An Application of Canonical Decomposition to TDOA Estimation for 3D Wireless Sensor Network Node Localization

Abstract. In this paper, a time difference of arrival (TDOA) estimation algorithm is proposed by extending canonical correlation decomposition (CCD) to measure multipath delays for determining node positions in Ultra-wideband wireless sensor networks (WSNs). Multilateral localization based on 3D Chan algorithm is employed to solve the non-linear problems for performance improvement. The method can perform well in the environment with unknown noise. The effectiveness is validated by simulations and the effects of noise and the number of anchor nodes are analyzed.

Streszczenie. Przedstawiono algorytm TDOA (time difference of arrival) do określania wielościeżkowych opóźnień w szerokopasmowej bezprzewodowej sieci czujników. Metoda jest przydatna w środowisku o nieznanych szumach. (**Zastosowanie kanonicznej dekompozycji do oszacowania parametru TDOA i lokalizacji węzłów w bezprzewodowej sieci czujników**)

Keywords: wireless sensor network, canonical correlation decomposition, MUSIC algorithm, time difference of arrival, ultra-wideband, node localization, 3D Chan algorithm.

Słowa kluczowe: bezprzewodowa sieć czujników, różnica czasu przybycia.

Introduction

Recently, ranging and localization are becoming key techniques in wireless sensor networks (WSN) [1-3]. There are several methods that can be used to determine the distances, such as time of arrival (TOA), time difference of arrival (TDOA), direction of arrival (DOA) and received signal strength (RSS). RSS methods need little cost, however, the ranging resolution is greatly influenced by the environment. DOA methods need multiple antennas, and is not preferred in low hardware cost and simple system. TOA and TDOA methods measure the distance between nodes using signal propagation time. TDOA methods is more preferred because they are impressively accurate under line-of-sight conditions without an accurate synchronization between the transmitter and receiver clocks. Some rangebased localization methods have been discussed. However, most algorithms are only applicable for two-dimensional (2D) localization, leading to the limitation in actual environment. Specifically, in the environment of multipath and unknown noise, these traditional localization algorithms suffer from severe performance degradation.

In this paper, we put forward a novel range-based node localization method for ultra wideband (UWB) based WSN. In the ranging for WSN localization, a multipath time delay estimation algorithm is developed based on the multiple signal classification (MUSIC) algorithm [4,5] employing canonical correlation decomposition (CCD) technology [6-8]. CCD is a high-resolution method originally used in the direction of arrival (DOA) estimation in unknown correlated noise. In the proposed paper, we investigate its application in TDOA measurement for 3D WSN node localization. CCD-based delay estimation algorithm is proposed to measure the TDOA of an unknown node by two anchor nodes. The proposed method can work well in a colored noise situations, and give a quantitatively controllable performance. Furthermore, three-dimensional (3D) Chan algorithm with multilateral localization instead of trilateral localization is employed to determine the physical coordinates of a group of sensor nodes. Combined with trilateral localization, it much enhances the accuracy. Using 3D Chan algorithm [9], the nonlinear equations can be accuately solved to obtain 3D positions of the nodes.

Received Signals from Two Anchors

The fine time-resolution of UWB signals provides potentially accurate ranging for WSN location. Assume that UWB signals are transmitted from an unknown node to two anchor nodes by L_1 and L_2 paths, respectively. The received signals in the *q*-th ($q=1,2,\cdots,Q$) snapshot can be expressed as

(1)
$$\begin{cases} y_1^{(q)}(t) = \sum_{l=1}^{L_1} \beta_{ll}^{(q)} s(t - \tau_{1l}) + v_1^{(q)}(t) \\ y_2^{(q)}(t) = \sum_{l=1}^{L_2} \beta_{2l}^{(q)} s(t - \tau_{2l}) + v_2^{(q)}(t) \end{cases}$$

where s(t) is a typical UWB signal, and here we use a basic Gaussian pulse waveform,

(2)
$$s(t) = A_p e^{-2\pi t^2/\tau_p^2}$$

where A_p is the amplitude of the UWB pulse, τ_p is a parameter to determine the UWB pulse width. β_{1l}, β_{2l} and τ_{1l}, τ_{2l} represent the complex fading amplitudes and multipath time delays of the *l*-th path arriving at the anchor node 1 and 2, respectively, where τ_{11}, τ_{21} denote the first path delays, and $|\tau_{21} - \tau_{11}|$ is the time difference of arrival.

