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# Fiber Orientation and Rheology Behavior of the Laminar Suspension Flow through a 2-D Curved Expansion Duct

***Abstract**. By coupling the fiber extra stress into the laminar Newtonian flow equations, the fiber orientation and the fiber suspension rheology in a 2D curved expansion duct is numerically investigated. The orientation distribution probability function is numerically solved to obtain the bulk orientation distributions and the extra stress distributions. The results show that the overall distribution of the flow field parameters, such as velocity, pressure have not been significantly affected in the most regions of the duct. But for the shear stress and normal stress difference, the non-equilibrium is significant. In the inlet region, the fibers rotate frequently due to the nonuniform flow shear effects, the orientation becomes more random and are less aligned with the local streamlines. But in the downstream region, with flow shear becomes uniform, the non-equilibrium is less obvious. In addition, with the fiber extra viscosity increasing, the shear stress and normal stress difference increase correspondingly*

***Streszczenie.** Przeanalizowano numerycznie orientację włókna i naprężenia zawieszenia w zastosowaniach do reologii. Stwierdzono, że takie parametry jak ciśnienie i szybkość przepływu nie decydują o parametrach kanału. Natomiast istotny wpływ mają naprężenia. (**Właściwości reologiczne laminarnego przepływu pola przez kanał zawieszenia 2D**)*

**Keywords:** fiber suspension; orientation; extra stress; curved expansion duct.
**Słowa kluczowe:** zawieszenie włóknowe, reologia

## Introduction

The Fiber orientation and rheology behavior in the fiber suspensions have direct impact on the products of the chemical processing procedures, i.e., the pulp making and the composite moulding industry. And the fiber extra stress distributions greatly influence the fiber suspension flowing efficiency. Primarily, with the assumption that the presentation of the fiber will not alter the ambient flow structures, the equation (Jeffery, 1922) was used to obtain the trajectory and the orientation evolution. (Givler, 1983) numerically predicted the fiber orientation by the Jeffery equation. (Jackson *et al.*, 1984) numerically investigated the fiber orientation in molding parts, and the orientation distributions were in good agreement with the experimental results. (Folgar & Tucker, 1983) developed a mathematical model for fiber orientation prediction. (Jackson *et al.*, 1986) predicted the fiber orientation in compression moldings, and found in sheet molding the results were in good agreement with experiments.

(Advani *et al.*, 1987) introduced a set of orientation tensors to describe the probability distribution function of fiber orientation. (Lin *et al.*, 2010) numerically solved the orientation distribution function by FVM. (Kamal *et al.*, 1989) predicted the two-dimensional orientational behavior of short fibers in the simple flows. They numerically solved the Fokker-Planck equation and obtained the shear stress and the normal stress difference. (Chiba & Nakamura, 1998) numerically simulated the fiber suspension in a backward facing step channel by coupling flow field with the fiber orientation distribution obtained through integrating Jeffery equation along streamlines. (Lipscomb *et al.*, 1988) numerically and experimentally studied the flow through the axisymmetric contractions, they argued that the presentation of fibers changed a bit the flow structure. (Chiba *et al.*, 2001) studied the suspension flow in paralleled plate channel, they found near the inlet region, the fiber suspension flow changed significantly apart from the Newtonian situation. (Zhang *et al.*, 2009; Zhang & Lin, 2010) numerically investigated the rheological properties in fiber suspensions through curved expansion duct, however, the effects of the fibers on the bulk flow parameters, i.e., the velocity and the pressure had not been explored.

The laminar fiber suspension through a curved expansion duct is considered in the present work. The flow field coupled with fibers is solved by FVM. The probability function of fiber orientation(the Fokker-Planck equation) is solved to obtain the fiber orientation distribution. The shear and normal stress difference are calculated by the (Batchelor, 1970) model and compared with the Newtonian flows. The fiber extra viscosity which mainly dominates the extra force is varied in the calculations to taking account of the extra effects of the fibers.

