID system.

**Streszczenie.** W artykule przedstawiono równania różniczkowe opisujące dynamikę silnika synchronicznego bazujące na MRAS (model reference adaptive system). Następnie opracowano system kontroli szybkości bazujący na sterowniku PID i sterowniku adaptacyjnym. (Identyfikacja i symulacja prędkości wirnika maszyny synchronicznej bazujące na MRAS)

**Keywords:** MRAS, Synchronous Motor, Speed identification, Simulation.

**Słowa kluczowe:** silnik synchroniczny, MRAS – model reference adaptive system

## Introduction

The closed-loop control of speed is essential in motor control system with high performance. Speed signal is acquired through velocity transducer which increases the complexity and costs, decreases the reliability of system, making system maintenance difficult[1-3]. With the development of the electronic technology, ways to identify motor speed are increasing [4-5], there are direct computing method based on motor model, model reference adaptive method, observer estimate method, rotor tooth harmonic method, etc. This paper adopts MRAS to double synchronous motor. This system has fast self-adaptability speed, and it finished the speed identification of synchronous motor preferably.

## Basic structures of MRAS

Model reference control is an explicit self-adaptive control technology, which has considerable attractiveness in early development of self-adaptive control.

State variables constitute the self-adaptive control rule of MRAS. Assume the state equation of controlled object is

$$(1) \quad \dot{x}_p = A_p(t)x_p + B_p(t)u$$

where:  $x$  –n-dimensional state vector,  $u$  –m-dimensional control vector,  $A_p(t)$ ,  $B_p(t)$  –data matrices.

The state equation of reference model is

$$(2) \quad \dot{x}_m = A_m x_m + B_m r$$

where:  $X_m$  –n-dimensional state vector,  $r$  –m-dimensional reference input vector,  $A_m$ ,  $B_m$  –stable matrices.

Adopting feedforward and feedback self-adaptive control system, we get,

$$(3) \quad u = K(t)r + F(t)x_p$$

Put Eq. 3 into Eq. 2,

$$(4) \quad \dot{x}_p = [A_p(t) + B_p(t)F(t)]x_p + B_p(t)K(t)r$$

Assume that,

$$(5) \quad A_p(t) + B_p(t)F(t) = A_0(t)$$

$$B_p(t)K(t) = B_0(t)$$

Because  $K$  and  $F$  are the functions of  $\varepsilon$ ,  $A_0(t)$  and  $B_0(t)$  can be expressed as,

$$(6) \quad A_0(t) = A_0(\varepsilon, t) \quad B_0(t) = B_0(\varepsilon, t)$$

So Eq. (4) changes into

$$(7) \quad \dot{x}_k = A_0(\varepsilon, t)x_k + B_0(\varepsilon, t)r$$

$$\dot{x}_k = A_0(\varepsilon, t)x_k + B_0(\varepsilon, t)r$$

where:  $x_k$  –state variable of adjustable system.

Then  $A_0(t)$  and  $B_0(t)$  can accord the self-adaptive rule.

This paper adopts self-adaptive control algorithm of model reference to control the speed.

## Self-adaptive speed controller consisted of system state variable

### The basic control principle

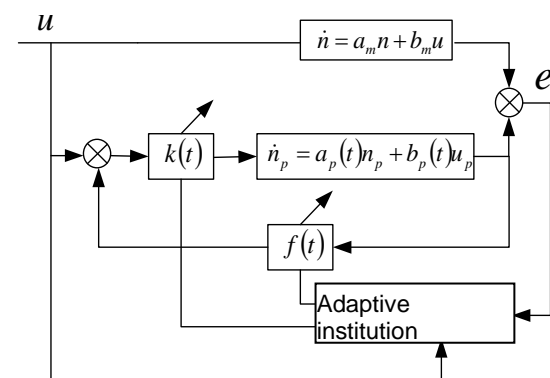


Fig. 1 .Two electromotors adaptive speed synchronically controller

Adaptive speed controller based on model reference shows in Fig.1. Control principle as follows: ideal motor in rated load is as reference model. Motor operating is as variable model. Variable error of system state is as the input of adaptive control. Adjust the feed forward gain  $k$  and the feedback gain coefficient  $f$  of variable model based on adaptive algorithm and make sure that generalized error vector converge to zero and the convergence of variable model to reference model. Then guarantee the consistency of variable model response characteristics [6-8].