We sample the received signals and assume the sampling number in the q-th snapshot to be N. The received data models of the two anchor nodes can be written as

(3)
$$\begin{cases} \mathbf{Y}_{1}^{(q)} = \mathbf{S}_{1}(\tau)\mathbf{B}_{1}^{(q)} + \mathbf{V}_{1}^{(q)} \\ \mathbf{Y}_{2}^{(q)} = \mathbf{S}_{2}(\tau)\mathbf{B}_{2}^{(q)} + \mathbf{V}_{2}^{(q)} \end{cases}$$

where.

$$\mathbf{S}_{1}(\tau) = \begin{bmatrix} s(T_{s} - \tau_{11}), s(T_{s} - \tau_{12}), \cdots, s(T_{s} - \tau_{1L_{1}}) \\ s(2T_{s} - \tau_{11}), s(2T_{s} - \tau_{12}), \cdots, s(2T_{s} - \tau_{1L_{1}}) \\ \cdots \\ s(NT_{s} - \tau_{11}), s(NT_{s} - \tau_{12}), \cdots, s(NT_{s} - \tau_{1L_{1}}) \end{bmatrix},$$

$$\mathbf{S}_{2}(\tau) = \begin{bmatrix} s(T_{s} - \tau_{21}), s(T_{s} - \tau_{22}), \cdots, s(T_{s} - \tau_{2L_{1}}) \\ s(2T_{s} - \tau_{21}), s(2T_{s} - \tau_{22}), \cdots, s(2T_{s} - \tau_{2L_{1}}) \\ \vdots \\ s(NT_{s} - \tau_{21}), s(NT_{s} - \tau_{22}), \cdots, s(NT_{s} - \tau_{2L_{1}}) \\ \vdots \\ s(NT_{s} - \tau_{21}), s(NT_{s} - \tau_{22}), \cdots, s(NT_{s} - \tau_{2L_{1}}) \end{bmatrix}$$

are two matrices of multipath delay signals.

$$\begin{split} \mathbf{B}_{1}^{(q)} = [\beta_{11}^{(q)}, \cdots, \beta_{lL_{1}}^{(q)}]^{T} \text{ and } \mathbf{B}_{2} = [\beta_{21}^{(q)}, \cdots, \beta_{2L_{2}}^{(q)}]^{T} \text{ are} \\ \text{two matrices of complex fading. } \mathbf{V}_{1}^{(q)} \text{ and } \mathbf{V}_{2}^{(q)} \text{ are } N \times 1 \\ \text{matrices of noise. } T_{s} \text{ is the sampling period.} \end{split}$$

CCD-based Multipath Delay Estimation Algorithm

In the section, MUSIC with CCD algorithm is employed for multipath delay estimation. The process is as follows: Firstly, a composite matrix $\bm{Y}^{(q)}$ is defined as

(4)
$$\mathbf{Y}^{(q)} = \begin{bmatrix} \mathbf{Y}_1^{(q)} \\ \mathbf{Y}_2^{(q)} \end{bmatrix}$$

Then its covariance matrix can be obtained as

(5)
$$\mathbf{R}_{Y} = E\left\{\mathbf{Y}^{(q)}\mathbf{Y}^{(q)H}\right\} = \begin{bmatrix} \mathbf{R}_{Y_{11}}\mathbf{R}_{Y_{12}} \\ \mathbf{R}_{Y_{21}}\mathbf{R}_{Y_{22}} \end{bmatrix}$$

where $\mathbf{R}_{Y_{11}}$, $\mathbf{R}_{Y_{12}}$, $\mathbf{R}_{Y_{21}}$, $\mathbf{R}_{Y_{22}}$ are four $N \times N$ submatrices of \mathbf{R}_{Y} . Since the two anchor nodes are separeted, the noise $\mathbf{V}_{1}^{(q)}$ and $\mathbf{V}_{2}^{(q)}$ are uncorrelated. Therefore, the noise items in $\mathbf{R}_{Y_{12}}$ and $\mathbf{R}_{Y_{21}}$ can be eliminated. Then, from the singular value decomposition (SVD) of the matrix $\mathbf{R}_{Y_{11}}^{-\frac{1}{2}} \mathbf{R}_{Y_{12}} \mathbf{R}_{Y_{22}}^{-\frac{1}{2}}$, we can obtain

(6)
$$\mathbf{R}_{Y_{11}}^{-\frac{1}{2}} \mathbf{R}_{Y_{12}} \mathbf{R}_{Y_{22}}^{-\frac{1}{2}} = \mathbf{U}_1 \Gamma \mathbf{U}_2$$