## Numerical methods for laminar flows

The basic equations governing the steady laminar flow through a curved expansion duct are:

(1) $$\nabla \cdot \boldsymbol{u} = 0$$

(2) $$\nabla\left(\rho \boldsymbol{u}\boldsymbol{u}\right) = -\nabla p + \rho f + \mu \nabla^2 \boldsymbol{u}$$

Where, $u$ is the velocity vector, $p$ is pressure(Pa), $\rho$ is the density (kg m$^{-3}$), $\mu$ is the dynamic viscosity (Pa·s), f is the body force.

For an irregular geometry, the boundary-fitted non-orthogonal grids generated by the Laplace equation, which is written as follows:

(3) $$\begin{cases} \nabla^2 \xi = \xi_{xx} + \xi_{yy} = 0 \\ \nabla^2 \eta = \eta_{xx} + \eta_{yy} = 0 \end{cases}$$

The inverse function of Eq.(3) is :

(4) $$\begin{cases} \alpha \dfrac{\partial^2 x}{\partial \xi^2} - 2\beta \dfrac{\partial^2 x}{\partial \xi \partial \eta} + \gamma \dfrac{\partial^2 x}{\partial \eta^2} = 0 \\ \alpha \dfrac{\partial^2 y}{\partial \xi^2} - 2\beta \dfrac{\partial^2 y}{\partial \xi \partial \eta} + \gamma \dfrac{\partial^2 y}{\partial \eta^2} = 0 \end{cases}$$

where

$$\alpha = \left(\frac{\partial x}{\partial \eta}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2 , \qquad \beta = \frac{\partial x}{\partial \xi}\frac{\partial x}{\partial \eta} + \frac{\partial y}{\partial \xi}\frac{\partial y}{\partial \eta} ,$$

$$\gamma = \left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \xi}\right)^2$$

By solving the above equations, the boundary fitted grids are generated. The geometry and corresponding 16x40 cells are depicted in Fig.1.

The FVM(finite volume method) based on non-orthogonal grid and the collocated grid variables arrangement is employed. Taking the U-momentum equation to clarify the numerical procedure, which takes the following forms:

(5) $$a_p U_p = \sum_{nb} a_{nb} U_{nb} + (1-\alpha_u) a_p U_p^0 - \delta y_p (P_e - P_w)$$

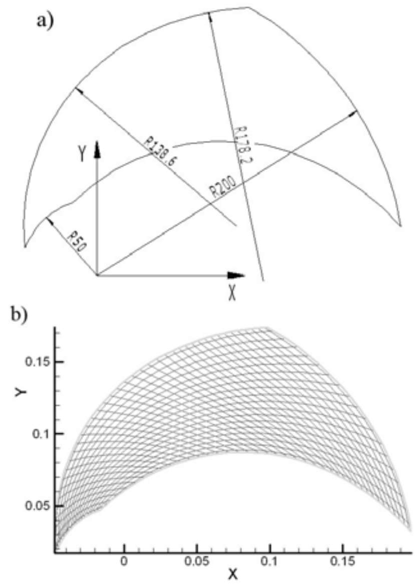



Fig.1. Geometry and grids (unit: mm)

To solve the pressure equation, the pressure-velocity coupling is implemented through the following strategy. Firstly, guess an initial field as(*U*,V**), then the incorrect field leads to a mass flux residual:

(6) $$S_m = \rho U_e^* \delta y - \rho U_w^* \delta y + \rho V_n^* \delta x - \rho V_s^* \delta x$$

Secondly, introduce the velocity and pressure correction by solving the following pressure correction equation:

(7) $$a_P P_P' = \sum_{nb} a_{nb} P_{nb}' - S_m$$

Then the velocity correction can be calculated:

(8) $$U_e' = -\overline{\left(\frac{1}{a_P}\right)_e} \delta y_e (P_E' - P_P')$$

With the above procedure, the velocity and pressure field can be obtained.

**Fiber suspension**

According to (Batchelor, 1970), the constitutive equation of a semi-dilute fiber suspension is:

(9) $$\sigma = 2\mu \mathrm{E} + \mu_p \left[ \langle \mathrm{pppp} \rangle - \frac{1}{3} \mathrm{I} \langle \mathrm{pp} \rangle \right] : \mathrm{E}$$

Where $\mathrm{E}=(\nabla u+\nabla u^{\mathrm{T}})/2$ is the strain rate, $\mu_{\mathrm{p}}=1/(6\pi nL^3\mu\varepsilon)$ is the fiber extra viscosity, p is the unit vector along the fiber central axis and is defined as follows:

(10) $$\mathrm{p} = (p_1, p_2, p_3) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

The vector p and its orientation are depicted in the Fig.2.