### The Institution of System Mathematical Model

The object is two excitation motor of linear vibration screen dual exciters motor. According to the ideal induction motor model introduced in literature [9-10], the motion equations of asynchronous motor rotor can expressed as follows:

$$(8) \quad J\dot{\omega} = H(\omega_0 - \omega) - M_1$$

$$H = (3/2)P^2 L_m \tau_0 |\bar{i}_m|^2$$

where:  $L_m = a_m L_{12}$ ,  $\tau_0 = L_m / r_2$ ,  $J$  –rotational inertia of the rotor,  $\omega, \omega_0$  –rotor angular speed and synchronous mechanical angular speed,  $M_1$  –rotor resistance torque,  $P$  –magnetic poles number of the rotor,  $a_m$  –arbitrary constant,  $L_{12}$  –mutual inductance between stator and rotor,  $r_2$  –rotor winding resistance,  $\bar{i}_m$  –equivalent field current.

The mathematical model of frequency converters - asynchronous motor is gained after treatment as follow:

$$(9) \quad J_G \dot{n} + K_m n = Ku - M_1$$

where:  $n$  –rotor speed,  $f$  –applied voltage frequency,  $J_G$  –rotor rotational inertia,  $J_G = \pi J / 30 = GD^2 / 375$ ,  $K_m$  –stiffness coefficient,  $K_m = \pi H / 30$ ,  $K_f$  –the constant of frequency converters,  $K_f = 2\pi H / p$ .

Transform Eq. 9 to

$$(10) \quad \dot{n} = -\frac{K_m}{J_G} n + \frac{K}{J_G} u - \frac{M_1}{J_G}$$

For the actual controlled motor, the change of load will effect the rotary inertia and resistance torque. So the motor mathematical model in actual operation can express as,

$$(11) \quad \dot{n}_p = \frac{K_m}{J_{Gp}(t)} n_p + \frac{K}{J_{Gp}(t)} u_p - \frac{M_{p1}(t)}{J_{Gp}(t)}$$

### Self-adaptive algorithms based on reference model

As shown in Fig. 1, introduce feed forward gain  $k(t)$  and feedback gain  $f(t)$  in controlled system. They constitute adjustable model with controlled object. The system model of AC servo systems in actual can be expressed as,

$$(12) \quad \dot{n}_p = \left[ \frac{K_m}{J_{Gp}(t)} + f(t) \frac{K}{J_{Gp}(t)} \right] n_p + \frac{K}{J_{Gp}(t)} k(t)r - \frac{M_{p1}(t)}{J_{Gp}(t)}$$

Assume the state error is

$$(13) \quad e(t) = n_m(t) - n_p(t)$$

Assume that the object parameters change is more slow than adaptive adjustment process, so we can regard.

$\frac{K_m}{J_{Gp}(t)}$ ,  $\frac{M_{1p}(t)}{J_{Gp}(t)}$ ,  $\frac{K}{J_{Gp}(t)}$  as constant. Get differential equation from Eq.10 and Eq.12

$$(14) \quad \dot{e} = -\frac{K_m}{J_G} e - \phi(t)n_p + \phi(t)r$$

where:  $\phi(t) = \frac{K_m}{J_G} - \frac{K_m}{J_{Gp}(t)} - \frac{K}{J_{Gp}(t)} f(t)$ ,  $\phi(t) = \frac{K}{J_G} u - \frac{K}{J_{Gp}(t)} k(t)$ .

Choose the quadratic function,

$$(15) \quad V(e, \phi, \phi) = \frac{1}{2} \left[ \frac{K}{J_{Gp}(t)} \operatorname{sgn} \left( \frac{K}{J_{Gp}(t)} \right) e^2 + \frac{1}{\lambda_1} \phi^2 + \phi^2 \frac{1}{\lambda_2^2} \right]$$

As possible liapunov function. Obviously, the function is positive definite in generalized error space. we can get by

differential coefficient of functions  $\dot{V}(e, \phi, \phi)$ , and Eq.14 as follow,

$$(16) \quad \dot{V}(e, \phi, \phi) = -\frac{K}{J_{Gp}(t)} \operatorname{sgn} \left( \frac{K}{J_{Gp}(t)} \right) \frac{K_m}{J_G} e^2$$

$$+ \phi \left[ \frac{1}{\lambda_1} \dot{\phi} - \frac{K}{J_{Gp}(t)} \operatorname{sgn} \left( \frac{K}{J_{Gp}(t)} \right) e n_p \right] + \phi \left[ \frac{1}{\lambda_2} \dot{\phi} + \frac{K}{J_{Gp}(t)} \operatorname{sgn} \left( \frac{K}{J_{Gp}(t)} \right) e r \right]$$