Define

(7)
$$\mathbf{L}_{1} = \mathbf{R}_{Y_{11}}^{-\frac{1}{2}} \mathbf{U}_{1}, \qquad \mathbf{L}_{2} = \mathbf{R}_{Y_{22}}^{-\frac{1}{2}} \mathbf{U}_{2},$$

 $\frac{1}{2}$

(8)
$$\mathbf{R}_1 = \mathbf{R}_{Y_{11}}^2 \mathbf{U}_1, \qquad \mathbf{R}_2 = \mathbf{R}_{Y_{22}}^2 \mathbf{U}_2$$

Here $\mathbf{L}_{1}^{H}\mathbf{R}_{1} = \mathbf{L}_{2}^{H}\mathbf{R}_{2} = \mathbf{I}$. Decompose $\mathbf{L}_{1} \triangleq [\mathbf{L}_{1s} | \mathbf{L}_{1v}]$, $\mathbf{L}_{2} \triangleq [\mathbf{L}_{2s} | \mathbf{L}_{2v}]$, $\mathbf{R}_{1} \triangleq [\mathbf{R}_{1s} | \mathbf{R}_{1v}]$, $\mathbf{R}_{2} \triangleq [\mathbf{R}_{2s} | \mathbf{R}_{2v}]$, and we define

(9) $\mathbf{P}_{1\nu} = \mathbf{L}_{1\nu} \mathbf{R}_{1\nu}^{H}, \qquad \mathbf{P}_{2\nu} = \mathbf{L}_{2\nu} \mathbf{R}_{2\nu}^{H}$

Refer to the derivation process in literature [8], we can obtain

(10)
$$\mathbf{P}_{1\nu}^{H}\mathbf{S}_{1}(\tau) = 0$$
, $\mathbf{P}_{2\nu}^{H}\mathbf{S}_{2}(\tau) = 0$

Therefore, when the MUSIC pseudo spectrum achieves peak values, the corresponding abscissas are the estimated multipath time delays. The curve function of the MUSIC pseudo spectrum is

(11)
$$Peak_1 = \frac{1}{\mathbf{s}(\tau)^H P_{1\nu} P_{1\nu}^H \mathbf{s}(\tau)}$$

(12)
$$Peak_2 = \frac{1}{\mathbf{s}(\tau)^H P_{2\nu} P_{2\nu}^H \mathbf{s}(\tau)}$$

where $\mathbf{s}(\tau) = \left[s(T_s - \tau), s(2T_s - \tau), \cdots, s(NT_s - \tau)\right]^T$.

Three-dimensional Chan Algorithm for Localization

We assume that the coordinate of anchor nodes is $(X_i, Y_i, Z_i)^T$, i = 0, 1, 2, 3, where i = 0 represents the main anchor node, and i = 1, 2, 3 represent the other three

anchor nodes. r_i denotes the distance between the unknown node and the *i*-th anchor node, and Δr_i denotes the difference of the distances from the unknown node to *i*-th anchor node and the main anchor node.

(13)
$$\begin{cases} r_0^2 = (x - X_0)^2 + (y - Y_0)^2 + (z - Z_0)^2 \\ r_i^2 = (x - X_i)^2 + (y - Y_i)^2 + (z - Z_i)^2 \quad i = 1, 2, 3 \\ \Delta r_i = r_i - r_0 = cd_i \end{cases}$$

where *c* is velocity of light, $d_i = \tau_i - \tau_0$ is the TDOA from the unknown node to the *i*-th anchor node and to the main anchor node.

From equation (13), we have

(14)
$$(X_0 - X_i)x + (Y_0 - Y_i)y + (Z_0 - Z_i) = k_i + r_0 \cdot \Delta r_i$$
where
$$k_i = \frac{1}{2} [\Delta r_i^2 + (X_0^2 + Y_0^2 + Z_0^2) - (X_i^2 + Y_i^2 + Z_i^2)], \quad i = 1, 2, 3$$

To solve the non-linear equations (14), the solutions r_0

are
$$r_0 = \frac{-b \pm \sqrt{b^2 - ac}}{a}$$
, where
(15)
$$\begin{cases} a = n_1^2 + n_2^2 + n_3^2 - 1 \\ b = (m_1 - x_0)n_1 + (m_2 - y_0)n_2 + (m_3 - z_0)n_3 \\ c = (m_1 - x_0)^2 + (m_2 - y_0)^2 + (m_3 - z_0)^2 \end{cases}$$

Multilateral 3D Localization Process

According to the estimation of multiple time delay for an node, the TDOA and distances can be determined by multiple anchor nodes. An example of the geometry relationship of the cross points of spheres using 4 anchor nodes is shown in fig.1. We use multilateral localization to calculate the 3D positions of an unknown node.