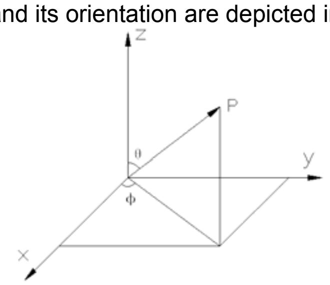



Fig.2. Definition of the unit vector p

Rewrite the above equation in Cartesian form:

(11) $$\sigma_{ij} = 2\mu e_{ij} + \mu_p \left[ \langle p_i p_j p_k p_l \rangle - \frac{1}{3} \delta_{ij} \langle p_k p_l \rangle \right] : e_{kl}$$

Define the extra stress as:

(12) $$\sigma_{ij,p} = \mu_p \left[ \langle p_i p_j p_k p_l \rangle - \frac{1}{3} \delta_{ij} \langle p_k p_l \rangle \right] : e_{kl}$$

Define 〈pppp〉 and 〈pp〉 as follows:

(13) $$a_{ijkl} = \langle \mathrm{pppp} \rangle = \langle p_i p_j p_k p_l \rangle = \oint p_i p_j p_k p_l \psi(\mathrm{p}) dp$$

(14) $$a_{ij} = \langle \mathrm{pp} \rangle = \langle p_i p_j \rangle = \oint p_i p_j \psi(\mathrm{p}) dp$$

Where $\psi$(p) is the orientation distribution function. In 2D case, $\psi(\mathrm{p})=\psi(\theta)$, within the range($-\pi/2, \pi/2$), $\psi$(p) satisfies:

(15) $$\int_{-\pi/2}^{\pi/2} \psi(\theta) \sin\theta d\theta = 1$$

Then the equation (2) becomes:

(16) $$\frac{\partial(\rho u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) + \frac{\partial(\sigma_{ij,p})}{\partial x_j}$$

The the extra stress is expressed as follows:

(17) $$\sigma_{ij,p} = \mu_p \left[ a_{ijkl} e_{kl} - \frac{1}{3} a_{kl} e_{kl} \delta_{ij} \right]$$

The fluid parameters are $\rho=1.0Kg/m^3$, $\mu=10^{-4}Pa\cdot s$. It is assumed that the fiber is a rod-like slender body with $r_p=50$, $\varepsilon=[\ln(2r_p)]^{-1}$. For semi-dilute fiber suspension, we define $nL^3=8.8$, thus the volume density $c=(\pi nd^2L)/4=[\pi nL^3(1/r_p)^2]/4=0.276\%$, then the extra viscosity $r_p=(6\pi nL^3\mu\varepsilon)/6\approx\mu$. The inlet velocity is set to be tangential to the wall and with uniform $u=0.05$(m/s). Thus the inlet $\mathrm{Re}=(\rho uR)/\mu=25$.

To account for fiber-fiber interaction, the orientation distribution function is numerically solved, and the overall fiber orientation distributions are obtained. A reduced differential equation of orientation angle $\phi$ has been given by (Givler *et al.*, 1987):

(18) $$\dot{\phi} = \varphi + B\left[ d_{xy}\cos 2\phi - \frac{1}{2}(d_{xx} - d_{yy})\sin 2\phi \right] = f(\phi_t)$$

where

$d_{xx} = \partial u/\partial x$ , $d_{yy} = \partial v/\partial y$ , $d_{xy} = (\partial v/\partial x + \partial u/\partial y)/2$ , $\varphi = (\partial v/\partial x - \partial u/\partial y)/2$ .

$B$ is the shape factor, $r_p=L/d$ is fiber aspect ratio.