where:  $\operatorname{sgn} \left( \frac{K}{J_{Gp}(t)} \right) = 1$

To make  $\dot{V}(e, \phi, \phi)$  negative definite, choose  $\dot{\phi}(t)$ ,  $\dot{\phi}(t)$  as follows to make the second and the third parameters of Eq.16 to zero, that is

$$(17) \quad \dot{\phi} = -\lambda_1 \frac{K}{J_{Gp}(t)} e n_p(t), \quad \dot{\phi} = \lambda_2 \frac{K}{J_{Gp}(t)} e r$$

We can get that the adaptive control law of the parameters is

$$(18) \quad \dot{f}(t) = -\lambda_1 e(t) n_p(t), \quad \dot{k}(t) = \lambda_2 e(t) r(t)$$

Discretize Eq.18 we can get,

$$(19) \quad f(n+1) = f(n) - \lambda_1 e(n) n_p(n) \tau$$

$$k(n+1) = k(n) + \lambda_2 e(n) r(n+1) \tau$$

where:  $\tau$  –sampling period,  $\lambda_1, \lambda_2$  –convergence step-size for  $f(t)$  and  $k(t)$ .

Increasing  $\lambda_1$  and  $\lambda_2$ , the convergences of  $f(t)$ ,  $k(t)$  accelerated, but the oscillation by converging curves intensified, which is unfavourable to the system stability.

We can get the input control law of controlled motor from Eq.13 and Eq.19

$$(20) \quad u(n+1) = k(n+1) [r(n) - f(n+1) n(n)_p]$$

### System simulation And Results Analysis

#### PID Adjustment simulation

Construct a PID simulation model in Simulink, as shown in Fig.2.