Fig.1. The geometric relationship in 3D localization

Let *t* be the time of emitting signals by the unknown node, and t_i be the time of receiving signals by the *i*-th anchor node. Then

(16)
$$\sqrt{(x-X_i)^2 + (y-Y_i)^2 + (z-Z_i)^2} = c(t_i-t) = c\tau_i$$
.

Here the number of anchor nodes is no less than 4 for multilateral localization. Fig. 2 shows the flow chart of localization process using 4 anchor nodes.



Fig.2. Localization Process

Simulations

CCD-based multipath delay estimation

Due to the multipath effect, we assume that the propagation multipath delays of the unknown node to two anchor nodes are [0.5, 0.6, 0.7] ns and [0.2 0.25] ns, respectively. The sampling period is T_s =0.01ns. The sampling number is *N*=100 in one snapshot and the number of snapshots is Q = 200 to ensure high-resolution. The background noises are colored noise of AR model. Fig. 3 (a) shows the waveform of transmitted signal and fig. 3 (b)(c) shows the estimated time delays arriving at two anchor nodes, SNR=10dB.

a) Transmitted UWB pulses.



b) Multipath delay estimation at anchor node 1.



c) Multipath delay estimation at anchor node 2.



Fig.3. Simulation results for CCD-based multipath delay estimation

The simulations confirm good resolution performance of the proposed algorithm in a multipath environment.

Position Computation using 3D Chan Algorithm

In fig. 4, the number of anchor nodes is 8, and R (the coverage radius of anchor nodes) is 10*m*. In a $40m \times 40m \times 40m$ space, we randomly generate 10 and 100 unknown nodes in fig. 4 (a) and (b), respectively. The nodes are located using 3D Chan algorithm by the TDOA estimation. Fig. 4 shows the results under colored Gaussian noise, SNR=10dB. It shows that the coordinates of unknown nodes can be estimated with high accuracy in complex noise environment even with low density of anchor nodes.

a) 10 nodes



b) 100 nodes



Fig.4. 3D Node Localization

Accuracy of the Proposed Algorithm

We define *Relative error* as follows,

(17) Relative error =
$$\frac{1}{R} \sqrt{\sum_{m=1}^{N_{mode}} [(x_m - \hat{x}_m)^2 + (y_m - \hat{y}_m)^2 + (z_m - \hat{z}_m)^2]}$$

where *Relativeerror* denotes the location relative error.

 (x_m, y_m, z_m) and $(\hat{x}_m, \hat{y}_m, \hat{z}_m)$ denotes the coordinates of the real and the estimated positions for the *m*-th unknown node, respectively. N_{node} denotes the number of unknown nodes. We generate 100 unknown nodes randomly in the environment added with colored noise of AR model, then localize them and calculate *Relativeerror*. In fig. 5 (a), SNR is 10 dB and the number of anchor nodes varies from 5 to 8. In fig. 5 (b), SNR ranges from -10dB to 15dB and the number of anchor nodes and localize them using traditional MUSIC-based estimation algorithm and the proposed algorithm in colored noise environment.

a) Effect of the number of anchor nodes



b) Effect of SNR



Fig.5. Effect of different factors on accuracy

The simulations show better performance of the proposed algorithm in colored noise, compared with the traditional MUSIC algorithm. In addition, the accuracy can be promoted by increasing the number of anchor nodes.

Summary

In this paper, we design a novel 3D localization scheme for UWB wireless sensor network. It mainly utilizes CCD-based TDOA estimation, multilateral localization and 3D Chan algorithms. As we known, accuate node localization in multipath and complex noise is an existing problem in most localization algorithms of WSN. Our algorithm has some key superiorities, such as high accuracy, robust inhibition on multipath effect and colored noise. The CCD-based TDOA measurement algorithm is effective in multipath and colored noise environment, and by the use of multilateral localization instead of trilateral localization, the accuracy of the proposed method is improved.

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Authors: Hong Jiang, Associate Professor, Nanhu Road 5372, College of Communication Engineering, Jilin University, Changchun, P. R. China, E-mail: <u>jiangh@jlu.edu.cn</u>; Chang Liu, graduate student, College of Communication Engineering, Jilin University, Changchun, P. R. China, E-mail: <u>lanniec@163.com</u>; Xin Sun, graduate student, College of Communication Engineering, Jilin University, Changchun, P. R. China, E-mail: <u>sunxin0529@163.com</u>.