According to (Folgar & Tucker, 1983), the orientation distribution function, or the Fokker-Planck equation is:

(19) $$\frac{D\psi}{Dt} = -\frac{\partial(\psi\dot{\phi})}{\partial\phi}$$

(20) $$\dot{\phi} = \varphi + d_{xy}\cos 2\phi - \frac{1}{2}(d_{xx} - d_{yy})\sin 2\phi - \frac{C_I\dot{\gamma}}{\psi}\frac{\partial\psi}{\partial\phi}$$

where, $\dot{\gamma}$ is fluid strain rate , $C_I$ is the Fokker-Planck constant.

It is assumed that the suspension is homogeneous, thus the above equation becomes:

(21) $$\frac{\partial\psi}{\partial t} = C_I\dot{\gamma}\frac{\partial^2\psi}{\partial\phi^2} - \frac{\partial}{\partial\phi}\left[\psi\left(\varphi + d_{xy}\cos 2\phi - \frac{1}{2}(d_{xx} - d_{yy})\sin 2\phi\right)\right]$$

The equation can be solved by time-marching method. With the periodic boundary condition $\psi(\phi)=\psi(\phi+\pi)=1/\pi$, the above equation is numerically solved.

## Numerical results and analysis

The velocity field is depicted in Fig.3a. And the Fig.3b is adapted form (Zhang *et al.*, 2009). In the most parts of the domain, the steady state orientation is nearly aligned with the flow direction, but in the region close to inlet and centerline of the duct, the fiber orientations become less aligned with the local streamlines. In the inlet region, from left to right wall, the fiber flips rapidly. This is mainly due to the the rapid change of the flow shear effect in this region.

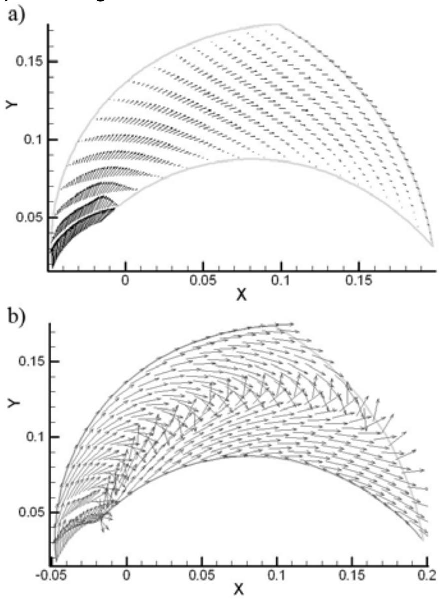
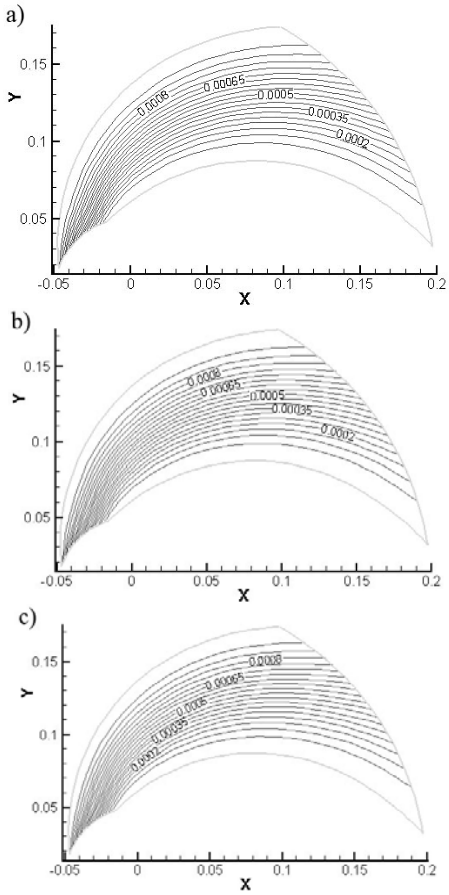



Fig.3. Steady velocity field and the fiber orientation distribution in a curved expansion duct

In the present work, the calculations are performed with different extra viscosities, such as 0, 1.E-4, 2.E-4 and 3.E-4. The effects of the extra viscosity on the flow field is shown in Fig.4.