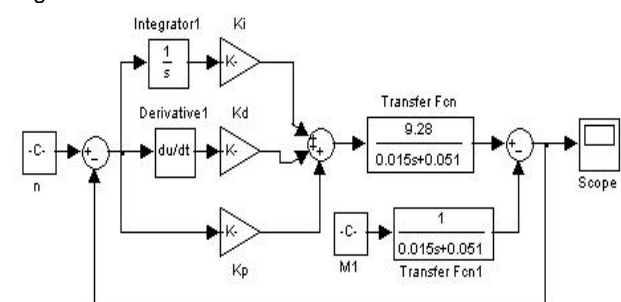


Fig.2.emulational model of PID

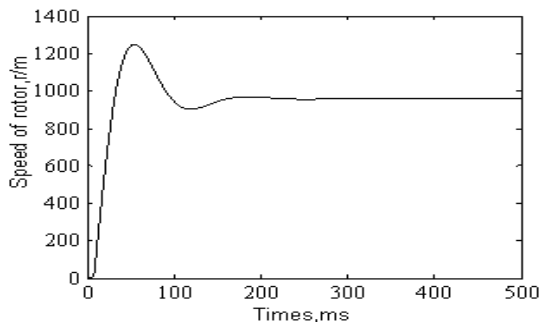


Fig.3. emulational curve of PID control

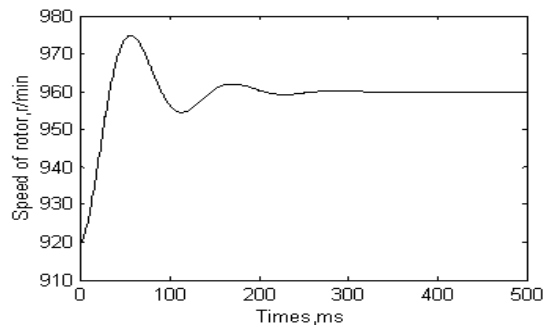


Fig.4. emulational model of MRAS controlled

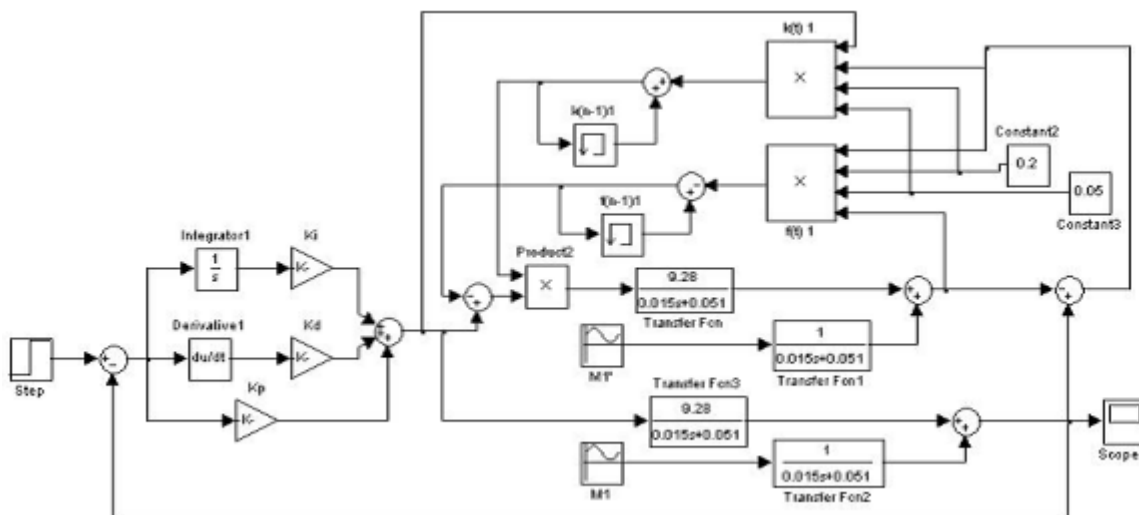


Fig. 5. MRAS's controlled emulational curve

The simulation curve after the model running is shown in Fig.3.

### simulation on the control section of Model referenced adaptive speed

- Build an simulation model as Fig. 4.
- The simulation curve after running the model aboved is shown as Fig.5.
- Comparing Fig.3 and Fig.5 we can see, adopting MRAS is better than common PID system and the system overshoot is smaller.

### Summary

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### Conclusion

Rotational speed difference is an important factor that effects synchronism and stability of vibratory machines. This paper establishes the motor dynamical differential equation. Then adopting MRAS to control two rotors. In this paper, speed adaptive control method of liapunov stability theory is adopted, combining PID control, to control the system. The two regulators based on MRAS and PID are partly simulated. From comparing the simulation curves of two controllers we can see, adopting the system based on MRAS can get better control result.

### REFERENCES

[1] Fig. 5. Model referenced adaptive speed's controlled emulational curve

[2] B.C. Wen, C.Y. Zhao, D.P. Su et al., Vibration Synchronization and Controlled Synchronization [M], Science Press, Beijing, 2003.

[3] B.C. Wen and F.Q. Liu, Theory of Vibrating Machines and Its Applications[M], Machine Press, Beijing, 1982.

[4] B.C. Wen, J. Fan, C.Y. Zhao et al., Vibratory Synchronization and Controlled Synchronization in Engineering[M], Science Press, Beijing, 2009.

[5] Meng G, Gasch R. Stability and stability degree of a cracked flexible rotor supported on journal bearing. ASME Journal of Vibration and Acoustics [J]. 2000, 122: 116-125

[6] Gasch R. A survey of the dynamic behavior of a simple rotating shaft with a transverse crack. Journal of sound and vibration [J]. 1993, 160(2): 313-332

[7] Yu Hongjie, Lu Hexiang, Qiu Chuanhang. Dynamic behavior analysis of elastic shaft rotor with a crack supported on the unsteady and non-linear oil film [J]. Acta Mechanica Sinica, 2002, 23(4): 439-445

[8] Li Zhenping, Luo Yuegang, Yao Hongliang, etc. Dynamics of rotor-bearing system with coupling faults of crack and rub-impact [J]. Chinese Journal of Applied Mechanics, 2003, 20(3):136-141

[9] Luo Yuegang, Wen Bangchun. Stability of the two-span rotor-bearing system periodic with coupling faults of crack and rub-impact [J]. Chinese Journal of Mechanical Engineering, 2008, 44(4):123-129

[10] Liu Changli, Zheng Jianrong, Zhou Wei, etc. On the bifurcation and stability of periodic motion of rotor-bearing systems with crack and pedestal looseness fault [J]. Journal of Vibration and Shock, 2007, 26(11):13-17

**Authors:** Zhaohui REN, Chaofeng Li and Chunyu Zhao are with College of mechanical Engineering and Automation ,Northeastern Universtiy, Shenyang, China. E-mail: [zhhren@mail.neu.edu.cn](mailto:zhhren@mail.neu.edu.cn); [chfli@mail.neu.edu.cn](mailto:chfli@mail.neu.edu.cn); [13889179917@126.com](mailto:13889179917@126.com)