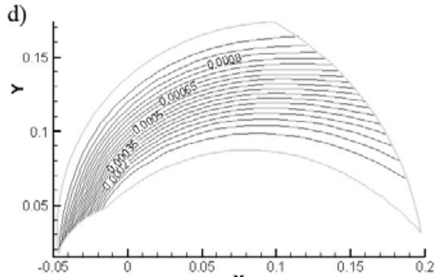




Fig.4. Streamlines at different extra viscosities (a) 0; (b) 1E-4; (c) 2E-4; (d) 3E-4

Although the difference among the streamlines is not obvious. It can be found that with the extra viscosity increasing, the streamlines becomes more closer to the centerline of the duct.

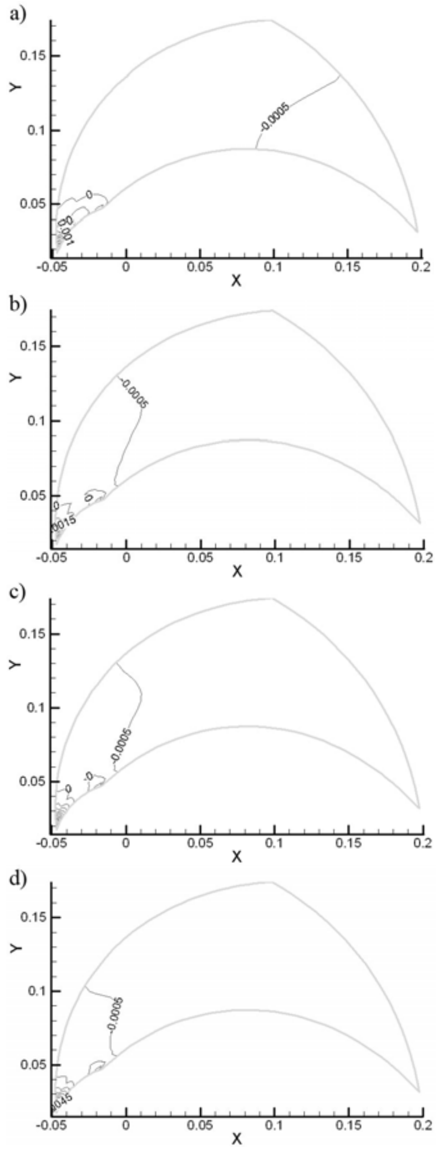



Fig.5. Pressure distribution at different extra viscosities (a) 0; (b) 1E-4; (c) 2E-4; (d) 3E-4

For the pressure distribution in Fig.5, small difference is shown at different extra viscosities. With extra viscosity increasing, the large pressure gradient region in the inlet does not change significantly, but in the small gradient region, with the extra viscosity increasing, the gradient becomes larger.

The shear stress and the normal stress difference equations are:

(22) $$\sigma_{12} = \mu\left(\partial v / \partial x + \partial u / \partial y\right) + \sigma_{12,p}$$

(23) $$\sigma_{11} - \sigma_{22} = \mu\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + \sigma_{11,p} - \sigma_{22,p}$$

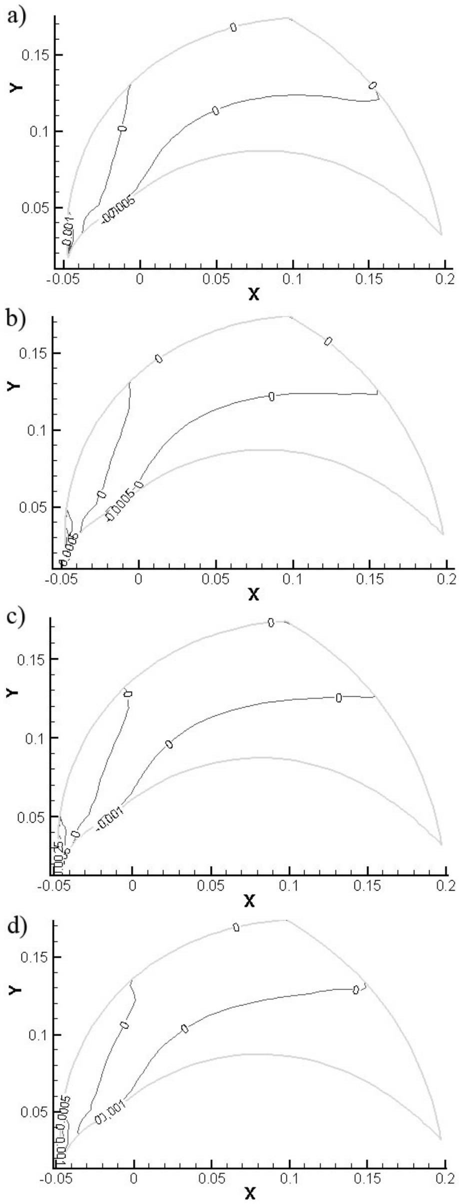
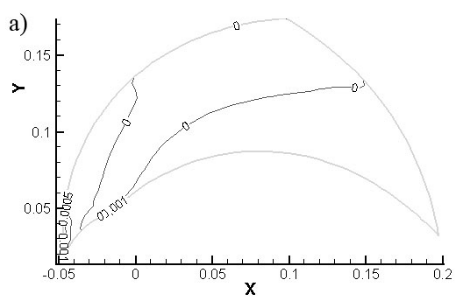



Fig.6. Shear stress distribution at different extra viscosities (a) 0; (b) 1E-4; (c) 2E-4; (d) 3E-4



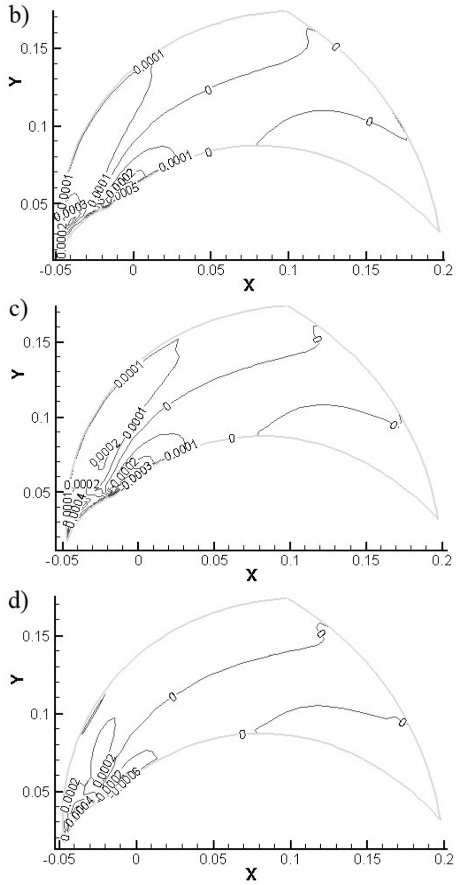




Fig.7. Normal stress difference distribution at different extra viscosities (a) 0; (b) 1E-4; (c) 2E-4; (d) 3E-4

As shown in Fig.6 and Fig.7, the extra viscosity doesn't influence the shear stress significantly, as well as the normal stress difference. It can be seen that the shear stress and the normal stress difference are slightly increasing with the extra viscosity. But their distributions have not changed significantly. It can be concluded that the presentation of the fibers only takes effects on a small parts of the flow fields, especially in the inlet region and the central regions.

**Conclusions**

For the suspension through a curved expansion duct, undergoing strong negative pressure gradients, the flow field is different from those shear and extensional flows. The results show that, in the inlet region, under the strong shear force enforced by the flow field, the fibers rotate quickly and become less aligned with the local streamlines. And the extra shear stress distribution is concentrated in the inlet region. While in the downstream region, with the velocity gradient becoming more uniform, the overall distribution of fiber orientation becomes more aligned with the local flow direction, and the rheology properties becomes less significant. Due to the presentation of the fibers, the streamlines become closer to the central regions than the Newtonian situations, and with viscosity increasing, the extra shear stress correspondingly increases, while the shear stress distribution remains unchanged. It can be seen that, the extra effects of the fibers are equivalent to increasing the effective viscosity of the fiber suspension. These results may be instructive to understand the flow characteristics within the curved expansion duct flows.

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